ARRAYS

In many applications it is necessary to design very directive antennas (antennas with high gain) to meet the demand of long distance communication. This can be accomplished by increasing the electrical size of the antenna or by making an assembly of antennas of same elements.

An antenna which is composed of multiple antenna elements is known as an ARRAY. In general the elements of an array are identical.

The total field of the array is determined by the vector addition of the fields radiated by the individual elements.

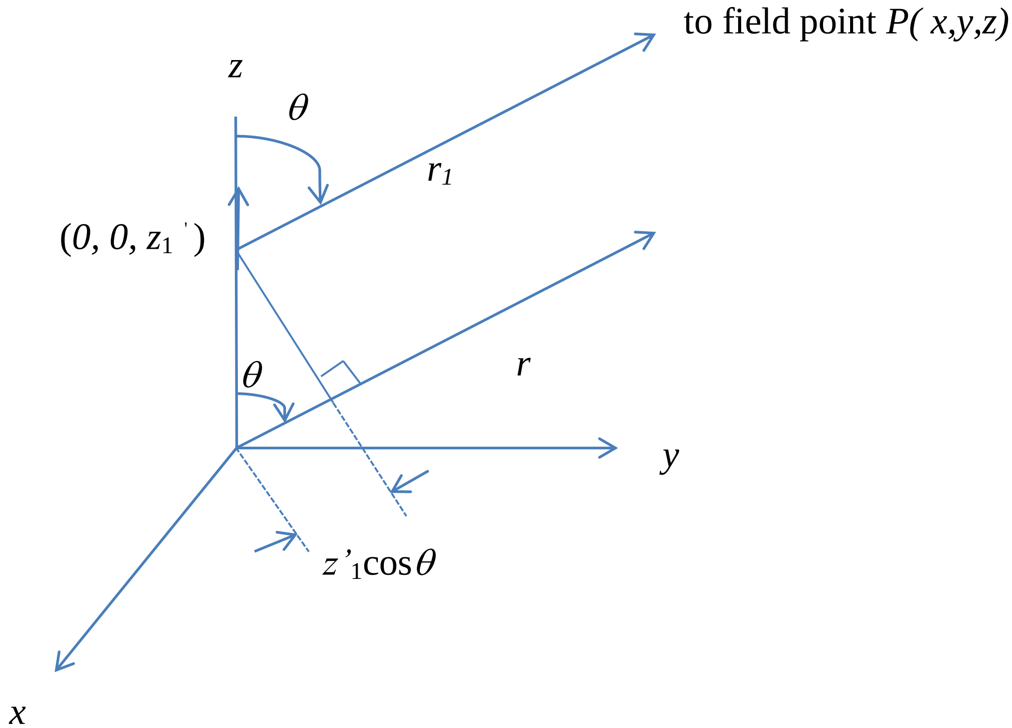
The following factors can be used to shape the overall pattern of the antenna:

1) The geometry of the array (Types) (linear, circular, rectangular, etc.)
2) The relative displacement between the elements,
3) The excitation phase of the individual elements,
4) The relative pattern of the individual elements.

Let us consider a current source over a length of several wavelengths long. Let this long line current distribution be produced by having several $\lambda/2$ dipoles arranged end-to-end and excite by in-phase currents. Each dipole may be excited with different amplitude and phase because they are independent. This arrangement is known as a linear array. Each antenna is called an element of the array.

LINEAR ARRAY AND PATTERN MULTIPLICATION

Consider an infinitesimal dipole of length $d\ell$ kept at point $(0,0,z_1^{'})$ in free space.



$$A_z = \frac{\mu}{4\pi} I_1 d\ell \frac{e^{-jkr_1}}{r_1}$$

When the field point is at a large distance, then

$r_1 \approx r$ for amplitude

$r_1 \approx r - z'_1 \cos\theta$ for phase

Then,

$$E_{\theta_1} = j\eta \frac{kI_1\ell}{4\pi}\sin\theta\left\{\frac{e^{-jkr}}{r}\right\}e^{jkz'_1\cos\theta}$$

Now, let us consider N such infinitesimal , z-directed current elements kept along the z-axis at points $z'_1, z'_2, \ldots\ldots\ldots, z'_N$.

Let the currents in these dipoles be $I_1, I_2, \ldots\ldots\ldots, I_N$

All currents have the same frequency.

Using superposition, for $r \to \infty$:

$$E_\theta = \left(E_\theta\right)_1 + \left(E_\theta\right)_2 + \ldots\ldots + \left(E_\theta\right)_N$$

$$E_\theta = j\eta k\frac{d\ell}{4\pi}\sin\theta\left[I_1\frac{e^{-jkr_1}}{r_1} + I_2\frac{e^{-jkr_2}}{r_2} + \ldots\ldots + I_1\frac{e^{-jkr_N}}{r_N}\right]$$

Doing the following approximations:

$$r_n \approx r;\quad n = 1, 2, \ldots\ldots, N \quad \text{for amplitude}$$

$$r_n \approx r - z'_n\cos\theta;\quad n = 1, 2, \ldots\ldots, N \quad \text{for phase}$$

The total field is then:

$$E_\theta = j\eta\frac{kd\ell}{4\pi}\frac{e^{-jkr}}{r}\sin\theta\sum_{n=1}^{N} I_n e^{jkz'_n\cos\theta}$$

Element Pattern= $j\eta \frac{kd\ell}{4\pi} \frac{e^{-jkr}}{r} \sin\theta$ the electric field produced by an infinitely small dipole excited by a unit current kept at the origin.

Array Factor= $\sum_{n=1}^{N} I_n e^{jkz'_n \cos\theta}$

Pattern Multiplication:

Array Pattern= element pattern X array factor

It can be shown that the pattern multiplication theorem is applicable to any array of identical, equi-oriented antenna elements.

Equi-oriented means translation not rotation.

It is assumed that there is no iteraction between the elements, which may result in altering the individual radiation patterns.

The overall pattern is mainly controlled by the array factor, because the element pattern is generally a very broadpattern.

$$AF = \sum_{n=1}^{N} I_n e^{jkz'_n \cos\theta}$$

The AF depends on the excitation currents (both amplitude and phase) and positions of the elements.

Two-Element Array

Consider two isitropic antennas radiating equal power. Antenna separation is ‘d’.

Dipole 1: $\left(0,0,-\frac{d}{2}\right)$

Dipole 2: $\left(0,0,\frac{d}{2}\right)$

r is the distance from the origin to point P

r_1 is the distance from dipole 1 to point P

r_2 is the distance from dipole 2 to point P

$$r_1 \approx r + \frac{d}{2}\cos\theta$$
$$r_2 \approx r - \frac{d}{2}\cos\theta$$

The dipoles have equal magnitude but they are out of phase by α . Dipole 1 is at zero phase, and dipole 2 is at α . i.e. $e^{j0}, e^{j\alpha}$

$$E_1 = E_o\, e^{-jkr_1} = E_o\, e^{-jkr} e^{-j[k\frac{d}{2}\cos\theta]}$$

$$E_2 = E_o\, e^{-jkr_2} e^{j\alpha} = E_o\, e^{-jkr} e^{j[k\frac{d}{2}\cos\theta+\alpha]}$$

$$E_{tot} = E_o\, e^{-jkr}[e^{-j[k\frac{d}{2}\cos\theta]} + e^{j[k\frac{d}{2}\cos\theta+\alpha]}]$$

$$E_{tot} = E_o\, e^{-jkr} e^{-j\frac{\alpha}{2}}[e^{-j[k\frac{d}{2}\cos\theta+\frac{\alpha}{2}]} + e^{j[k\frac{d}{2}\cos\theta+\frac{\alpha}{2}]}]$$

$$E_{tot} = E_o e^{-jkr} e^{-j\frac{\alpha}{2}}[2\cos\left(\frac{kd}{2}\cos\theta + \frac{\alpha}{2}\right)]$$

$$\left|E_{tot}\right| = 2E_o[\cos\left(\frac{kd}{2}\cos\theta + \frac{\alpha}{2}\right)]$$ E for a two element array.

Radiation Pattern for different α and d

a) Two elements in phase and $d = \frac{\lambda}{2}$

$$\left|E_{tot}\right| = 2E_o[\cos\left(\frac{kd}{2}\cos\theta + \frac{\alpha}{2}\right)]$$

$$k = \frac{2\pi}{\lambda}, d = \frac{\lambda}{2} \quad kd = \pi, \; \alpha = 0$$

$$\left|E_{tot}\right| = 2E_o[\cos\left(\frac{\pi}{2}\cos\theta\right)]$$

Maximum occurs when:

$$\cos\left(\frac{\pi}{2}\cos\theta_{\max}\right) = \mp 1$$

$$\frac{\pi}{2}\cos\theta_{\max} = n\pi$$

$$\cos\theta_{\max} = 2n$$

$$\cos\theta_{\max} = 0$$

$$\theta_{\max} = \frac{\pi}{2}$$

Since $0 \le \theta \le \pi$ the only possible solution is n=0.

<u>Minimum (zero)</u>

$$\left|E_{tot}\right| = 2E_o[\cos\left(\frac{\pi}{2}\cos\theta\right)] = 0$$

$$\cos\left(\frac{\pi}{2}\cos\theta_{\min}\right) = 0$$

$\theta_{\min} = 0$ and π.

b) Consider the case where $\alpha = \pi$ and $d = \frac{\lambda}{2}$

$$\left|E_{tot}\right| = 2E_o[\cos\left(\frac{kd}{2}\cos\theta + \frac{\alpha}{2}\right)]$$

$$kd = \pi \qquad d = \frac{\pi}{2}$$

$$\left|E_{tot}\right| = 2E_o[\cos\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right)]$$

Maximum:

$$\cos\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right) = \mp 1$$

$$\theta_{\max} = \pi$$

$$\theta_{\max} = 0$$

Minimum

$$\cos\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right) = 0$$

$$\theta_{\min} = \frac{\pi}{2}$$

c) Consider the case where $\alpha = 0$ and $d = \lambda$

$$\left|E_{tot}\right| = 2E_o[\cos\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right)]$$

$$\left|E_{tot}\right| = 2E_o[\cos\left(\pi\cos\theta\right)]$$

Maximum

$$\cos\left(\pi\cos\theta_{\max}\right) = \pm 1$$

$$\theta_{\max} = 0$$

$$\theta_{\max} = \frac{\pi}{2}$$

Minimum

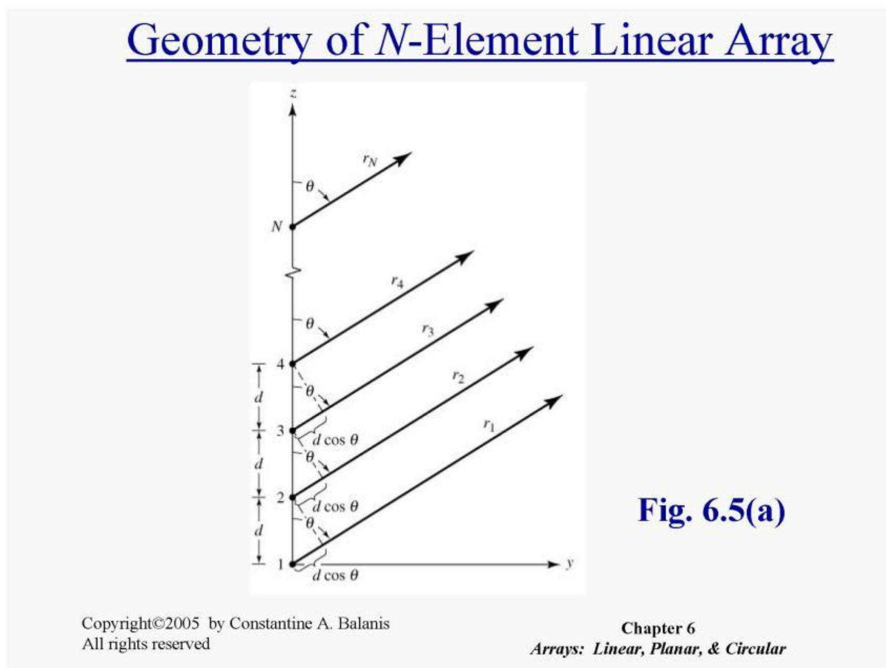
$$\cos\left(\pi\cos\theta_{\min}\right) = 0$$

$$\theta_{\min} = 60^o$$

$$\theta_{\min} = 120^o$$

UNIFORM ARRAYS

Consider an array of N point sources placed along the z-axis with the first element at the origin. The distance between any two consecutive elements is d . The excitation elements have equal magnitude and a progressive phase shift of α .



Such an array is called a uniform array.

The phase of the element at the origin is zero. Then, $\alpha, 2\alpha, 3\alpha, ...$ where α is the phase difference between the suggestive elements.

$r_1 = r$
$r_2 = r - 2d\cos\theta$
$r_3 = r - 3d\cos\theta$
.
.
.
$r_N = r - (N-1)d\cos\theta$

For the first element we have:

$$\bar{E} = E_o e^{-jkr}$$

The total electric field:

$$E_{tot} = E_o e^{-jkr} + E_o e^{-jkr_1 + j\alpha} + ... + E_o e^{-jkr_N + j(N-1)\alpha}$$

$$E_{tot} = E_o e^{-jkr}(1 + e^{j(kd\cos\theta+\alpha)} + e^{j2(kd\cos\theta+\alpha)} + ... + e^{j(N-1)(kd\cos\theta+\alpha)})$$

$$E_T = E_o.AF$$

$$AF = 1 + e^{j(kd\cos\theta+\alpha)} + ... + e^{j(N-1)(kd\cos\theta+\alpha)}$$

Defining

$$\psi = kd\cos\theta + \alpha$$

$$AF = 1 + e^{j\psi} + e^{j2\psi} + ... + e^{j(N-1)\psi} \quad ....(1)$$

Multiply (1) with $e^{j\psi}$

$$AFe^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + ... + e^{jN\psi} \quad ....(2)$$

Subtract (1) from (2):

$$AF(e^{j\psi} - 1) = e^{jN\psi} - 1$$

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1}$$

$$AF = \frac{e^{j\frac{N\psi}{2}}\left(e^{j\frac{N\psi}{2}} - e^{-j\frac{N\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}\right)}$$

$$AF = e^{j(N-1)\frac{\psi}{2}} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

Magnitude:

$$|AF| = \left|\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right| = \left|\frac{\sin\left(\frac{N\alpha}{2} + \frac{Nkd\cos\theta}{2}\right)}{\sin\left(\frac{\alpha}{2} + \frac{kd}{2}\cos\theta\right)}\right|$$

Where $\psi = kd\cos\theta + \alpha$

For $\psi = 0$,

$$|AF| = \frac{0}{0} = \left.\left|\frac{\frac{N}{2}\cos\left(\frac{N\psi}{2}\right)}{\frac{1}{2}\cos\psi}\right|\right|_{\psi=0} = N$$

The normalized

$$AF = \frac{|AF|}{N}$$

$$|AF_n| = \left| \frac{\sin\left(\frac{N\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)} \right|$$

Analysis of N Element Array

<u>Maximum (Peak)</u>

It occurs when $\sin\frac{\psi}{2} \to 0$ and $\sin\frac{N\psi}{2} \to 0$

So it occurs when:

$$\sin(\frac{\psi}{2}) = 0$$

$$\frac{\psi}{2} = n\pi$$

$$\psi = 2n\pi$$

$$k\,d\cos\theta_{\max} + \beta = 2n\pi$$

$$k\,d\cos\theta_{\max} = 2n\pi - \beta$$

$$\theta_{\max} = \cos^{-1}\left(\frac{1}{kd}(2n\pi - \beta)\right)$$

When n=0,

$\theta_{\max} = \cos^{-1}\left(\frac{-\beta}{kd})\right)$ This max is the main max.

Minimum (zeros):

$$AF = \frac{\sin\left(\frac{N\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}$$

Zeros occur when $\sin\left(\frac{N\psi}{2}\right) = 0$ except when there is a max.

So occurs when

$$\frac{N\psi}{2} = n\pi$$
$$\psi = \frac{2n\pi}{N}$$

except when there is a max.

Sidelobe Peaks

$$AF = \frac{\sin\left(\frac{N\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}$$

No sidelobe occurs at first maxima of $\sin\left(\frac{N\psi}{2}\right)$.

Sidelobe peaks occurs when:

$$\sin\left(\frac{N\psi}{2}\right) = \pm 1$$

$\frac{N\psi}{2} = (2n+1)\frac{\pi}{2}$ except for the first peak to the left and right of the peak