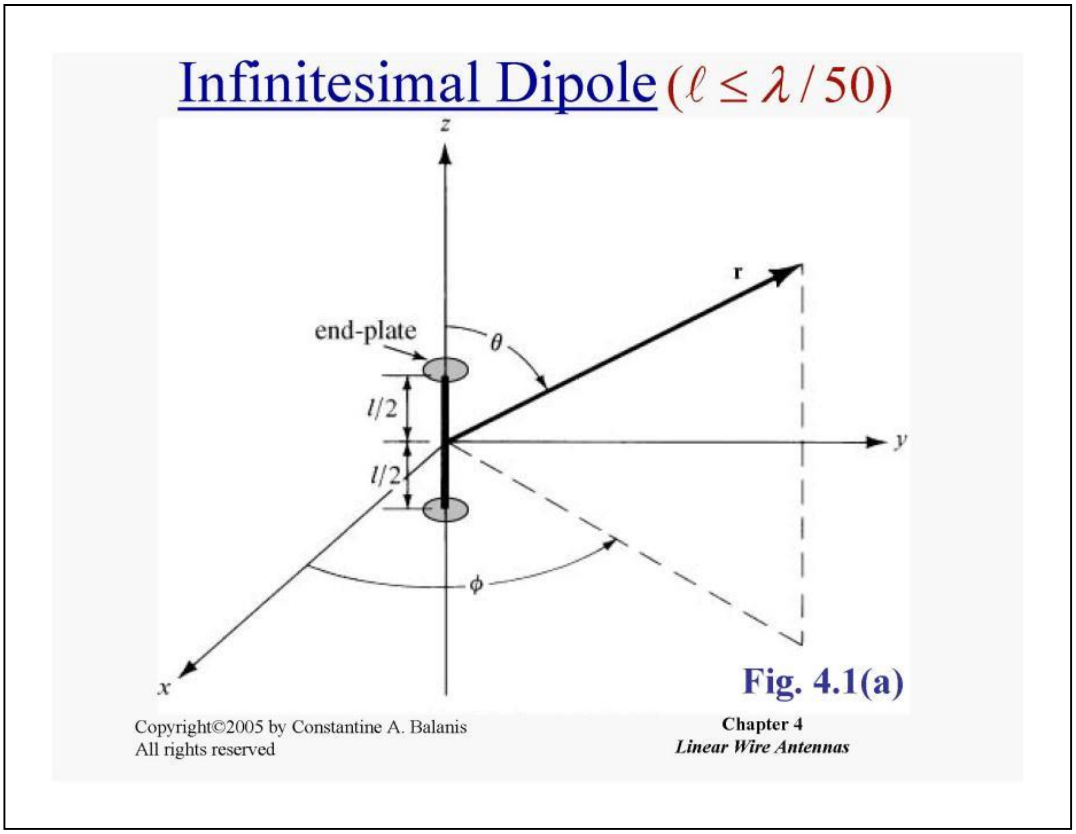
LINEAR WIRE ANTENNAS

Infinitesimal Dipole - Hertzian Dipole



A current element $I\ell$ refers to a filamentary current I flowing along an elemental length ℓ . This is approximated when a current I flows in a very short of thin wire, if the length ℓ is so short (??) that the current is essentially constant along the length.

Modeling:

Consider a short wire for which

$$\ell << \lambda$$

Plates at the ends of the dipole provide capacitive load.

Since $\ell << \lambda$, then $\dfrac{\ell}{\lambda} << 1$ or $\dfrac{\lambda}{\ell} >> 1$

Consider $\ell = 1m$ (physical length).

For $f_1 = 1MHz$, $\lambda_1 = \dfrac{c}{f_1} = \dfrac{3X10^8}{10^6} = 300m$

For $f_2 = 1GHz$, $\lambda_2 = \dfrac{c}{f_1} = \dfrac{3X10^8}{10^9} = 30cm$

$$k_1\ell = \frac{2\pi}{\lambda_1} = \left(\frac{1}{300}\right)2\pi = 0.02094$$

$$k_2\ell = \frac{2\pi}{\lambda_2} = \left(\frac{1}{0.3}\right)2\pi = 20.94$$

We see that:

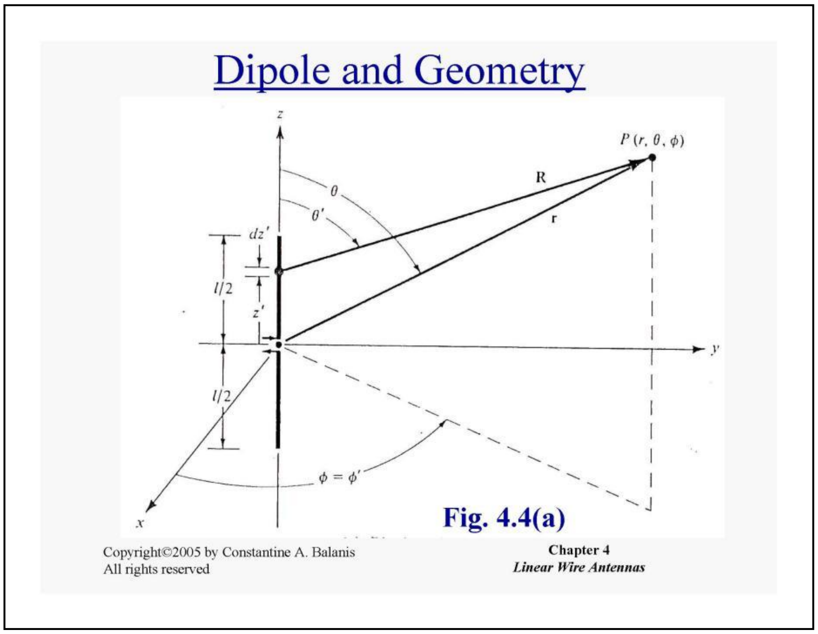
$$k_1\ell << 1$$

$$k_2\ell >> 1$$

The short length and the presence of the plates result in a uniform current I along the entire wire.

$\ell = 1m$ length is very short at f_1. **Current is uniform**. This length is not very short at f_2. **Current is not uniform**.

For the purpose of analysis we may consider that the short dipole appears as follows:



Define the current element: $I\,d\bar{\ell} = I\,d\ell\,\hat{a}_z$

It is uniform along $d\ell$.

It has sinusoidal time variation $\sin\omega t$ ($e^{j\omega t}$).

Electric and Magnetic Fields

We know that

$$\bar{B} = \nabla \times \bar{A}$$

$$\bar{H} = \frac{1}{\mu}\nabla \times \bar{A}$$

where $\bar{A}$ is the magnetic vector potential.

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi}\int_v \frac{e^{-jkR}}{R}\left(\bar{J}dv'\right)$$

$$\bar{J}dv' \rightarrow \bar{K}ds' \rightarrow Id\bar{\ell}'$$

So, for Hertzian Dipole:

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi}\int_C Id\bar{l}^{'}\,\frac{e^{-jkR}}{R}$$

Maxwell's Equations

$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t} \qquad \nabla\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}$$

$$\nabla.\bar{D} = \rho_v \qquad \nabla.\bar{B} = 0$$

So, if we know $Id\bar{\ell}$, we can obtain, $\bar{A}\rightarrow\bar{B}\rightarrow\bar{H}\rightarrow\bar{E}$

Calculate $\bar{A}$:

$$\bar{A}(x,y,z) = \frac{\mu}{4\pi}\int_C I\left(x^{'},y^{'},z^{'}\right)\frac{e^{-jkR}}{R}d\bar{l}^{'}$$

where

$\bar{A}(\bar{r}) = \bar{A}(x,y,z)$ is the magnetic vector potential at the point (x,y,z)

(x,y,z) coordinates are used for observation point,

$(x^{'},y^{'},z^{'})$ coordinates are used for the source point,

$\bar{r}$ is the vector from the origin to the observation point.

$\bar{r}^{'}$ is the vector from the origin to the source point.

$$R = \left|\bar{r}-\bar{r}^{1}\right| = \left[\left(x-x^{'}\right)^2+\left(y-y^{'}\right)^2+\left(z-z^{'}\right)^2\right]^{1/2}$$

$d\bar{l}^{'}$ is the infinitesimal length element along the source.

Since the source current is very small electrically, we may consider it to be just on the origin, O.

So, $I =$ constant w.r.t. (x', y', z'),

$R \approx r$.

$d\bar{l}' = \hat{a}_z dz'$ $\quad k = \omega\sqrt{\varepsilon\mu}$. Then,

$$\bar{A}(x, y, z) = \frac{\mu}{4\pi} I \frac{e^{-jkr}}{r} \int_{-\ell/2}^{\ell/2} dz' \hat{a}_z = \frac{\mu I \ell}{4\pi} \frac{e^{-jkr}}{r} \hat{a}_z$$

$$r = \left(x^2 + y^2 + z^2\right)^{1/2}$$

$$\bar{A} = \frac{\mu I \ell}{4\pi} \frac{e^{-jkr}}{r} \hat{a}_z$$

$\bar{A}$ is along the z-axis. It has only the z-component in the Cartesian coordinates.

In the spherical coordinates:

$$A_r = A_z \cos\theta$$

$$A_\theta = -A_z \sin\theta$$

$$A_\phi = 0$$

The magnetic flux density:

$$\bar{B} = \nabla X \bar{A}$$

$$\bar{H} = \frac{1}{\mu} \nabla X \bar{A} = \frac{\hat{a}_\phi}{r\mu} \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

Conducting the derivative operations:

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = j\frac{kI\ell}{4\pi r}\sin\theta\left(1+\frac{1}{jkr}\right)e^{-jkr}$$

Calculate the electric field intensity:

$$\nabla X\bar{H} = j\omega\varepsilon\bar{E}$$

$$\bar{E} = \frac{\nabla X\bar{H}}{j\omega\varepsilon}$$

Which results:

$$E_r = \eta\frac{I\ell}{2\pi r^2}\cos\theta\left(1+\frac{1}{jkr}\right)e^{-jkr}$$

$$E_\theta = j\eta\frac{kI\ell}{4\pi r}\sin\theta\left(1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right)e^{-jkr}$$

$$E_\phi = 0$$

Let $\frac{kI\ell}{4\pi} = C$

Then,

$$H_\phi = jC\sin\theta\left(\frac{1}{r}+\frac{1}{jkr^2}\right)e^{-jkr}$$

$$H_{\phi} = jkC\sin\theta\left(\frac{1}{kr} + \frac{1}{j(kr)^2}\right)e^{-jkr}$$

Let

$$jkC = D$$

$$H_{\phi} = \left(\frac{D\sin\theta}{kr} + \frac{D\sin\theta}{j(kr)^2}\right)e^{-jkr}$$

E_r has $\frac{1}{r^2}$ and $\frac{1}{r^3}$

E_{θ} has $\frac{1}{r}$, $\frac{1}{r^2}$ and $\frac{1}{r^3}$

A z-directed current element (infinitesimal) kept at the origin has only the H_{ϕ}, E_r and E_{θ} components and further the fields have components that decay as $\frac{1}{r}$, $\frac{1}{r^2}$ and $\frac{1}{r^3}$ away from the current element.

Thus, these expressions form a convenient basis for classifying the fields of any antenna depending on the nature of decay of the antenna.

$$E_r = \eta\frac{k^2 I\ell\cos\theta}{2\pi}e^{-jkr}\left[\frac{1}{(kr)^2} + \frac{1}{j(kr)^3}\right]$$

$$E_{\theta} = j\eta\frac{k^2 I\ell\sin\theta}{4\pi}e^{-jkr}\left[\frac{1}{(kr)} + \frac{1}{j(kr)^2} - \frac{1}{(kr)^3}\right]$$

For large values of kr, i.e., $kr >> 1$ (or $\frac{2\pi}{\lambda} r >> 1$ or $r >> \lambda$ the terms containing $\frac{1}{(kr)^2}$ and $\frac{1}{(kr)^3}$ decay much faster than $\frac{1}{kr}$. Therefore, at large distances from the Hertzian dipole, we have only:

$$E_\theta = j\eta \frac{kI\ell \sin\theta}{4\pi} \frac{e^{-jkr}}{r}$$

$$H_\phi = \frac{jkI\ell \sin\theta}{4\pi} \frac{e^{-jkr}}{r}$$

Fields are in time phase.

$$\frac{E_\theta}{H_\phi} = \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \approx 377\Omega$$ in air, can be considered as uniform plane wave.

POWER / RADIATION RASISTANCE

INFINITESIMAL DIPOLE

$$\bar{E} = E_r \hat{a}_r + E_\theta \hat{a}_\theta$$

$$\bar{H} = H_\phi \hat{a}_\phi$$

Complex Poynting Vector:

$$W = \frac{1}{2}\left(\bar{E} X \bar{H}^*\right)$$

$$\bar{E} X \bar{H}^* = \left(E_r \hat{a}_r + E_\theta \hat{a}_\theta\right) X \; H_\phi{}^* \hat{a}_\phi = -E_r H_\phi^* \hat{a}_\theta + E_\theta H_\phi^* \hat{a}_r$$

$$W = \frac{1}{2}\left(E_\theta H^*{}_\phi \hat{a}_r - E_r H^*{}_\phi \hat{a}_\theta\right)$$

Fields:

$$E_r = \eta \frac{I\ell}{2\pi r^2} \cos\theta \left(1 + \frac{1}{jkr}\right) e^{-jkr}$$

$$E_\theta = j\eta \frac{k^2 I \ell \sin\theta}{4\pi} \left[\frac{1}{(kr)} + \frac{1}{j(kr)^2} - \frac{1}{(kr)^3}\right] e^{-jkr}$$

$$H_\phi = j \frac{kI\ell}{4\pi r} \sin\theta \left(1 + \frac{1}{jkr}\right) e^{-jkr}$$

$$W_r = \frac{1}{2}\left(E_\theta H_\phi^*\right) = \frac{\eta}{8}\left|\frac{I\ell}{\lambda}\right|^2 \left(\frac{\sin^2\theta}{r^2}\right)\left[1 - j\frac{1}{(kr)^3}\right]$$

$$W_\theta = j\eta \frac{k|I\ell|^2 \cos\theta \sin\theta}{16\pi^2 r^3}\left[1 + \frac{1}{(kr)^2}\right]$$

Complex Power Moving in Radial Direction

$$P = \oiint_S \bar{W} . \hat{a}_n ds$$

$$P = \int_0^{2\pi}\int_0^{\pi} \left(\hat{a}_r W_r + \hat{a}_\theta W_\theta\right) . \hat{a}_r r^2 \sin\theta d\theta d\phi$$

$$P = \int_0^{2\pi}\int_0^{\pi} W_r r^2 \sin\theta d\theta d\phi = \eta \frac{\pi}{3}\left|\frac{I\ell}{\lambda}\right|^2\left[1 - j\frac{1}{(kr)^3}\right] \quad \text{Watt.}$$

W_θ is purely imaginary, so it will not contribute to any real power.

Consider

$$P = \frac{1}{2}\int\left(\bar{E}X\bar{H}^*\right).\hat{a}_n ds = \eta\left(\frac{\pi}{3}\right)\left|\frac{I\ell}{\lambda}\right|^2\left[1 - j\frac{1}{(kr)^3}\right]$$

$$P = P_{rad} + j2\omega\left(\tilde{w}_m - \tilde{w}_e\right)$$

P = Power (in radial direction)

P_{rad} = Time average radiated power (in radial direction)

$\tilde{w}_m$ =Time average magnetic energy density (in the radial direction)

$\tilde{w}_e$ =Time average electric energy density (in the radial direction)

$2\omega\left(\tilde{w}_m - \tilde{w}_e\right)$ = Time averaged imaginary (reactive) power (in the radial dir).

Comparison gives:

$$P_{rad} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I\ell}{\lambda}\right|^2$$

We associate a resistance to P_{rad} (radiation resistance) as:

$$P_{rad} = \frac{1}{2}|I|^2 R_r = \eta\left(\frac{\pi}{3}\right)\left|\frac{I\ell}{\lambda}\right|^2$$

$$R_r = \eta\left(\frac{2\pi}{3}\right)\left|\frac{\ell}{\lambda}\right|^2 \quad \Omega$$

$$R_r = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2$$

$$2\omega\left(\tilde{w}_m - \tilde{w}_e\right) = -\eta\left(\frac{\pi}{3}\right)\left|\frac{I\ell}{\lambda}\right|^2 \frac{1}{(kr)^3}$$

It is clear from the above equation that the radial electric energy must be larger than radial magnetic energy.

For (large values of kr) $\quad kr >> 1,\ 2\omega\left(\tilde{w}_m - \tilde{w}_e\right) \to 0$

And

$$2\omega\left(\tilde{w}_m - \tilde{w}_e\right) = 0 \quad , r = \infty$$

DIRECTIVE GAIN

$$D_g = \frac{4\pi U}{P_{rad}}$$

The procedure is:

$$\bar{E} \to W \to U \to D_g$$

$$W = \frac{1}{2}\frac{\left|E^2\right|}{\eta}$$

$$U = r^2 W$$

$$E_\theta = j\eta \frac{kI\ell \sin\theta}{4\pi} \frac{e^{-jkr}}{r}$$

$$U = \frac{\eta}{2} \frac{k^2 I\ell}{(4\pi)^2} \sin^2\theta$$

$$\mathrm{P}_{\mathrm{r}ad} = \eta \left(\frac{\pi}{3}\right)\left(\frac{I\ell}{\lambda}\right)^2$$

Directive gain:

$$D_g = 1.5 \sin^2\theta$$

Directivity= $D_{\max} = 1.5$

Radiation Pattern:

$$U = U_{\max} \sin^2\theta$$