



## EENG 428 Introduction to Robotics

### Tutorial 2

#### Question 1

Given a coordinate frame  $\mathbf{T}$  and  $\mathbf{T}_{new}$  which is the new location of the frame after a differential transformation

$$\mathbf{T}_{new} = \begin{bmatrix} -1 & -0.2 & -0.1 & 2 \\ -0.1 & 0 & 1 & -1.5 \\ -0.2 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Determine the differential translation  $\mathbf{d}$  and rotation  $\delta$  made with respect to  $\mathbf{T}$ .
- What is the equivalent translation and rotation with respect to base coordinate frame?

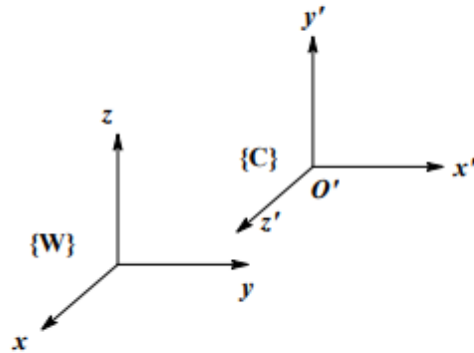
## Question 2

The hand frame of a robot and the corresponding Jacobian are given. For the given differential changes of the joints, compute the change in the hand frame, its new location and corresponding  $\Delta$ .

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & 18.66 \\ 0 & 0 & -1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^0J = \begin{bmatrix} -18.66 & -10 & 0 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}, \quad dq = \begin{bmatrix} 0 \\ 0.1 \\ 0.2 \\ -0.2 \end{bmatrix}$$

### Question 3

The world coordinate frame  $W(x, y, z)$  and the coordinate frame  $C(x', y', z')$  are given as shown below. The origin  $O'$  of frame  $C$  is given by  $O' = [1 \ 4 \ 2]^T$  with respect to world coordinate system.



- (a) Find the representation of frame  $C$  with respect to world coordinate frame  $W$  ( ${}^W T_C$ ).

$${}^W T_C = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (b) Calculate  ${}^W D$ ,  ${}^W \Delta$ ,  $d{}^W T_C$ ,  ${}^C D$ ,  ${}^C \Delta$ , and  ${}^C M_W$  corresponding to a differential translation 0.5 unit in  $x'$ -direction, differential rotations 0.2 radians around the  $y'$ -axis and  $-0.1$  radian around the  $z'$ -axis of the frame  $C$ .

#### **Question 4**

For a three-link cylindrical manipulator, derive the Jacobian with respect to base coordinate frame.

The link parameters of the manipulator shown in the following table.

Link	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1	$a_1$	0	0	$\theta_1^*$
2	$a_2$	180	0	$\theta_2^*$
3	0	0	$d_3^*$	0

### Question 5

For a two-link planar manipulator, it is given that

$$T_2=A_1A_2 \text{ where } A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1C_1 \\ S_1 & C_1 & 0 & l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is desired that at the end of manipulator we have the force and moment vectors given

$$\text{by } {}^2f = \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix} \quad {}^2M = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Determine the joint torques  $\tau_1$  and  $\tau_2$  required.

### **Question 6**

A fifth-order polynomial is to be used to control the motions of the joints of a robot in joint-space. Find the coefficients of a fifth-order polynomial that allow a joint to go from initial angle of  $30^\circ$  to a final angle of  $90^\circ$  in 6 seconds, while the initial and final velocities are zero and initial acceleration and final decelerations are  $10 \text{ degrees/sec}^2$  and plot the position, velocity and acceleration curves for the motion.

### **Question 7**

Joint 1 of a 6-axis robot is to go from an initial angle of  $\theta_i = 30^\circ$  to the final angle of  $\theta_f = 120^\circ$  in 4 seconds with a cruising velocity of  $\omega_1 = 30^\circ/sec$ . Find the necessary blending time for a trajectory with linear segments and parabolic blends and plot the joint positions, velocities, and accelerations.