Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG428 Introduction to Robotics

Instructor:

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Midterm EXAMINATION
Spring 2018-19

April 12, 2019

Duration: 100 minutes

Number of Problems: 4

Good Luck

STUDENT'S	
NUMBER	
NAME	SOLUTIONS
SURNAME	

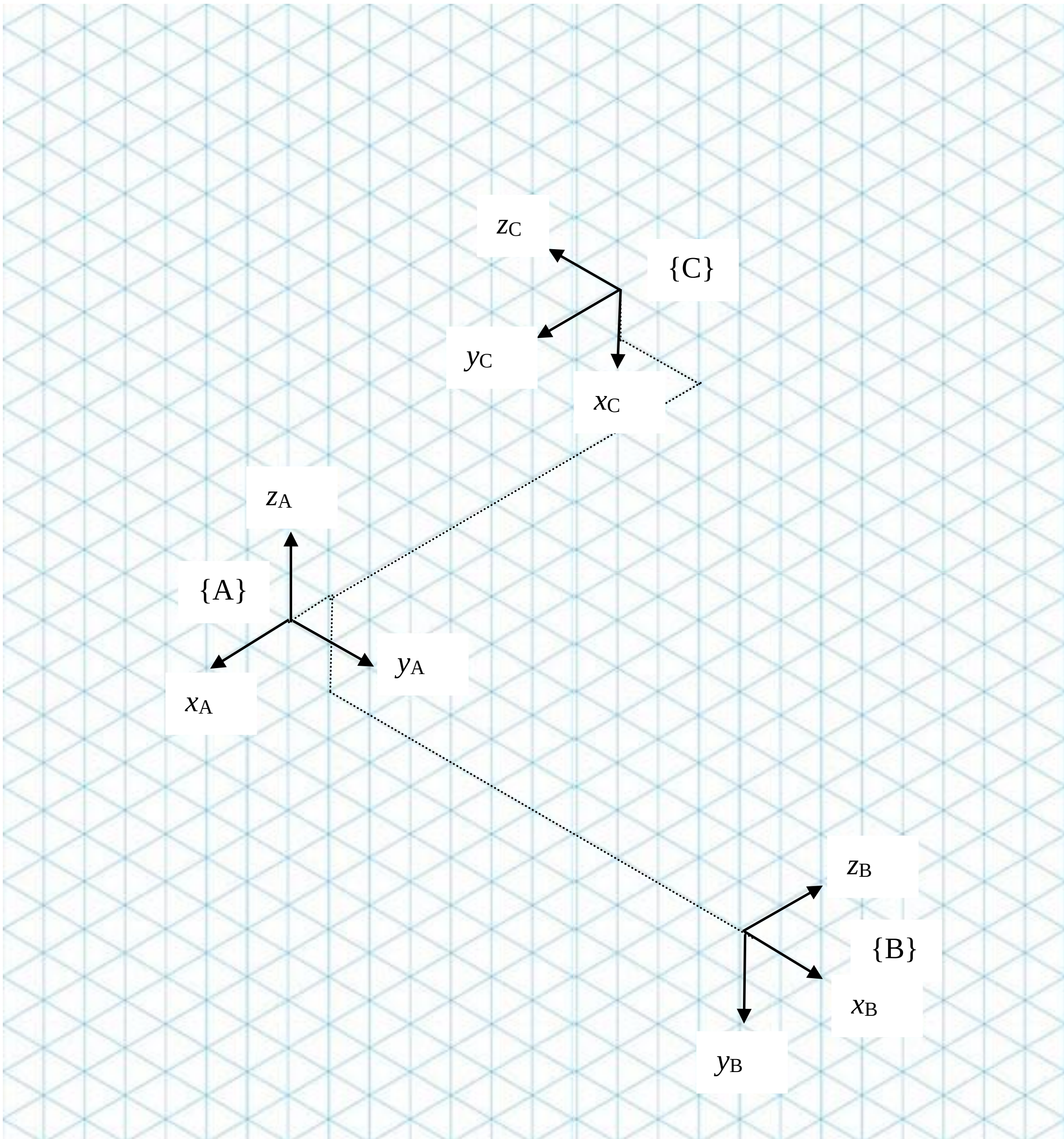
Problem		**Points**
1		25
2		25
3		25
4		25
TOTAL		100

Problem 1

The frames B and C are given relative to the frame A.

a) Draw the frames {A}, {B}, and {C} using the isometric grid given below.
b) Express ${}^{A}B$ in terms of two rotational and one translational transformation.
c) By inspection, determine ${}^{C}B$

$$ {}^{A}B = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & 10 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{A}C = \begin{bmatrix} 0 & 1 & 0 & -10 \\ 0 & 0 & -1 & -2 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



$$ {}^{A}B = Trans(-1,10,-2)Rot(z,90)Rot(x,-90) $$

$$ {}^{C}B = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & 9 \\ -1 & 0 & 0 & -12 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Problem 2

Assume that Roll, Pitch, Yaw (**RPY**) angles used with a robot are 90°, 90°, 90° respectively. Determine what angles should be used to achieve the same result if **Euler Angles** are used instead.

Given:

Some General Analytical Inverse Kinematics Formulas	
IF	THEN
$\cos\theta = b$	$\theta = a\tan 2\left(\pm\sqrt{1-b^2}, b\right)$; *i.e,.* both θ and $-\theta$

RPY(90,90,90)=Rot(z,90)Rot(y,90)Rot(x,90)

$$RPY(90,90,90) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Euler(\theta_1,\theta_2,\theta_3) = Rot(z,\theta_1)Rot(y,\theta_2)Rot(z,\theta_3) == \begin{bmatrix} C_1C_2C_3 - S_1S_3 & -C_1C_2S_3 - S_1C_3 & C_1S_2 & 0 \\ S_1C_2C_3 - C_1S_3 & -S_1C_2S_3 + C_1C_3 & S_1S_2 & 0 \\ -S_2C_3 & S_2S_3 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Cos}\,\theta_2 = 0 \Rightarrow \theta_{21} = 90, \theta_{22} = -90$$

$$\left.\begin{matrix} \mathrm{Cos}\,\theta_{11}\,\mathrm{Sin}\,\theta_{21} = 1 \\ \mathrm{Sin}\,\theta_{11}\,\mathrm{Sin}\,\theta_{21} = 0 \end{matrix}\right\} \theta_{11} = a\tan 2(0,1) = 0$$

$$\left.\begin{matrix} \mathrm{Cos}\,\theta_{12}\,\mathrm{Sin}\,\theta_{22} = 1 \\ \mathrm{Sin}\,\theta_{12}\,\mathrm{Sin}\,\theta_{22} = 0 \end{matrix}\right\} \theta_{12} = a\tan 2(0,-1) = 180$$

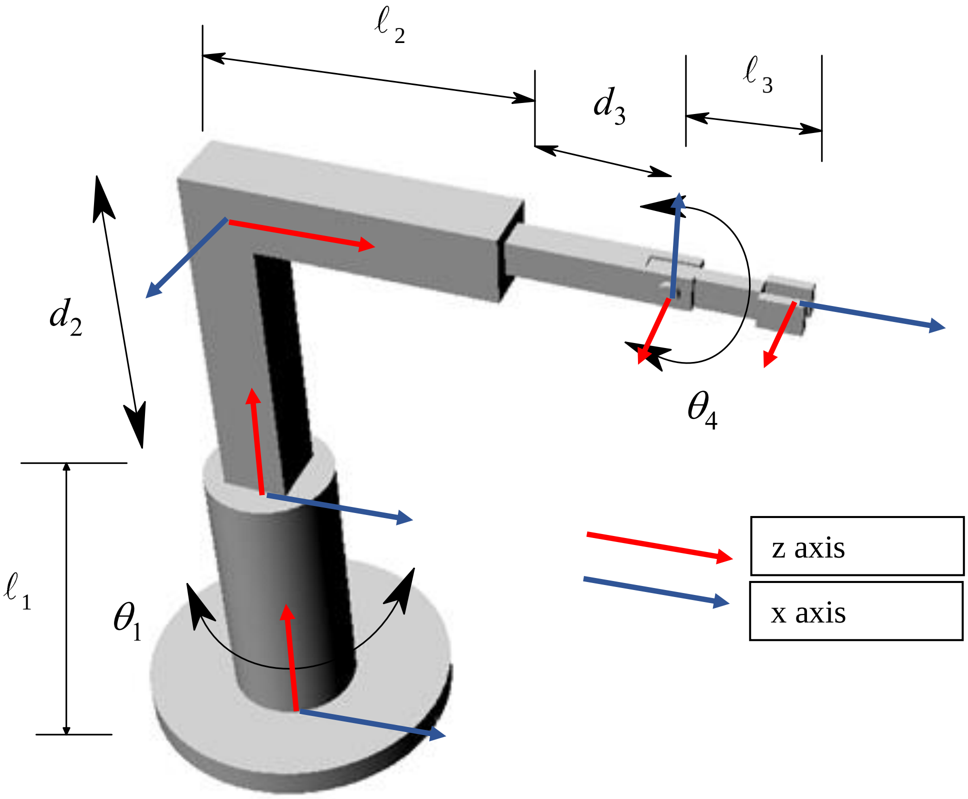
$$\left.\begin{matrix} \mathrm{Cos}\,\theta_{31}\,\mathrm{Sin}\,\theta_{21} = 1 \\ \mathrm{Sin}\,\theta_{31}\,\mathrm{Sin}\,\theta_{21} = 0 \end{matrix}\right\} \theta_{31} = a\tan 2(0,1) = 0$$

$$\left.\begin{matrix} \mathrm{Cos}\,\theta_{32}\,\mathrm{Sin}\,\theta_{22} = 1 \\ \mathrm{Sin}\,\theta_{32}\,\mathrm{Sin}\,\theta_{22} = 0 \end{matrix}\right\} \theta_{32} = a\tan 2(0,-1) = 180$$

Problem 3

For the given 4-DOF robot:

a) Assign appropriate frames for the Denavit-Hartenber representation.
b) Fill out the parameter table.
c) Derive the forward kinematic equation of the robot.



GIVEN:

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sin(\alpha \mp \beta) = \sin\alpha\cos\beta \mp \sin\beta\cos\alpha$$

$$\cos(\alpha \mp \beta) = \cos\alpha\cos\beta \pm \sin\beta\sin\alpha$$

Link Parameter Table:

Link$_i$	θ_i	d_i	a_i	α_i
1(0-1)	θ_1	ℓ_1	0	0
2(1-2)	-90	d_2	0	-90
3(2-3)	-90	$\ell_2 + d_3$	0	-90
4(3-4)	$\theta_4 - 90$	0	ℓ_3	0

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & \ell_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 + \ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} S_4 & S_4 & 0 & \ell_3 S_4 \\ -C_4 & S_4 & 0 & \ell_3 C_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4 = A_1 A_2 A_3 A_4 = \begin{bmatrix} C_1 C_4 & -S_4 C_1 & S_1 & (d_3 + \ell_2 + \ell_3 C_4) C_1 \\ C_4 S_1 & -S_1 S_4 & -C_1 & (d_3 + \ell_2 + \ell_3 C_4) S_1 \\ S_4 & C_4 & 0 & d_2 + \ell_1 + \ell_3 S_4 \\ 0 & 0 & & 1 \end{bmatrix}$$

Problem 4

The frame A was subjected to a differential rotation of $\boldsymbol{\delta} = \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix}$ radians relative to the **reference frame** and a differential translation of ${}^{A}\mathbf{d} = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix}$ units relative to **frame A.**

Find

a) the differential operator relative to frame **A**.
b) the differential operator relative to reference frame.
c) the differential transformation d**A**
d) the new location and orientation of the frame **A**.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 10 \\ 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}\delta_x = n.\delta = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix} = 0.1$$

$${}^{A}\delta_y = o.\delta = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix} = 0.1$$

$${}^{A}\delta_z = a.\delta = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix} = 0$$

$${}^{A}\Delta = \begin{bmatrix} 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & -0.1 & 0 \\ -0.1 & 0.1 & 0 & 0.2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Delta A = A^A\Delta \Rightarrow \Delta = A^A\Delta A^{-1}$$

$$\Delta = \begin{bmatrix} 0 & 0 & 1 & 10 \\ 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & -0.1 & 0 \\ -0.1 & 0.1 & 0 & 0.2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} -0.1 & 0.1 & 0 & 0.2 \\ 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -0.1 & 0.1 & 0.4 \\ 0.1 & 0 & 0 & -0.9 \\ -0.1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$dA = A^A\Delta = \begin{bmatrix} -0.1 & 0.1 & 0 & 0.2 \\ 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{new} = A + dA = \begin{bmatrix} 0 & 0 & 1 & 10 \\ 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -0.1 & 0.1 & 0 & 0.2 \\ 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{new} = \begin{bmatrix} -0.1 & 0.1 & 1 & 10.2 \\ 1 & 0 & 0.1 & 5.1 \\ 0 & 1 & -0.1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$