Why Model

We Concern Physical System

We Want a Representation

We have Something to Do on a Model

From Physical System to Transfer Function

Steps to Analysis Models

Find Inputs & Outputs

Look up Physical Laws

Derive Differential Equations

Perform Laplace Transform

Rearrange Equations

Get Solutions

Laplace Transform

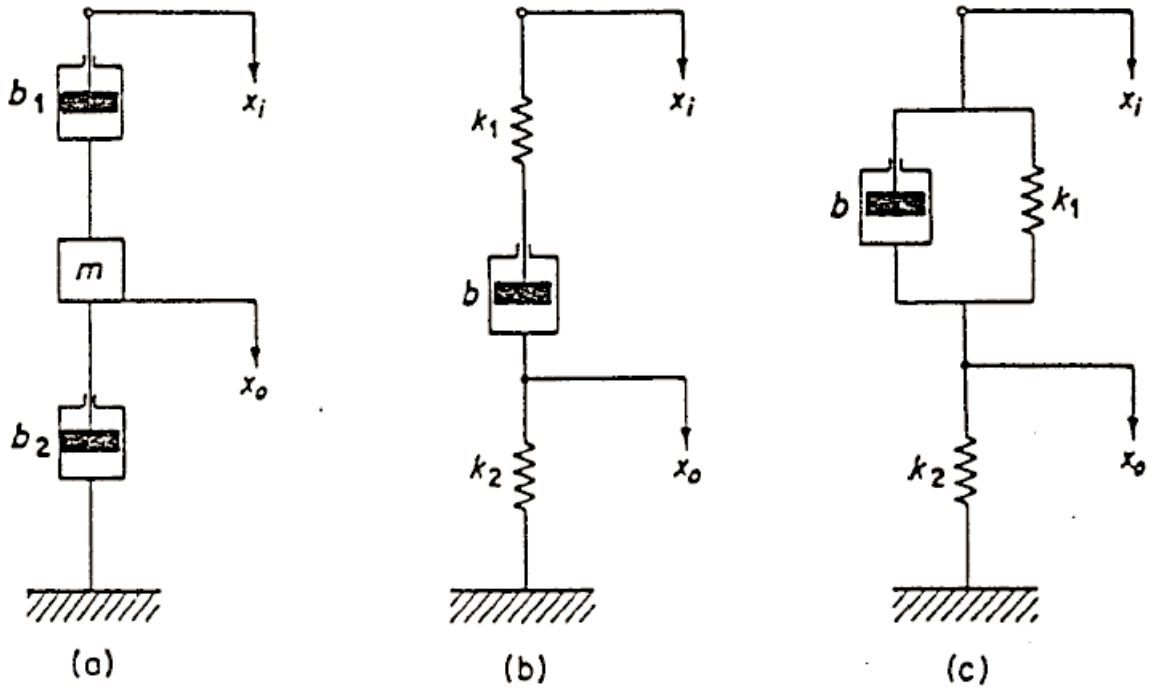
$$L[g(t)] \quad \Rightarrow \quad G(s)$$

$$L[\dot{g}(t)] \quad \Rightarrow \quad sG(s)$$

$$L[\ddot{g}(t)] \quad \Rightarrow \quad s^2G(s)$$

Example 1

Obtain the transfer function $X_o(s)/X_i(s)$ of each of the three mechanical systems.



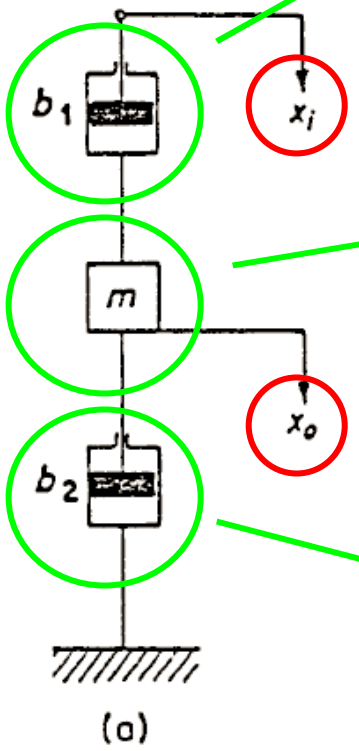
Example 1

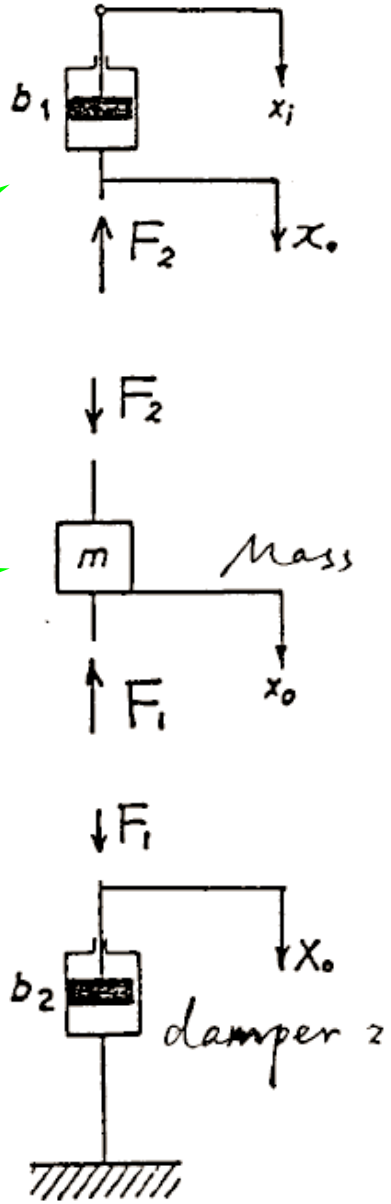
Review Some Physical Laws

- Mass
- Damper
- Spring

Example 1

(a) Solution:





$$F_2 = b_1(\dot{x}_i - \dot{x}_o)$$

$$F_2 - F_1 = m\ddot{x}_o$$

$$b_2\dot{x}_o = F_1$$

Example 1

$$\left.\begin{aligned} F_2 &= b_1(\dot{x}_i - \dot{x}_o) \\ F_2 - F_1 &= m\ddot{x}_o \\ b_2\dot{x}_o &= F_1 \end{aligned}\right\} \Rightarrow \quad b_1(\dot{x}_i - \dot{x}_o) - b_2\dot{x}_o = m\ddot{x}_o$$

By Laplace Transform

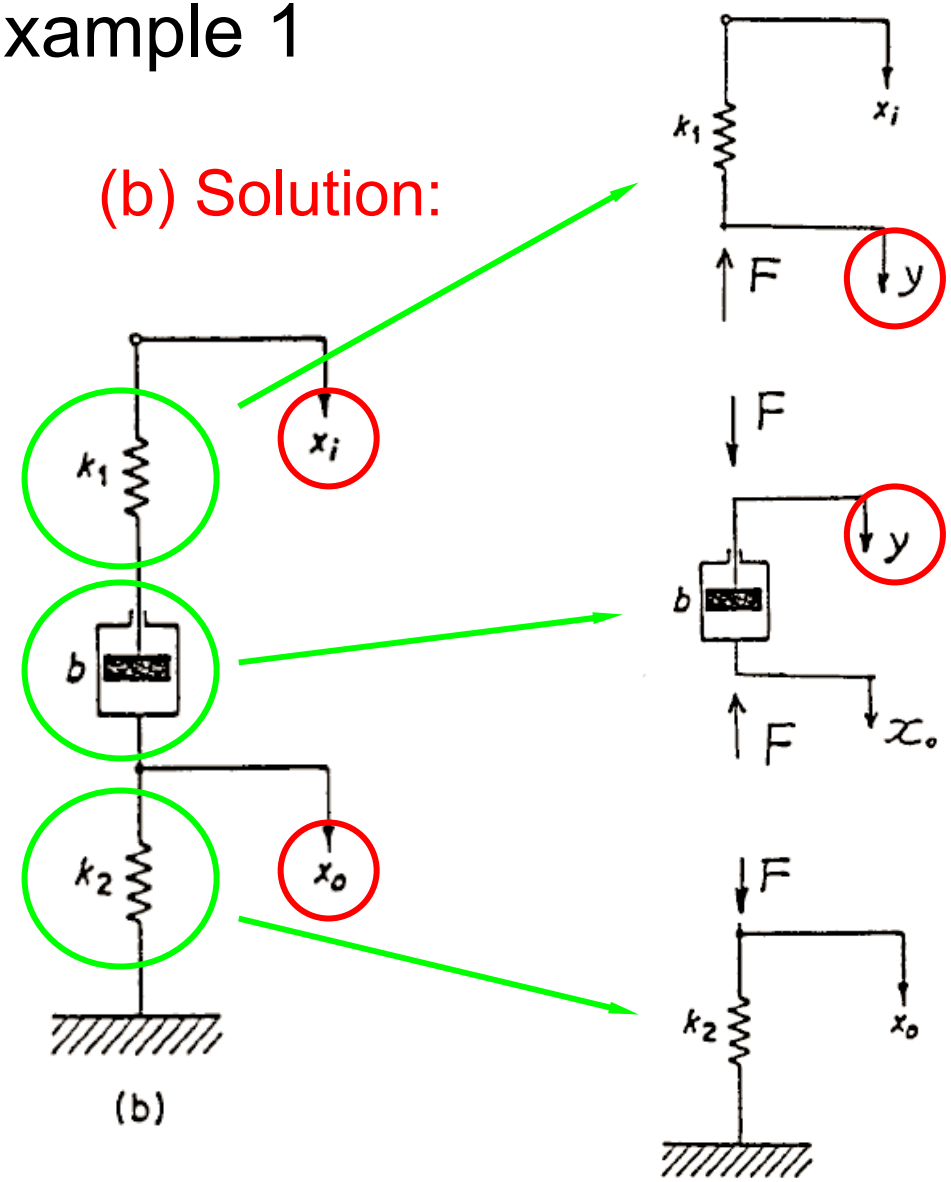
$$b_1[sX_i(s) - sX_o(s)] - b_2 sX_o(s) = ms^2 X_o(s)$$

$$(ms^2 + b_1 s + b_2 s)X_o(s) = b_1 sX_i(s)$$

$$\frac{X_o(s)}{X_i(s)} = \frac{b_1 s}{ms^2 + b_1 s + b_2 s} = \frac{b_1}{ms + b_1 + b_2}$$

Example 1

(b) Solution:



Temporary Used

$$F = k_1(x_i - y)$$

$$F = b(\dot{y} - \dot{x}_o)$$

$$F = k_2 x_o$$

Example 1

$$F = k_1(x_i - y)$$
$$F = b(\dot{y} - \dot{x}_o)$$
$$F = k_2 x_o$$

$$k_2 x_o = k_1(x_i - y)$$
$$k_2 x_o = b(\dot{y} - \dot{x}_o)$$

By Laplace Transform

$$k_2 X_o(s) = k_1[X_i(s) - Y(s)]$$
$$k_2 X_o(s) = b[sY(s) - sX_o(s)]$$

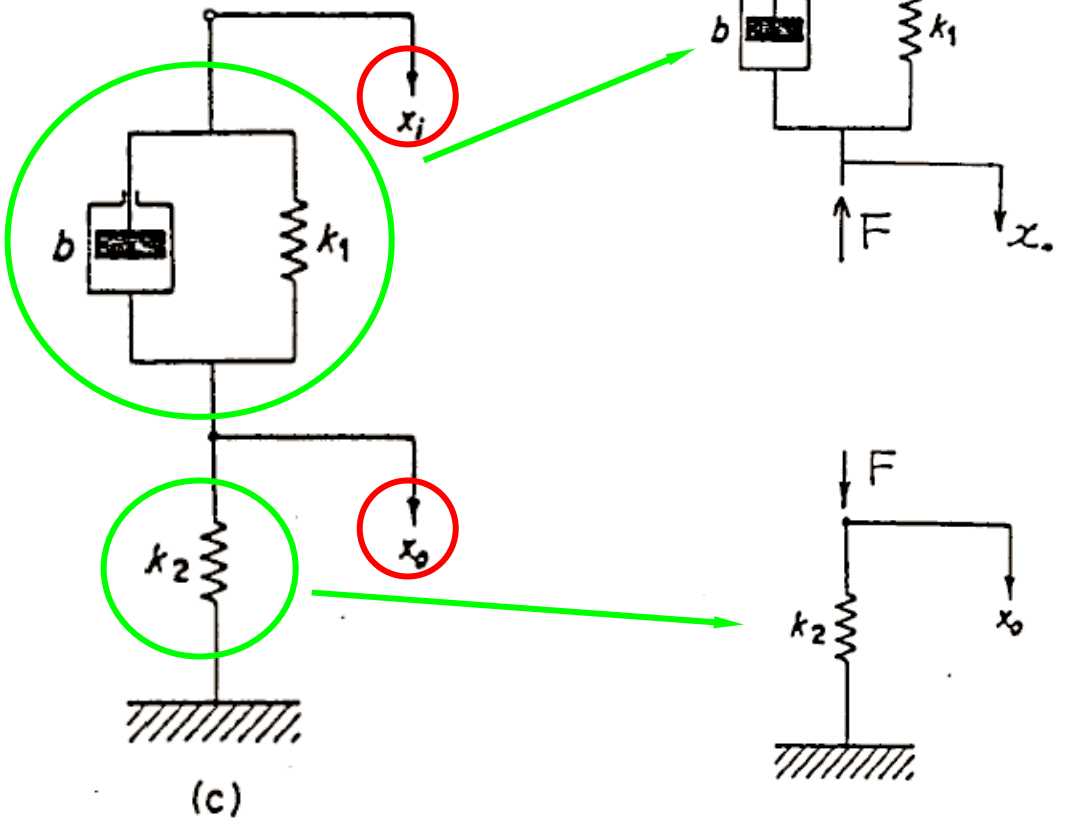
$$Y(s) = X_i(s) - k_2 X_o(s)/k_1$$
$$Y(s) = k_2 X_o(s)/bs + X_o(s)$$

$$X_i(s) - k_2 X_o(s)/k_1 = k_2 X_o(s)/bs + X_o(s)$$

$$\frac{X_o(s)}{X_i(s)} = \frac{1}{1 + k_2/k_1 + k_2/bs} = \frac{k_1 bs}{k_1 bs + k_2 bs + k_1 k_2}$$

Example 1

(c) Solution:



$$F = b(\dot{x}_i - \dot{x}_o) + k_1(x_i - x_o)$$

$$F = k_2 x_o$$

Example 1

$$F = b(\dot{x}_i - \dot{x}_o) + k_1(x_i - x_o)$$
$$F = k_2 x_o$$

$$\Rightarrow \quad k_2 x_o = b(\dot{x}_i - \dot{x}_o) + k_1(x_i - x_o)$$

By Laplace Transform

$$k_2 X_o(s) = b[sX_i(s) - sX_o(s)] + k_1[X_i(s) - X_o(s)]$$

$$(bs + k_1 + k_2) X_o(s) = (bs + k_1) X_i(s)$$

$$\frac{X_o(s)}{X_i(s)} = \frac{bs + k_1}{bs + k_1 + k_2}$$