The Fourier Transform of Exponential Functions $e^{j2\pi at}$

If $x(t) = e^{j2\pi At}$ => $X(f) = \int_{-\infty}^{\infty} e^{j2\pi At} e^{-j2\pi ft}\, dt = \int_{-\infty}^{\infty} e^{-j2\pi(f-A)t}\, dt = \delta(f-A)$

The Fourier Transform of $\sin(2\pi At)$ and $\cos(2\pi At)$ Functions

The Fourier Transform of a cosine function can be found using the above derivation.

Hence, expanding

$$g(t) = \cos(2\pi At) \text{ using Eulers expansion } g(t) = cos(2\pi At) = \frac{(e^{j2\pi At} + e^{-j2\pi At})}{2}$$

Then

$$G(f) = \int_{-\infty}^{\infty} \frac{(e^{j2\pi At} + e^{-j2\pi At})}{2} e^{-j2\pi ft}\, dt$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{j2\pi At} e^{-j2\pi ft}\, dt + \int_{-\infty}^{\infty} e^{-j2\pi At} e^{-j2\pi ft}\, dt\right]$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-j2\pi(f-A)t}\, dt + \int_{-\infty}^{\infty} e^{-j2\pi(f+A)t}\, dt\right]$$

$$= \frac{1}{2}[\delta(f-A) + \delta(f+A)]$$

The Fourier Transform of

$$s(t) = sin(2\pi At) = \frac{(e^{j2\pi At} - e^{-j2\pi At})}{2j} \text{ will be}$$

$$S(f) = \frac{1}{2j}[\delta(f-A) - \delta(f+A)]$$