Question	Mark
Q1 (33 pts)	
Q2 (33 pts)	
Q3 (34 pts)	
Total points	

Eastern Mediterranean University
Electrical and Electronic / Information Systems Engineering Programs
Signals and Systems (EENG226 / INFE226)
Mid Term 1 (Date: 29th Nov. 2018, 8:30, Duration: 90 min)
Instructor: Prof. Dr. Hasan AMCA

Student Name: ______ SOLUTIONS ___________ **Number:** ____________

Important Notes for **Students** and **Invigilators**

1. Please Answer **All** Questions.
2. You may **use only the main course textbook** (Signals and Systems by Simon Haykin) in the exam. No other materials are allowed.
3. Make sure that there is **no undesirable writing or note in the textbook**. If anything is written in the book, the books will be taken away and disciplinary action may follow.
4. **No mobile telephones or computers** are allowed in the exam room. Hence, strictly no soft copies of books or other materials.
5. Students are not allowed to leave the exam room within the first **30 minutes**. Late comers are welcome in the first 30 minutes.

Formula Sheet for Signals and Systems

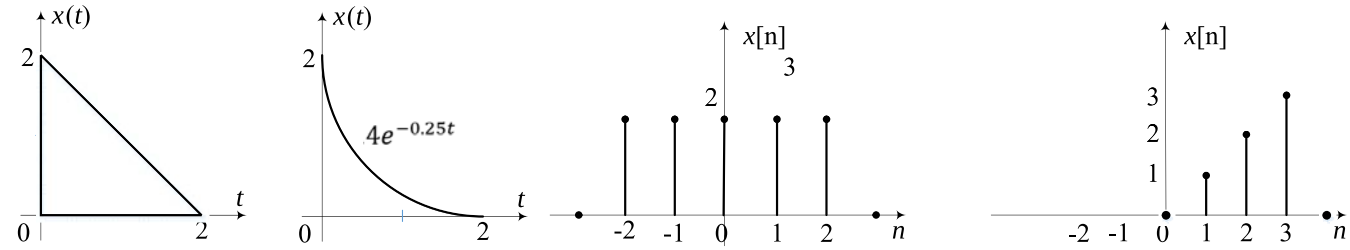
$cos^2(x)=\frac{1}{2}[1+\cos(2x)]$	$\sin^2(x)=\frac{1}{2}[1-\cos(2x)]$	$\sin(a+b)$ $=\sin(a)\cos(b)+\cos(a)\sin(b)$	$\frac{d}{dx}n=0$	$\frac{d}{dx}\cos(x)=\sin(x)$
$e^{ix}=\cos(x)+i\,sin(x)$	$e^x=\cosh(x)+sinh(x)$	$\cos(a+b)$ $=\cos(a)\cos(b)-\sin(a)\sin(b)$	$\frac{d}{dx}x=1$	$\frac{d}{dx}\sin(x)=cos(x)$
$\cos(x)=\frac{1}{2}(e^{ix}+e^{-ix})$	$\sin(x)=\frac{1}{2i}(e^{ix}-e^{-ix})$	$\sin(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$	$\frac{d}{dx}x^n=nx^{n-1}$	$\frac{d}{dx}\cos(x)=-\sin(x)$
$\cosh(x)=\frac{1}{2}(e^{-x}+e^x)$	$\sinh(x)=\frac{1}{2}(e^x-e^{-x})$	$\cos(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	$\frac{d}{dx}e^x=e^x$	$\frac{d}{dx}\tan(x)=sec^2(x)$
$\tan(x)=\sin(x)/\cos(x)$	$\cot(x)=1/tan(x)$	$\tanh(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{e^x-e^{-x}}{e^{-x}+e^x}$	$\frac{d}{dx}\ln x=\frac{1}{x}$	$\frac{d}{dx}\cot(x)=csc^2(x)$
$cos(2x)=2(\cos x)^2-1$ $=1-2(\sin x)^2$	$\sin(2x)=2\sin(x)\cos(x)$	$\coth(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{e^{-x}+e^x}{e^x-e^{-x}}$	$\frac{d}{dx}n^x=n^x\ln x$	$\frac{d}{dx}\arcsin(x)=\frac{1}{\sqrt{1-x^2}}$
$\sin(w_1t)+\sin(w_2t)=2\sin\left(\frac{w_1+w_2}{2}t\right)\cos\left(\frac{w_1-w_2}{2}t\right)=\pm\sqrt{2+2\cos((w_1-w_2)t)}\sin\left(\frac{w_1+w_2}{2}t\right)$				$\frac{d}{dx}\arccos(x)=-\frac{1}{\sqrt{1-x^2}}$
$\sec(x)=1/\cos(x)$	$\mathrm{cosec}(x)=1/\sin(x)$	$\frac{d}{dx}\mathrm{arcsec}(x)=\frac{1}{x\sqrt{x^2-1}}$	$e^{-1}=0.37$	$\frac{d}{dx}\arctan(x)=\frac{1}{1+x^2}$
$\mathrm{sech}(x)=1/\cosh(x)$	$cosech(x)=1/\sinh(x)$	$\frac{d}{dx}\mathrm{arccsc}(x)=-\frac{1}{x\sqrt{x^2-1}}$		$\frac{d}{dx}\mathrm{arccot}(x)=-\frac{1}{1+x^2}$
$(cosx)^2+(sinx)^2=1$	$\cos(2x)=(\cos x)^2-(\sin x)^2$	$P=\frac{1}{T}\int_0^T\lvert x(t)\rvert^2\,dt$	$E=\int_0^T\lvert x(t)\rvert^2\,dt$	
$x_e(t)=\frac{1}{2}\{x(t)+x(-t)\}$	$x_o(t)=\frac{1}{2}\{x(t)-x(-t)\}$			
$x_e[n]=\frac{1}{2}\{x[n]+x[-n]\}$	$x_o[n]=\frac{1}{2}\{x[n]-x[-n]\}$			

$x(at)\leftrightarrow\frac{1}{\lvert a\rvert}X\left(\frac{jw}{a}\right)$	$\frac{d}{dt}x(t)\leftrightarrow jwX(jw)$	$-jtx(t)\leftrightarrow\frac{d}{dw}X(jw)$	$x[n-n_0]\overset{DTFT}{\longleftrightarrow}e^{-j\Omega n_0}X(e^{-j\Omega})$	$x(t-t_0)\overset{FT}{\longleftrightarrow}e^{-jwt_0}X(jw)$
$x(at)\leftrightarrow\frac{1}{\lvert a\rvert}X\left(\frac{jw}{a}\right)$		$\int_{-\infty}^{\tau}x(\tau)\,d\tau=\frac{1}{jw}X(jw)$	$x[n-n_0]\overset{DTFS}{\longleftrightarrow}e^{-jk\Omega_0 n_0}X[k]$	$x(t-t_0)\overset{FS}{\longleftrightarrow}e^{-jkw_0t_0}X[k]$

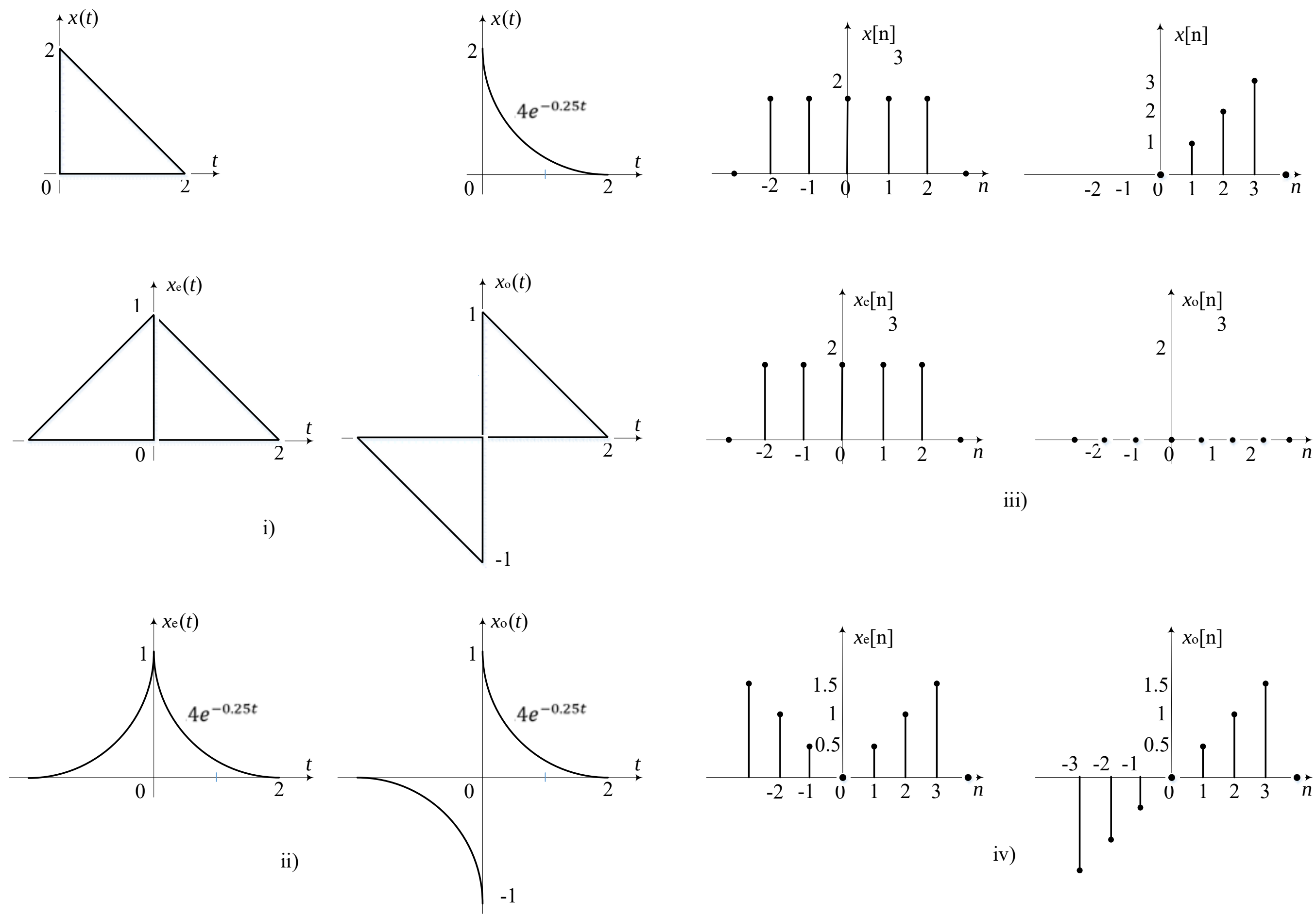
Periodic-Continuous (Fourier Series)		Non-Periodic-Continuous (Fourier Transform)	
$x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{jkw_0t}$	$X[k]=\frac{1}{T}\int_0^T x(t)e^{-jkw_0t}\,dt$	$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(jw)e^{jwt}\,dw$	$X(jw)=\int_{-\infty}^{\infty}x(t)e^{-jwt}\,dt$
Periodic-Discrete (DTFS)		**Non-Periodic-Discrete (DTFT)**	
$x[n]=\sum_{k=0}^{N-1}X[k]e^{jk\Omega_0 n}$	$X[k]=\frac{1}{N}\sum_{n=0}^{N-1}x[n]e^{-jk\Omega_0 n}$	$x[n]=\frac{2}{2\pi}\int_{-\pi}^{\pi}X(e^{j\Omega})e^{j\Omega n}\,d\Omega$	$X(e^{j\Omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n}$

$\int_{-\infty}^{\infty}e^{-j2\pi(f-A)t}\,dt=\delta(f-A)$	$\int dx=x+c$	$\int\sec(ax)\,dx$ $=\frac{1}{a}ln\lvert\sec(ax)+\tan(ax)\rvert+c$	$\int a^x dx=\frac{a^n}{\ln a}+c$
$\int\sin(x)dx=-\cos(x)+c$	$\int x^n dx=\frac{x^{n+1}}{n+1}+c$		$\int e^{ax}dx=\frac{e^{ax}}{a}+c$
$\int\cos(x)dx=sin(x)+c$	$\int u\,dv=u\,v-\int v\,du+c$	$\int cot(x)dx=ln\lvert\sin(x)\rvert+c$	$\int xe^{ax}dx=\frac{e^{ax}(ax-1)}{a^2}+c$
$\int\tan(x)dx=-ln\lvert\cos(x)\rvert+c$	$\int\frac{dx}{ax+b}=\frac{1}{a}ln\lvert ax+b\rvert+c$	$\int sec^2(ax)dx=\frac{1}{a}\tan(ax)+c$	$\int csc^2(ax)dx=-\frac{1}{a}\cot(ax)+c$

Q1. a) Sketch and label the even and odd components of each of the signals shown in the Figure. b) Find the energy in each of these signals in the Figure. Hint: The energy in $x(t)$ and $x(-t+2)$ is same.



(i) (ii) (iii) (iv)



The energy in

i) $E = \int_0^T |x(t)|^2\,dt = E = \int_0^2 t^2\,dt = \frac{t^3}{3}\Big]_0^2 = \frac{8}{3}$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt \qquad P = \frac{1}{T_0}\int_0^{T_0} [x(t)]^2\,dt$$

ii) $E = \int_0^T |x(t)|^2\,dt = \int_0^T |4e^{-0.25t}|^2\,dt$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

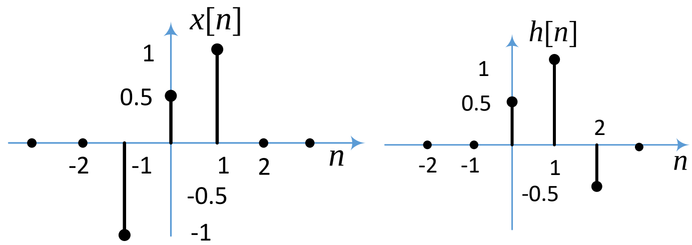
$$= 16\int_0^T |e^{-0.5t}|\,dt = \frac{16e^{-0.5t}}{-0.5}\Big]_0^2$$

$$= -32(e^{-1} - 1) = 20.2$$

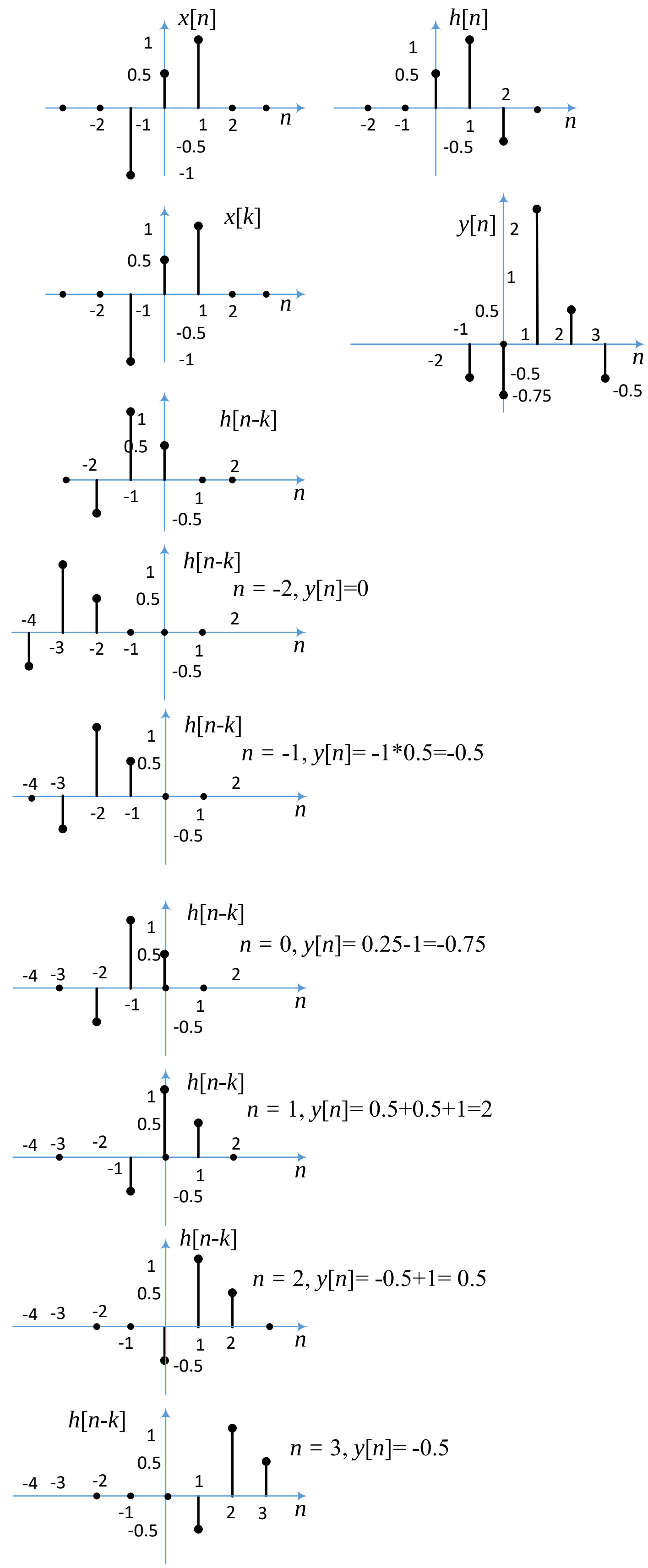
iii) $E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 5 \times 4 = 20,$

iv) $E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 1^2 + 2^2 + 3^2 = 14$

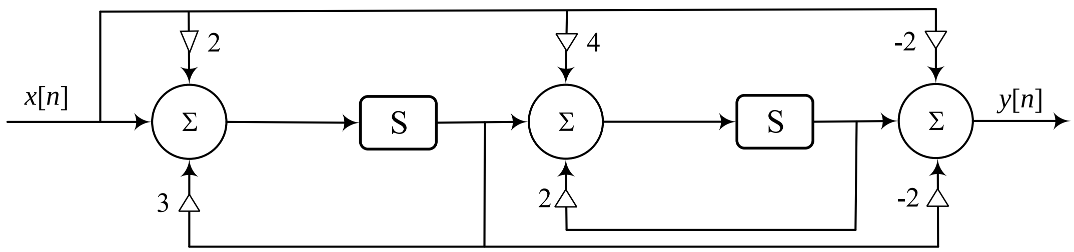
Q2) Find the convolution of the waveforms given in figure. Write down the convolution sum and solve the summation for critical values of discrete time n. Draw the convolution of $y[n] = x[n] * h[n]$.



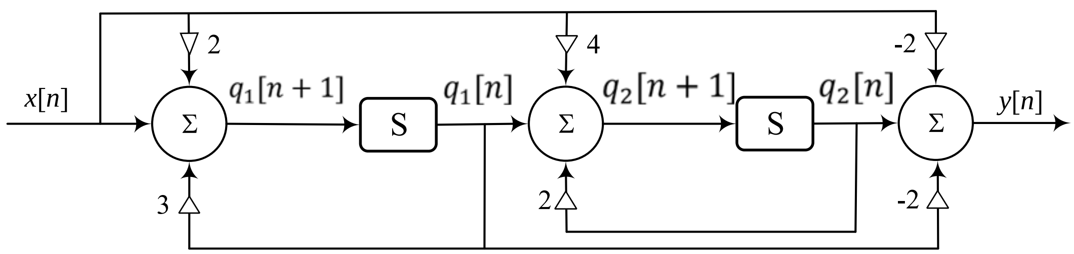
Solution:



Q3.a) If the state variables are the outputs of the unit delays, label the figure below by placing the state variables for the current state and the next state. b) Write down the state equations and the equation giving the output. Find the state variable description of the system shown. Please give details of your derivation.



Solution a)



Solution b) The block diagram indicates that the states are updated according to the equation

$$q_1[n+1] = 3q_1[n] + 0q_2[n] + 3x[n] \quad (1)$$

$$q_2[n+1] = 1q_1[n] + 2q_2[n] + 4x[n] \quad (2)$$

And the output is given by

$$y[n] = -2q_1[n] + 1q_2[n] - 2x[n] \quad (3)$$

We need to convert these equations into

$$q[n+1] = Aq[n] + bx[n] \quad (4)$$

$$y[n] = cq[n] + Dx[n] \quad (5)$$

Rearrange (1) and (2) to have

$$q[n+1] = \begin{bmatrix} q_1[n+1] \\ q_2[n+1] \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} x[n] \quad (6)$$

And

$$y[n] = \begin{bmatrix} -2 & 1 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} - 2x[n] \quad (7)$$

Comparing (4) and (6) then (5) and (7) we get

$$A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}, \; b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \qquad c = \begin{bmatrix} -2 & 1 \end{bmatrix}, \qquad D = -2$$