Review

1. Pay attention to basic terminologies and concepts, basic formulas, and basic skills.
2. Understand the examples and homework
3. Reading materials: book, lecture notes and homework

Review

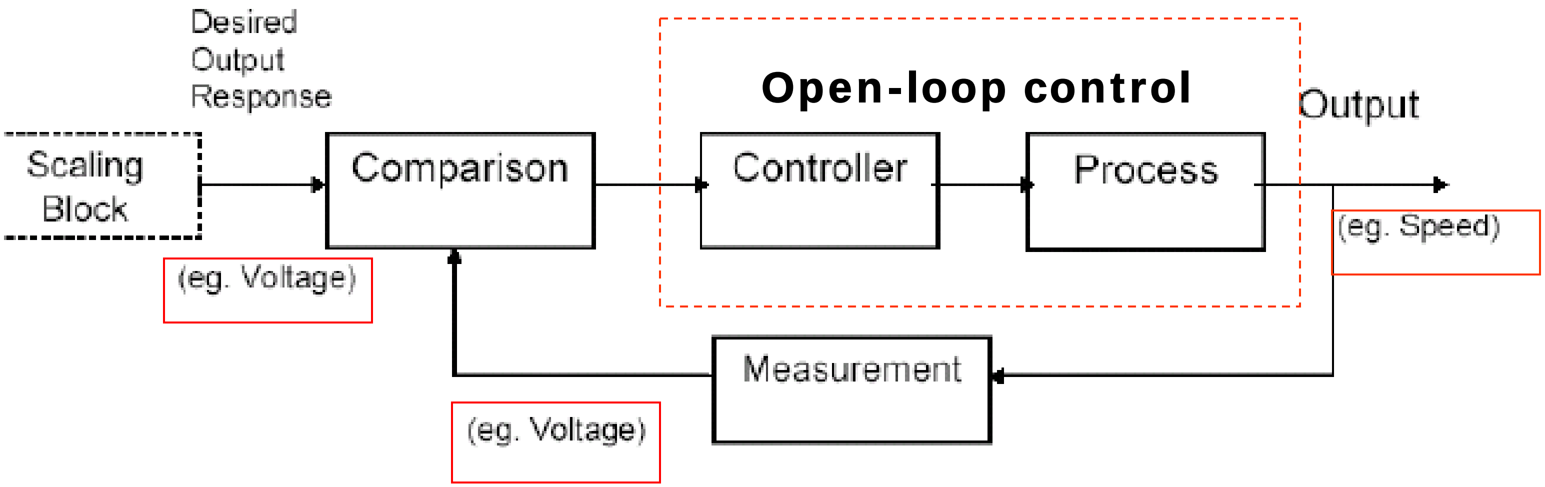
Chapter two:
Closed-loop control, basic components, modeling physical system, linearization, Laplace transformation, transfer function pole& zero, final value theorem, block diagram reduction, Mason' Gain formula

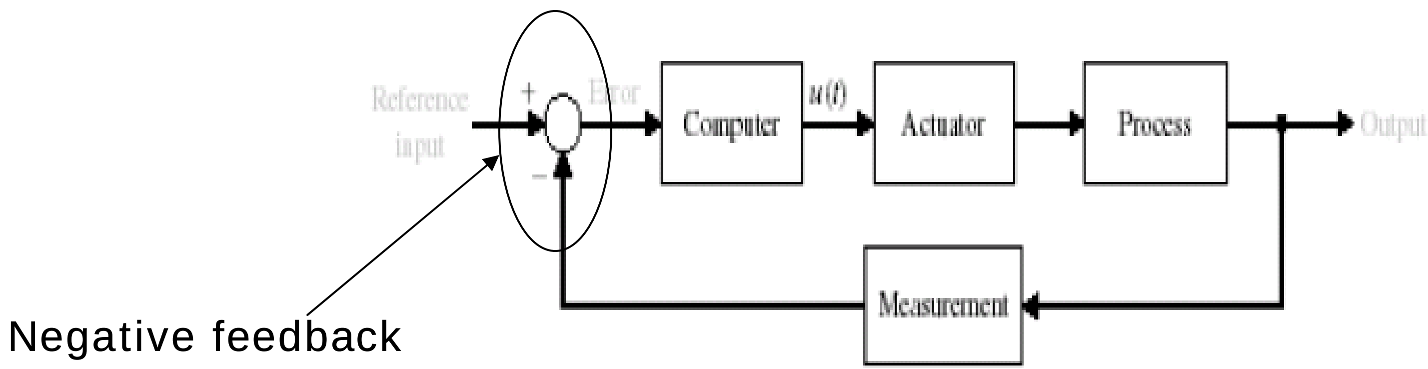
Chapter three:
state space equation; state transition matrix, controllability, observability

Chapter four:
Closed-loop system vs open loop, sensitivity function, disturbance response, parameter variation, steady state error

Chapter five:
Standard test inputs, step response for second order systems, dominant closed-loop poles for high order system, steady state error and system type, error constants for unit feedback system

Closed-Loop Control





Physical Laws commonly used in control

Newtown's Law:

The sum of forces on a body equal to zero; the sum of torques on a body equals zero.

$$\sum forces = 0.$$

Kirchhoff's voltage law:

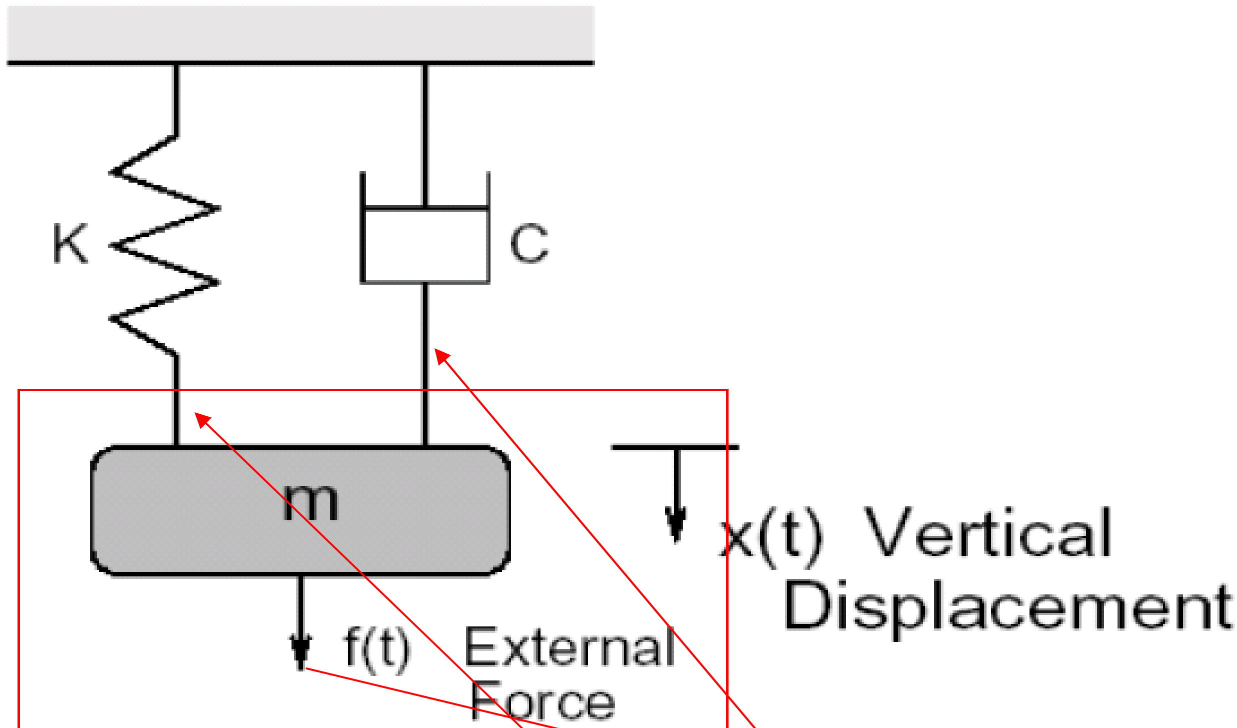
The sum of voltages around a closed path equals zero.

$$\sum voltages = 0.$$

Kirchhoff's current law:

The sum of electric currents flowing from a node equals zero. $\sum currents = 0.$

Spring-Mass-Damper System



$$m\ddot{x} = F(t) = -kx - c\dot{x} + f(t)$$

?

Spring-Mass-Damper System

$$m\ddot{x} = F(t) = -kx - c\dot{x} + f(t)$$

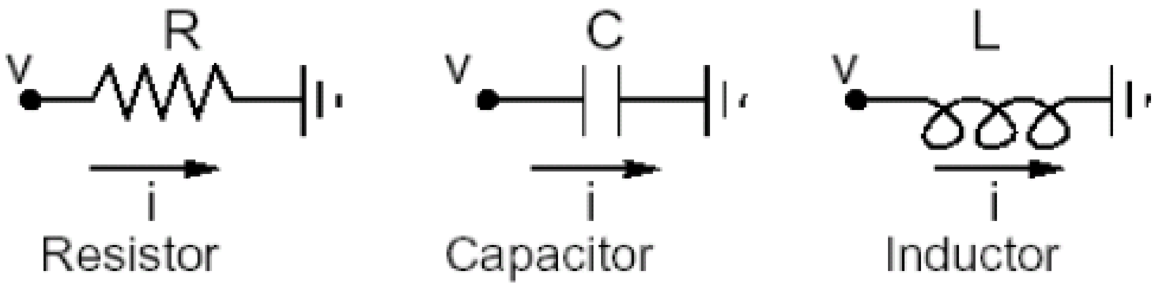
The signs of $-kx$ and $-c\dot{x}$ are negative because these forces oppose the motion of m.

- external force f causes the motion of m.

$$\Longrightarrow \quad m\ddot{x} + c\dot{x} + kx = f(t)$$

Note: (i) Second order system (energy storage), (ii) dc gain

Basic Electric Components

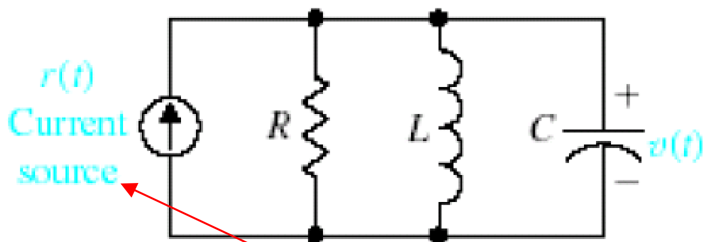


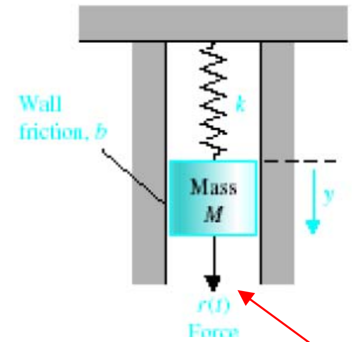
$v \longleftarrow$ voltage

$i \longleftarrow$ current

$\Longrightarrow$ Models : $v = iR, \qquad i = C\frac{dv}{dt}, \qquad v = L\frac{di}{dt}$

Analogous Systems





$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int_0^t v(t)dt = r(t)$$

$$M\frac{dv(t)}{dt} + bv(t) + k\int_0^t v(t)dt = r(t)$$

volatge

velocity

M=C   b=1/R   K=1/L

Velocity=Voltage   Force =current

Linearization of Nonlinear Models

- Although many systems are nonlinear, they can be approximated by linear systems for small deviations around a chosen operating point
- Note that the derived approximation will be valid only around this operating point
- Control theory for Linear Time-Invariant (LTI) systems is more complete and much simpler than that for nonlinear systems

The Laplace Transform

What is it?

The Laplace Transform

- Like the Fourier transform, the Laplace transform is an *integral transform*

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

$$s = \sigma + j\omega$$

Alternatively, the Laplace variable s can be considered to be the differential operator:

$$s = \frac{d(.)}{dt}$$

Pole & Zero

$$Y(s) = \frac{(Ms+b)y_0}{Ms^2+bs+k} = \frac{p(s)}{q(s)}. \quad (2.21)$$

The denominator polynomial $q(s)$, when set equal to zero, is called the **characteristic equation** because the roots of this equation determine the character of the time response of the system.

Poles: The roots of this characteristic equation $(q(s)=0)$ are also called the **poles** of the system.

Zeros: The roots of the numerator polynomial $(p(s)=0)$ are called the **zeros** of the system.

At the poles, $Y(s)=\infty$; at zeros, $Y(s)=0$.

Mass-spring-damper system

The s-plane plot of the poles and zeros of $Y(s)$ is shown in Fig. 2.9 where $\theta = \cos^{-1}\zeta$.

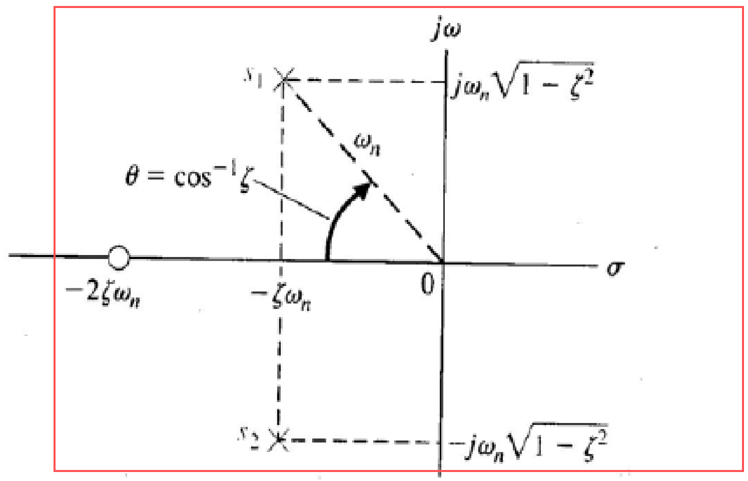
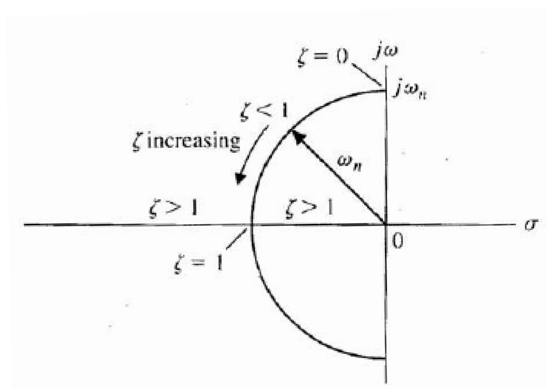


FIGURE 2.9 An *s*-plane plot of the poles and zeros of *Y*(*s*).



Final Value Theorem

Final value theorem is used to determine the steady-state or final value of the response of $y(t)$. It states that

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s), \qquad (2.28)$$

where a simple pole of $Y(s)$ at the origin is permitted, ***but poles on the imaginary axis and in the right half-s-plane and repeated poles at the origin are excluded***. In other words, final value theorem does not apply to these cases. Therefore, when using the final value theorem, one must check the poles of $Y(s)$ to see if the conditions are satisfied.

Transfer Function

- Defined as the ratio of the Laplace transform of the output to that of the input

G(s)=output(s)/ input(s)

- Describes dynamics of a LTI system

$$R(s) \longrightarrow \boxed{\frac{(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}} \longrightarrow C(s)$$

G(s)=C(s)/R(s)

With zero initial conditions.

Transfer Function

- Using the final value theorem, the static or d.c. gain of a transfer function $G(s)$ is given by $G(0)$

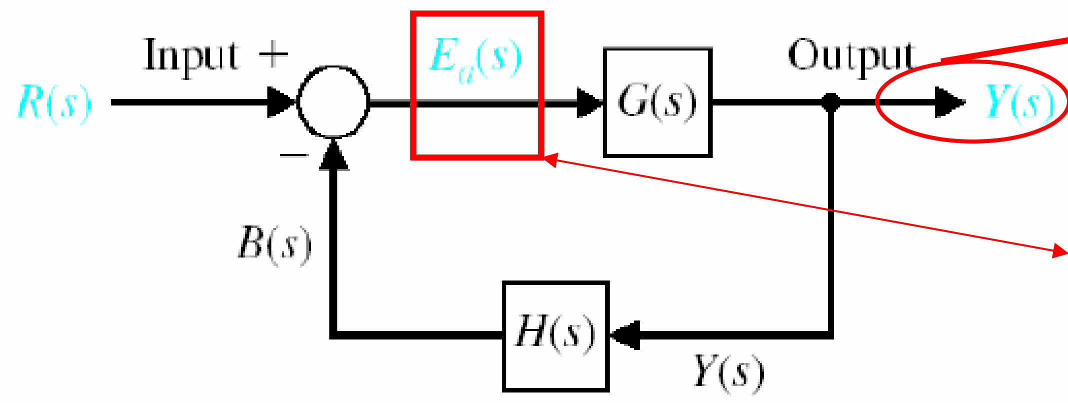
G(s)=output(s)/ input(s)

Let $G(s) = N(s)/D(s)$, then the roots of:

the characteristic equation $D(s) = 0$ are called the poles of the system

$N(s) = 0$ are called the zeros of the system

The Closed-Loop Transfer Function



$$Y(s) = G(s)E_a(s),$$

$$E_a(s) = R(s) - B(s)$$

$$= R(s) - H(s)Y(s).$$

$$Y(s) = G(s)E_a(s) = G(s)[R(s) - H(s)Y(s)].$$

$$Y(s)[1 + G(s)H(s)] = G(s)R(s).$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

- Note: when $H(s) \neq 1 \Rightarrow E_a(s) \neq E(s)$ $\quad E(s) = R(s) - Y(s)$

State-Variable Models

transfer function $\underset{\text{inverse Laplace transform}}{\overset{\text{Laplace transform}}{\Leftrightarrow}}$ the differential equations

The state-variable model, or state-space model

- A particular differential equation model
- Equations are written in a specific format expressed as *n* first-order coupled differential equations, but the choice of states is not unique
- All choices of state variables preserve the system's input-output relationship (that of the transfer function)

The Performance of Feedback Control Systems

- The ability to adjust the transient and steady-state performance is a distinct advantage of feedback control system.

- The performance of the control system is specified in terms of both the transient response and stead-state response.

  - The **transient response** is the response that disappears with time.
  - The **steady-state response** is that which exists a long time following any inputs signal initiation.

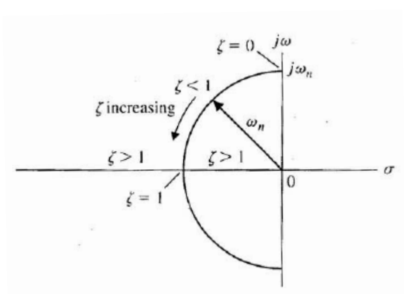
- The design specifications for control systems normally include several time-response indices for a specified input command as well as a desired steady-state accuracy.

Response for second order systems

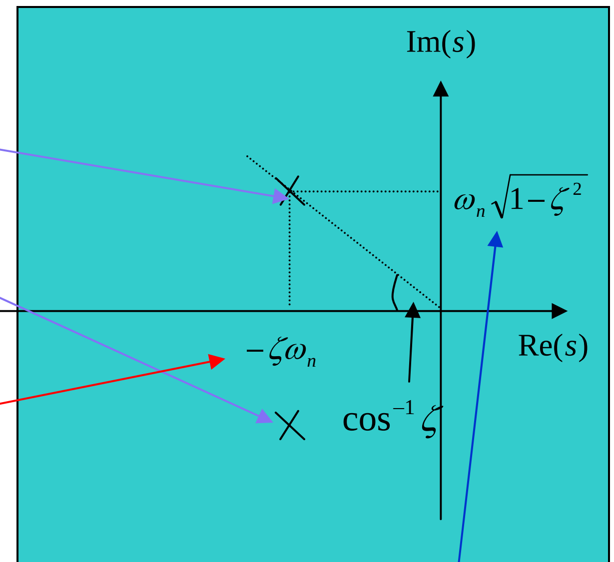
Transfer function: $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

Characteristic equation: $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

Poles: $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$



Time domain response requirements can be translated into pole positions



time constant

oscillation frequency

For higher order system, we have seen in chapter 2 that the response of a system is simply the sum of the response associated with each pole. The transient response is **dominated** by poles closer to the imaginary axis.

s-Plane and Transient Response