## Faculty of Engineering

**DEPARTMENT of ELECTRICAL AND ELECTRONIC ENGINEERING**

***EENG428 Introduction to Robotics***

**Instructor:**

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*Midterm EXAMINATION*

April 19, 2017

*Duration: 100 minutes*

Number of Problems: 4

*Good Luck*

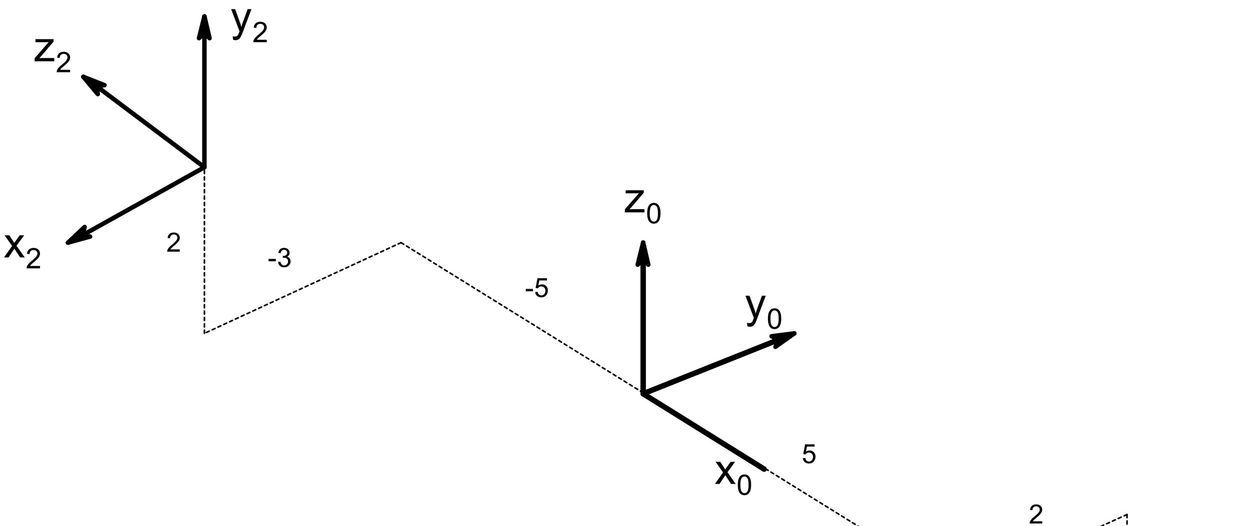
| **STUDENT'S** | |
|---|---|
| NUMBER | |
| NAME | SOLUTIONS |
| SURNAME | |

| Problem | | Points |
|---|---|---|
| 1 | | 25 |
| 2 | | 25 |
| 3 | | 25 |
| 4 | | 25 |
| *TOTAL* | | 100 |

## Problem 1

The orientations and positions of the frame 1 and the frame 2 are given relative to the frame 0 as ${}^0A_1$ and ${}^0A_2$, respectively. Plot the frames 1 and 2 and determine ${}^1A_2$ graphically and then mathematically.

$${}^0A_1 = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^0A_2 = \begin{bmatrix} 0 & 0 & -1 & -5 \\ -1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




$${}^1A_2 = \begin{bmatrix} 0 & -1 & 0 & -5 \\ 0 & 0 & -1 & -10 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0A_2 = {}^0A_1\,{}^1A_2$$

$$\left({}^0A_1\right)^{-1}\,{}^0A_2 = {}^1A_2$$

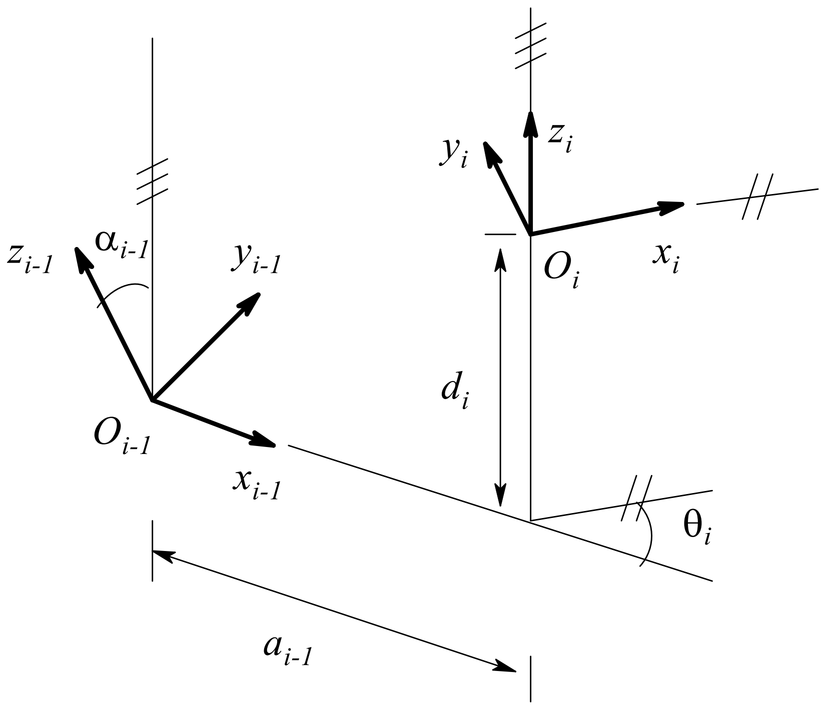
$$\left({}^{0}A_{1}\right)^{-1}=\begin{bmatrix}0 & 0 & -1 & -3\\ 1 & 0 & 0 & -5\\ 0 & -1 & 0 & 2\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$${}^{1}A_{2}=\begin{bmatrix}0 & 0 & -1 & -3\\ 1 & 0 & 0 & -5\\ 0 & -1 & 0 & 2\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & -1 & -5\\ -1 & 0 & 0 & -3\\ 0 & 1 & 0 & 2\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$${}^{1}A_{2}=\begin{bmatrix}0 & -1 & 0 & -5\\ 0 & 0 & -1 & -10\\ 1 & 0 & 0 & 5\\ 0 & 0 & 0 & 1\end{bmatrix}$$

## Problem 2

Determine the transformation ${}^{i-1}A_i$




$${}^{i-1}A_i = Rot(x,\alpha_{i-1})Trans(a_{i-1},0,0)Rot(z,\theta_i)Trans(0,0,d_i)$$

$${}^{i-1}A_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_{i-1} & -S\alpha_{i-1} & 0 \\ 0 & S\alpha_{i-1} & C\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{i-1}A_i = \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & C\alpha_{i-1} & -S\alpha_{i-1} & 0 \\ 0 & S\alpha_{i-1} & C\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{i-1}A_i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ C\alpha_{i-1}S\theta_i & C\alpha_{i-1}C\theta_i & -S\alpha_{i-1} & -d_iS\alpha_{i-1} \\ S\alpha_{i-1}S\theta_i & S\alpha_{i-1}C\theta_i & C\alpha_{i-1} & d_iC\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Problem 3

For the 3-DOF (RPR) robot shown in Fig.P3:

a) Assign the coordinate frames based on the D-H representation.
b) Fill out the parameter table.
c) Write all the *A* matrices.
d) Derive the forward kinematic equations, ${}^0T_H$ .

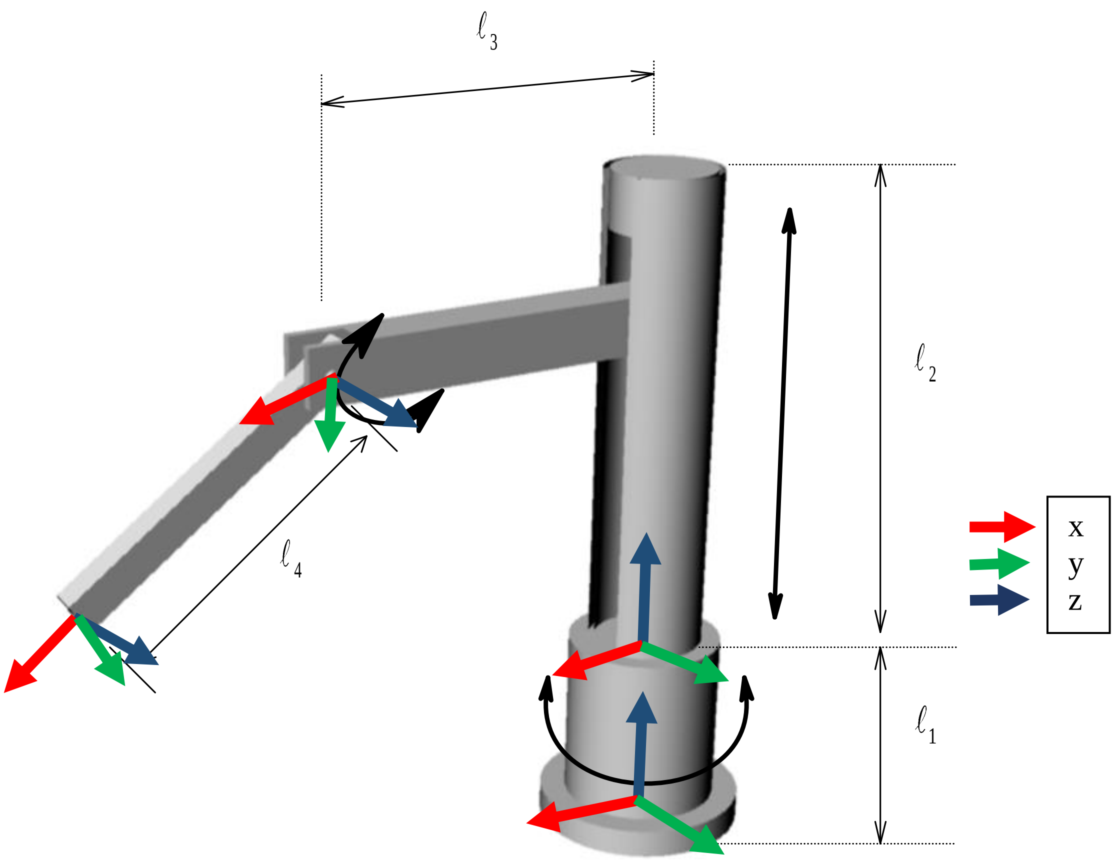



**Figure P 3: RPR robot**

**GIVEN:**

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Link Parameters table:

| **Link** | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| **1** | $\theta_1$ | $\ell_1$ | **0** | **0** |
| **2** | **0** | $\ell_2$ | $\ell_3$ | $-90$ |
| **3** | $\theta_3$ | **0** | $\ell_4$ | **0** |

$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & \ell_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & \ell_3 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & \ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & \ell_4 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & \ell_4 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_3 = A_1 A_2 A_3$$

$${}^0T_3 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & \ell_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \ell_3 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & \ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & \ell_4 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & \ell_4 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_3 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & \ell_3 C\theta_1 \\ S\theta_1 & 0 & C\theta_1 & \ell_3 S\theta_1 \\ 0 & -1 & 0 & \ell_1 + \ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & \ell_4 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & \ell_4 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_3 = \begin{bmatrix} C\theta_1 C\theta_3 & -C\theta_1 S\theta_3 & -S\theta_1 & \ell_3 C\theta_1 + \ell_4 C\theta_1 C\theta_3 \\ S\theta_1 C\theta_3 & -S\theta_1 S\theta_3 & C\theta_1 & \ell_3 S\theta_1 + \ell_4 S\theta_1 C\theta_3 \\ -S\theta_3 & -C\theta_3 & 0 & \ell_1 + \ell_2 - \ell_4 S\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Problem 4

The $A_i$ matrices for the 3 - DOF RRP manipulator is given below:

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ -\cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3 + 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If the hand frame relative to the base coordinate frame ${}^0T_3$ is given as:

$${}^0T_3 = \begin{bmatrix} 0.5198 & -0.4226 & -0.7424 & -8.7947 \\ 0.2424 & 0.9063 & -0.3462 & 1.4158 \\ 0.8192 & 0 & 0.5736 & 15.1622 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then determine the joint variables $\theta_1, \theta_2, \text{and}\, \ell_3$.

**GIVEN**

| Some General Analytical Inverse Kinematics Formulas | |
|---|---|
| IF | THEN |
| $\cos\theta = b$ | $\theta = A\tan 2\left(\pm\sqrt{1-b^2}, b\right)$ ;i.e., both $\theta$ and -$\theta$ |
| $\sin\theta = a$ | $\theta = A\tan 2\left(a, \pm\sqrt{1-a^2}\right)$ ;i.e., both $\theta$ and (180-$\theta$) |
| $\sin\theta = a$<br>$\cos\theta = b$ | $\theta = A\tan 2(a,b)$ |
| $a\cos\theta - b\sin\theta = 0$ | $= A\tan 2(a,b)$ *and* $\theta = A\tan 2(-a,-b)$ } i.e., both $\theta$ and ($\theta \pm 180$) |

$${}^0T_3 = A_1 A_2 A_3$$

$${}^0T_3 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ -\cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3 + 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{0}T_3 = \begin{bmatrix} C\theta_1 S\theta_2 & -S\theta_1 & -C\theta_1 C\theta_2 & -5S\theta_1 \\ S\theta_1 S\theta_2 & C\theta_1 & -S\theta_1 C\theta_2 & 5C\theta_1 \\ C\theta_2 & 0 & S\theta_2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3 + 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_3 = \begin{bmatrix} C\theta_1 S\theta_2 & -S\theta_1 & -C\theta_1 C\theta_2 & -(\ell_3 + 3) C\theta_1 C\theta_2 - 5S\theta_1 \\ S\theta_1 S\theta_2 & C\theta_1 & -S\theta_1 C\theta_2 & -(\ell_3 + 3) S\theta_1 C\theta_2 + 5C\theta_1 \\ C\theta_2 & 0 & S\theta_2 & (\ell_3 + 3) S\theta_2 + 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Given:

$$ {}^{0}T_3 = \begin{bmatrix} 0.5198 & -0.4226 & -0.7424 & -8.7947 \\ 0.2424 & 0.9063 & -0.3462 & 1.4158 \\ 0.8192 & 0 & 0.5736 & 15.1622 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \left. \begin{array}{l} C\theta_2 = 0.8192 \\ S\theta_2 = 0.5736 \end{array} \right\} \boxed{\theta_2 = a\tan 2(0.5736, 0.8192) = 35^o} $$

$$ \left. \begin{array}{l} S\theta_1 = 0.4226 \\ C\theta_1 = 0.9063 \end{array} \right\} \boxed{\theta_1 = a\tan 2\left(0.4226, 0.9063\right) = 25^o} $$

$$ (\ell_3 + 3) S\theta_2 + 10 = 15.1622 $$

$$ (\ell_3 + 3) S\theta_2 = 5.1622 $$

$$ \ell_3 = \frac{5.1622}{\sin(35)} - 3 $$

$$ \boxed{\ell_3 = 6} $$