LINEAR SMALL DIPOLE

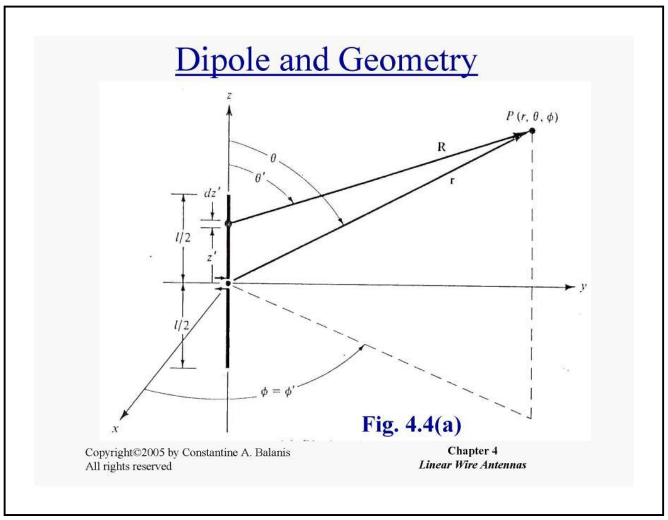
The length of infinitesimal dipole is usually taken as

$$\ell \le \frac{\lambda}{50}$$

The current distribution is uniform.

We call the wire antenna as small dipole if it's length is:

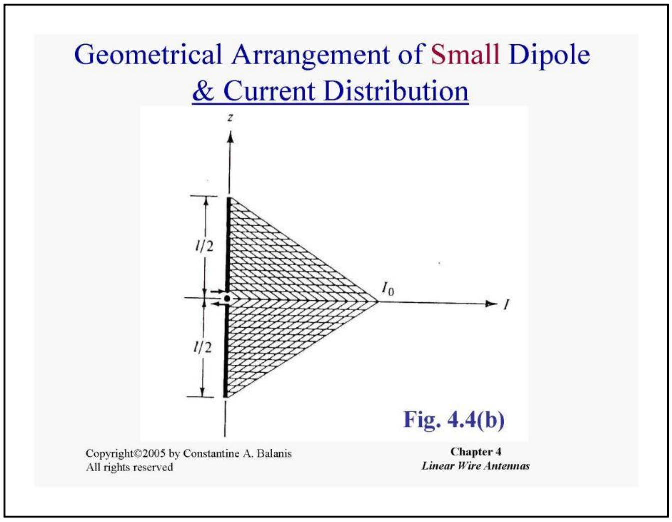
$$\frac{\lambda}{50} \le \ell \le \frac{\lambda}{10}$$



A better approximation for this case is a triangular variation. As the length of the dipole increases, the sinusoidal current variation becomes a better approximation.

$$I\left(x',y',z'\right)=\begin{cases} I_o\left(1-\frac{2}{\ell}z'\right) & 0\le z'\le\frac{\ell}{2} \\ I_o\left(1+\frac{2}{\ell}z'\right) & -\frac{\ell}{2}\le z'\le 0 \end{cases}$$

Flowing along the z-axis.



As $R \simeq r$ inside the integral, we can write the vector potential for the triangular current variation:

$$\bar{A}(x,y,z) = \frac{\mu_o}{4\pi}\left[\hat{a}_z \int_{-\ell/2}^{0} I_o\left(1+\frac{2}{\ell}z'\right)\frac{e^{-jkR}}{R}dz' + \hat{a}_z \int_{0}^{\ell/2} I_o\left(1-\frac{2}{\ell}z'\right)\frac{e^{-jkR}}{R}dz'\right]$$

$$\bar{A} = \hat{a}_z A_z = \hat{a}_z \frac{1}{2}\left[\frac{\mu_o I_o \ell e^{-jkr}}{4\pi r}\right]$$

This $\bar{A}$ is half of the $\bar{A}$ for infinitesimal dipole (Hertzian dipole).

As $\bar{H} = \frac{1}{\mu}\left(\nabla \times \bar{A}\right)$

$\bar{H}$ will be the same as before but divided by $\frac{1}{2}$. Similarly $\bar{E}$ $\frac{1}{2}$ of the Hertzian dipole (as $\left|\bar{E}\right| = \frac{\left|\bar{H}\right|}{\eta}$).

Field component for $kr \to \infty$ (in the far field):

$$E_\theta \approx j\eta \frac{kI_o \ell \sin\theta}{8\pi}\frac{e^{-jkr}}{r}$$

$$H_\phi \approx \frac{jkI_o \ell \sin\theta}{8\pi}\frac{e^{-jkr}}{r}$$

$$E_r = E_\phi = H_r = H_\theta = 0$$

As $E_\theta = E_{\max}\sin\theta$, field pattern and power pattern are the same with the infinitesimal dipole.

Power density is $W = \frac{1}{2}\frac{|E^2{}_\theta|}{\eta}$

So, $\frac{1}{4}$ of the Hertzian dipole. P_{rad}, U are also $\frac{1}{4}$ of the Hertzian dipole.

Radiation resistance of small dipole is:

$$R_r = \frac{2P_{rad}}{|I_o|^2}$$

Since field magnitudes are half of the infinitesimal dipole the radiated power is $\frac{1}{4}$ of the infinitesimal dipole.

$$R_r = \frac{2P_{rad}}{|I_o|^2} = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2$$

for the infinitesimal dipole. So:

$$R_r = 20\pi^2\left(\frac{\ell}{\lambda}\right)^2$$

for the small dipole.

Directivity

$D_g = 4\pi\frac{U}{\mathrm{P}_{rad}}$ is same as before since $U_{SD} = \frac{U_{HD}}{4}$ and $P_{rad\,SD} = \frac{P_{rad\,HD}}{4}$

$D = 1.5\sin^2\theta$

FINITE LENGTH DIPOLE (along z -axis)

Length of the dipole: $\ell > \frac{\lambda}{10}$

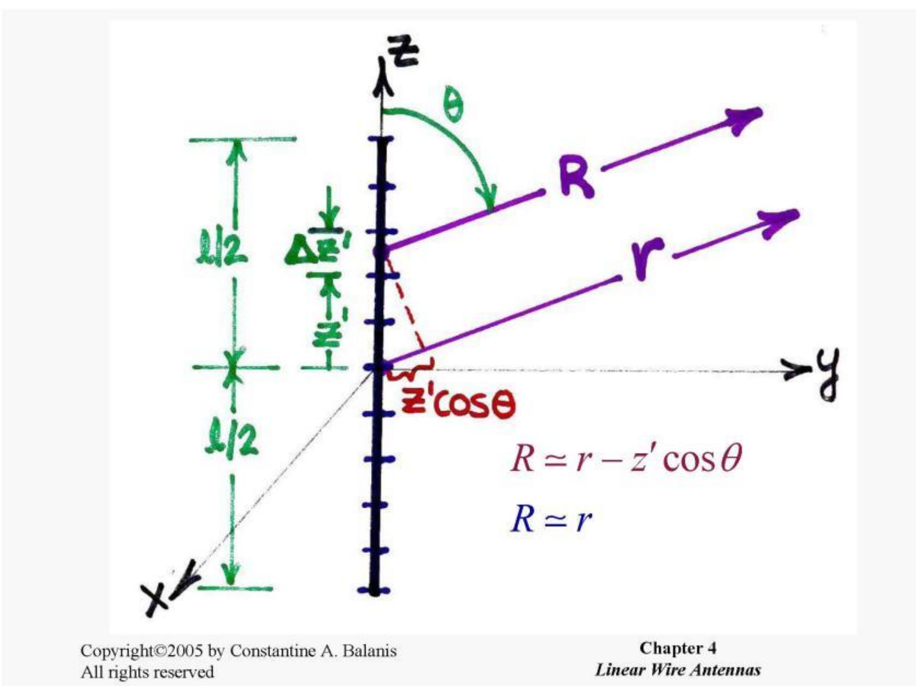
Assumption: The dipole has a negligible diameter. This is a good approximation if $d \ll \lambda$ (where *d* is the diameter).

Current Distribution:

If the dipole is very thin, a very good approximation for the distribution of current will be as:

$$I(z') = \begin{cases} I_o \sin\left[k\left(\frac{\ell}{2} - z'\right)\right] & 0 \le z' \le \frac{\ell}{2} \\ I_o \sin\left[k\left(\frac{\ell}{2} + z'\right)\right] & -\frac{\ell}{2} \le z' \le 0 \end{cases}$$

The antenna is center-fed and the current vanishes at the end points.



Radiated Fields

The finite dipole is subdivided into a number of infinitesimal dipoles of length $\Delta z'$.

Electric and magnetic field components of an Hertzian dipole of length dz', in the far-field:

$$dE_\theta \cong j\eta \frac{kI(z')e^{-jkR}}{4\pi R}\sin\theta dz'$$

$$dE_r \simeq dE_\phi = dH_r = dH_\theta = 0$$

$$dH_\phi \simeq j\frac{kI(z')}{4\pi R}\sin\theta dz'$$

Geometrical Approximation:

$R \simeq r - z'\cos\theta$ for phase terms

$R \simeq r$ for amplitude terms

We can then write:

$$E_\theta \simeq j\eta \frac{kI(z')e^{-jkr}}{4\pi r}\sin\theta e^{jkz'\cos\theta}dz'$$

$$E_\theta \simeq j\eta \frac{ke^{-jkr}}{4\pi r}\sin\theta\left[\int_{-\ell/2}^{\ell/2} I(z')e^{jkz'\cos\theta}dz'\right]$$

Notice that:

Total field= element factor X space factor

Element factor=pattern multiplication

Now, substitude the expression for $I(z')$ into the above integral:

$$E_\theta \simeq j\eta \frac{ke^{-jkr}}{4\pi r}\sin\theta\left[\int_{-\ell/2}^{0}\sin[k\left(\frac{\ell}{2}+z'\right)]e^{jkz'\cos\theta}dz'+\int_{0}^{\ell/2}\sin[k\left(\frac{\ell}{2}-z'\right)]e^{jkz'\cos\theta}dz'\right]$$

$$\int e^{\alpha x}\sin(\beta x+\gamma)dx = \frac{e^{\alpha x}}{\alpha^2+\beta^2}\left[\alpha\sin(\beta x+\gamma)-\beta\cos(\beta x+\gamma)\right]$$

With

$$\alpha = \pm jk\cos\theta$$

$$\beta = \pm k$$

$$\gamma = k\frac{\ell}{2}$$

We find

$$E_\theta \simeq j\eta \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]$$

Similarly

$$H_\phi \simeq \frac{E_\theta}{\eta}$$

Time averaged power density:

$$\bar{W}_{av} = \frac{1}{2}\mathrm{Re}\left[\bar{E}\times\bar{H}^*\right]$$

$$\bar{W}_{av} = \frac{1}{2}\mathrm{Re}\left[\hat{a}_\theta E_\theta \times \hat{a}_\phi H_\phi^{\ *}\right]$$

$$\bar{W}_{av} = \frac{1}{2}\mathrm{Re}\left[\hat{a}_\theta E_\theta \times \hat{a}_\phi \frac{E_\theta^{\ *}}{\eta}\right]$$

$$\bar{W}_{av} = \hat{a}_r \frac{1}{2\eta}\left|E_\theta\right|^2$$

$$W_{av} = \eta \frac{\left|I_o\right|^2}{8\pi^2 r^2}\left|\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right|^2$$

Radiation Intensity

$$U = r^2 W_{ar} = \eta \frac{\left|I_o\right|^2}{8\pi^2}\left|\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right|^2$$

Half Wave Dipole:

A dipole that:

$\ell = \frac{\lambda}{2} \quad \frac{k\ell}{2} = \frac{2\pi}{\lambda}\frac{1}{2}\frac{\lambda}{2} = \frac{\pi}{2}$

So, $\cos\left(\frac{k\ell}{2}\right) = 0$

$$E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

Normalized field pattern (call it as $f(\theta)$):

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

Field pattern:

$$\theta = 0 \rightarrow f(0) = \frac{0}{0}$$

$$\frac{-\sin\left(\frac{\pi}{2}\cos\theta\right)\frac{\pi}{2}(-)\sin\theta}{\cos\theta} = 0$$

Maximum of $f(\theta)$ can be found from:

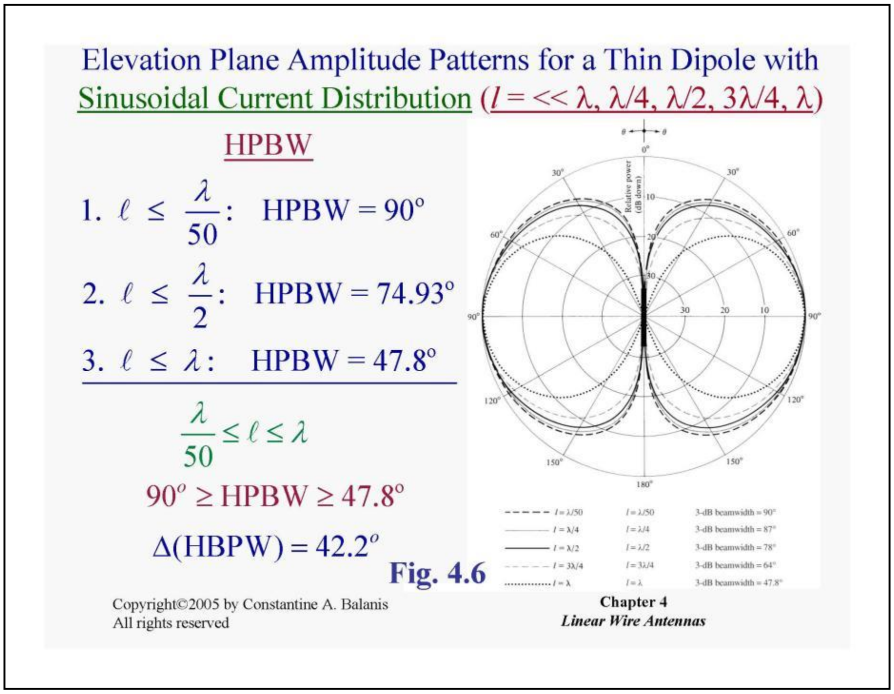
$$\frac{df(\theta)}{d\theta} = 0 \text{ is satisfied at } \theta = \frac{\pi}{2}$$

$$\frac{\cos\left[\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)\right]}{\sin\left(\frac{\pi}{2}\right)} = \frac{\cos 0}{\sin\left(\frac{\pi}{2}\right)} = 1$$

Half-power beamwidth:

$$f(\theta) = \frac{1}{\sqrt{2}} = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

This equation is zero at $\theta_1 = 51^o$ and $\theta_1 = 129^o$. Then,



Radiation Intensity for a half wave dipole:

$$U = r^2 W_{ar} = \eta \frac{|I_o|^2}{8\pi^2} \left| \frac{\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\left(\frac{k\ell}{2}\right)}{\sin\theta} \right|^2$$

$$U = r^2 W_{ar} = \eta \frac{|I_o|^2}{8\pi^2} \left| \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \right|$$

$$P_{rad} = \oint_{\Omega} U d\Omega = \eta \frac{I^2_o}{8\pi}\ 2.435$$

Directive Gain:

$$D_g = \frac{4\pi U}{P_{rad}} = \frac{4}{2.435}\left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right] = 1.64\left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]$$

$$D_{\max} = 1.64$$

Radiation pattern is similar to the Hertzian dipole.

Radiation Resistance:

$$R_r = \frac{2P_{rad}}{I_o^2} = \frac{2\eta I_o^2\, 2.435}{8\pi}\frac{1}{I_o^2} = \frac{2\eta\;\; 2.435}{8\pi}$$

$R_r \approx 73\Omega$ for half wave dipole.

LOOP ANTENNAS

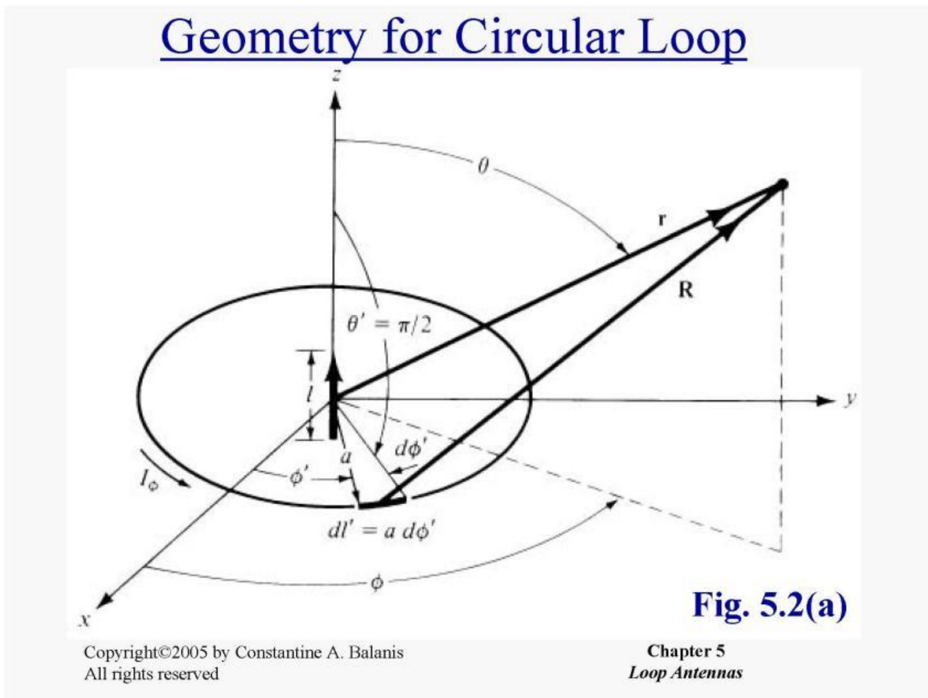
SMALL CIRCULAR LOOP

Not as commonly used as dipole antennas. Counterpart of Hertzian dipole.

Single Circular Loop

Fig. 5.1(a)

Chapter 5
Loop Antennas



$$\bar{A} = \frac{\mu}{4\pi}\int I\left(x',y',z'\right)\frac{e^{-jkR}}{R}d\bar{\ell}'$$

$d\bar{\ell}' = ad\phi'\hat{a}_{\phi}$ $\quad I = I_o$ =Constant w.r.t ϕ' anticlocwise

After expressing the distances and unit vectors in terms of appropriate coordinates, making relevant approximations we find:

$$\bar{A} = \hat{a}_{\phi}A_{\phi} = \hat{a}_{\phi}\frac{a^2\mu I_o}{4}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta$$

Since:

$$\bar{H} = \frac{1}{\mu}\left(\nabla X \bar{A}\right)$$

We get,

$$H_r = j\frac{ka^2 I_o \cos\theta}{2r^2}\left(1 + \frac{1}{jkr}\right)e^{-jkr}$$

$$H_\theta = -\frac{(ka)^2 I_o \sin\theta}{4r}\left(1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right)e^{-jkr}$$

$$H_\phi = 0$$

$$E_r = E_\theta = 0$$

$$E_\phi = \eta\frac{(ka)^2 I_o \sin\theta}{4\pi}\left(1 + \frac{1}{jkr}\right)e^{-jkr}$$

<u>FAR FIELDS</u>

Let $kr \to \infty$, then

$$E_\phi = \eta\frac{k^2 a^2 I_o}{4r}e^{-jkr}\sin\theta$$

$$H_\theta = -\frac{k^2 a^2 I_o}{4r}e^{-jkr}\sin\theta$$

Normalized field and power patterns are the same with infinitesimal dipole and small dipole along the z-axis.

Power Density and Radiation Resistance

The field expression of a small loop, as given above, are valid everywhere except at the origin.

$$\bar{W} = \frac{1}{2}\mathrm{Re}\left(\bar{E} X \bar{H}^*\right) = \frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\phi E_\phi\right) X \left(\hat{a}_r \bar{H}_r^* X \hat{a}_\theta \bar{H}_\theta^*\right)\right]$$

$$W_r = -\frac{1}{2} E_\phi H_\theta^* \quad \text{radial component}$$

$$W_r = \eta \frac{(ka)^4}{32 r^2} |I_o|^2 \sin^2\theta \ \left(W/m^2\right)$$

The radiation Intensity:

$$U = \eta \frac{(ka)^4}{32} |I_o|^2 \sin^2\theta$$

Now, if we intagrate W_r over a sphere of radius r, we get the complex power P_r:

$$P_r = \oiint_S W_r ds = \int\int W_r r^2 \sin\theta d\theta d\phi$$

$$P_r = \int_0^{2\pi}\int_0^{\pi} U \sin\theta d\theta d\phi$$

$$P_{rad} = \mathrm{Re}\left(P_r\right) = \eta\left(\frac{\pi}{12}\right)(ka)^4 |I_o|^2 \quad \text{Time average radiated power.}$$

$$P_{rad} = \frac{1}{2}|I_o|^2 R_r$$

Radiation resistance:

$$R_r = \eta\left(\frac{\pi}{6}\right)(ka)^4 = 20\pi^2(ka)^4$$

Directive Gain

$$D_g = 4\pi\frac{U}{P_{rad}} = 1.5\sin^2\theta$$

Directivity same as the Hertzian dipole:

$$D_{\max} = 1.5$$