Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EEE 223 Circuit Theory I

Fall 2005-06

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Midterm EXAMINATION

Dec 02, 2005

Duration : 100 minutes

Number of Problems: 4

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	
GROUP NO	

Problem		**Points**
1		25
2		25
3		25
4		25
TOTAL		100

1. (a) Use nodal analysis o find the branch currents i_1, i_2 and i_3 in the circuit in Fig.P1.
   (b) Check your solution for i_1, i_2 and i_3 by showing that the power dissipated in the circuit equals the power developed.

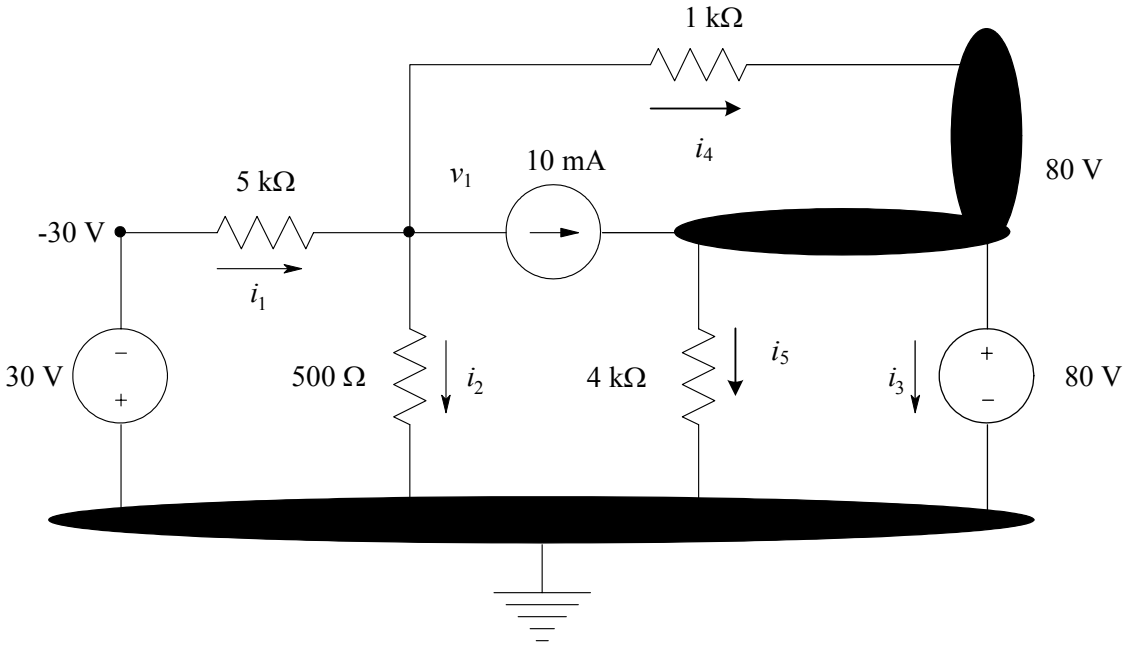


Figure P1

KCL at v_1:

$$-i_1 + i_2 + i_4 + 10m = 0$$

$$-\frac{-30 - v_1}{5k} + \frac{v_1}{0.5k} + \frac{v_1 - 80}{1k} = -10m$$

multipy both sides by 5k yields

$$30 + v_1 + 10v_1 + 5v_1 - 400 = -50$$

$$16v_1 = 320$$

$$v_1 = \frac{320}{16} = 20 \text{ V}$$

$$i_1 = \frac{-30 - 20}{5k} = -10mA$$

$$i_2 = \frac{20}{0.5k} = 40mA$$

$$i_3 = i_4 + 10m - i_5 = \frac{20 - 80}{1k} + 10m - \frac{80}{4k}$$

$$i_3 = -60m + 10m - 20m = -70mA$$

$$P_{30V} + P_{5k} + P_{0.5k} + P_{10m} + P_{1k} + P_{4k} + P_{80V} = 0$$

$$30(-10m) + (-10m)^2 5k + (40m)^2 0.5k +$$

$$\frac{(20-80)^2}{1k} + 10m(20-80) + \frac{80^2}{4k} + 80(-70m) = 0$$

$$-300m + 500m + 800m + 3600m - 600m$$

$$+1600m - 5600m = 0$$

2. For the circuit in Fig. P2, find the value of v_0 using mesh analysis.

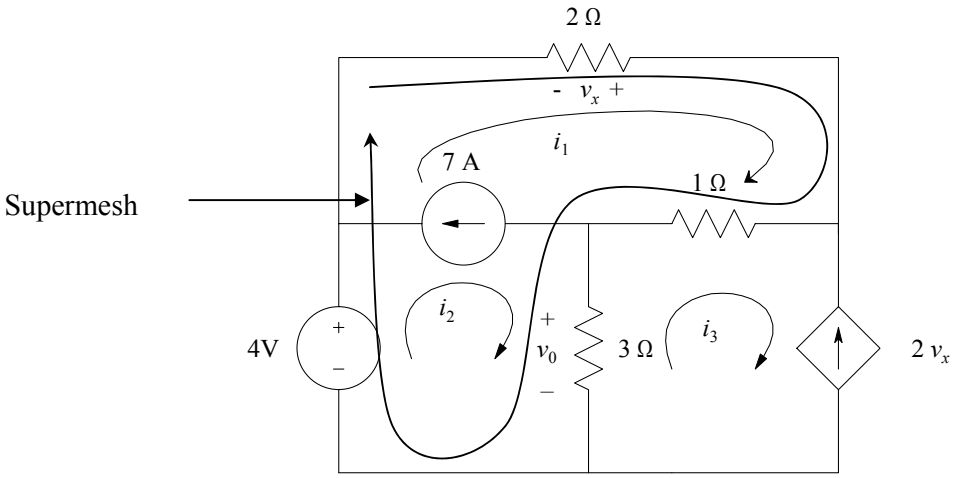


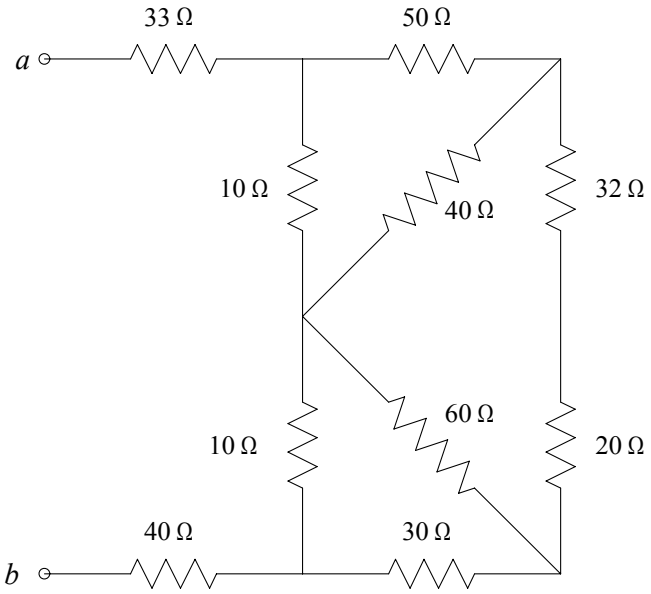
Figure P2

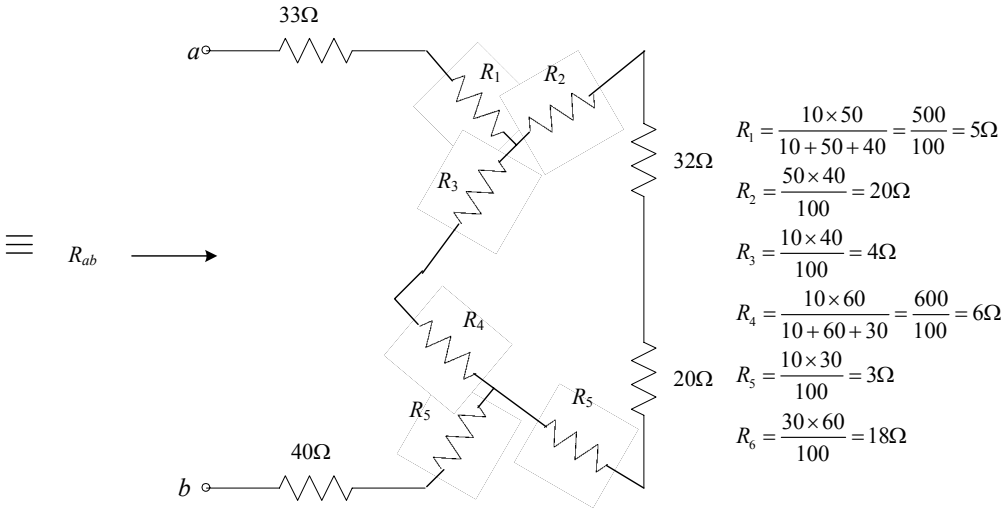
$$v_x = -2i_1$$
$$i_3 = -2v_x = -2\left(-2i_1\right) = 4i_1$$
$$i_1 - i_2 = 7$$
$$\boxed{i_2 = i_1 - 7}$$

KVL around the supermesh:

$$2i_1 + 1(i_1 - 4i_1) + 3(i_1 - 7 - 4i_1) - 4 = 0$$
$$\left(2 + 1 - 4 + 3 - 12\right)i_1 = 25$$
$$-10i_1 = 25$$
$$\boxed{i_1 = -2.5A}$$
$$v_0 = 3(i_1 - 7 - 4i_1)$$
$$\boxed{v_0 = 1.5\ V}$$

3. Find the equivalent resistance R_{ab} in the circuit in Fig. P3.





$$R_1 = \frac{10\times 50}{10+50+40} = \frac{500}{100} = 5\Omega$$
$$R_2 = \frac{50\times 40}{100} = 20\Omega$$
$$R_3 = \frac{10\times 40}{100} = 4\Omega$$
$$R_4 = \frac{10\times 60}{10+60+30} = \frac{600}{100} = 6\Omega$$
$$R_5 = \frac{10\times 30}{100} = 3\Omega$$
$$R_6 = \frac{30\times 60}{100} = 18\Omega$$

$$R_{ab} = 33 + R_1 + \left(R_3 + R_4\right) // \left(R_2 + 32 + 20 + R_6\right) + R_5 + 40$$
$$R_{ab} = 33 + 5 + \left(4 + 6\right) // (20 + 32 + 20 + 18) + 3 + 40$$
$$R_{ab} = 33 + 5 + 9 + 3 + 40 = 90\Omega$$

4. The variable resistor in the circuit in Fig. P4 is adjusted for maximum power transfer to R_0.

   a. Find the value of R_0.
   b. Find the maximum power that can be delivered to R_0.

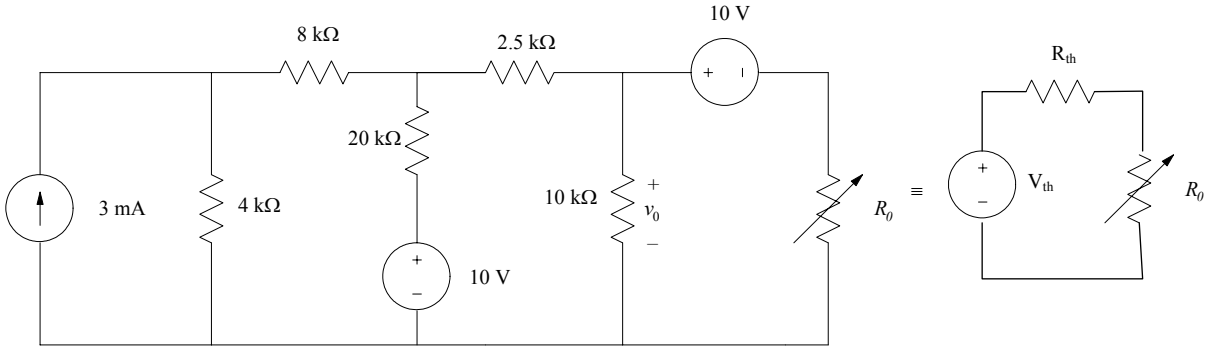
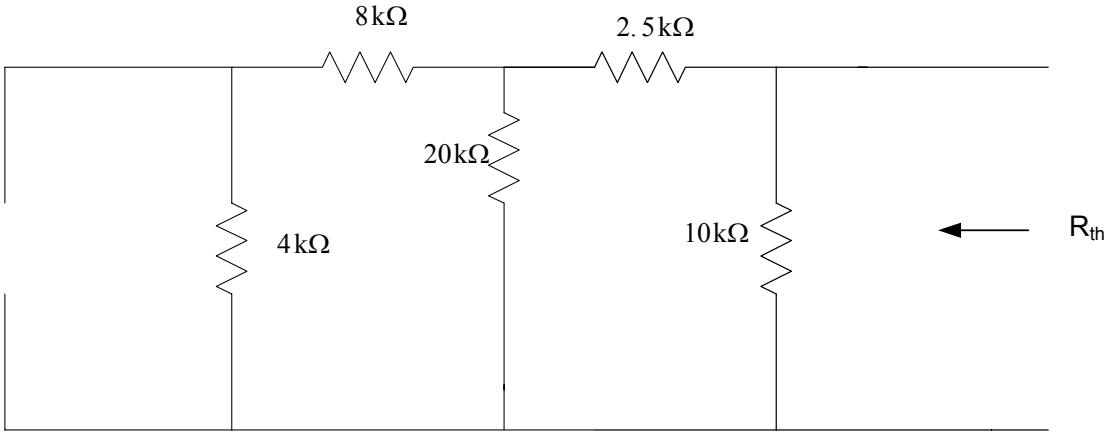


Figure P4

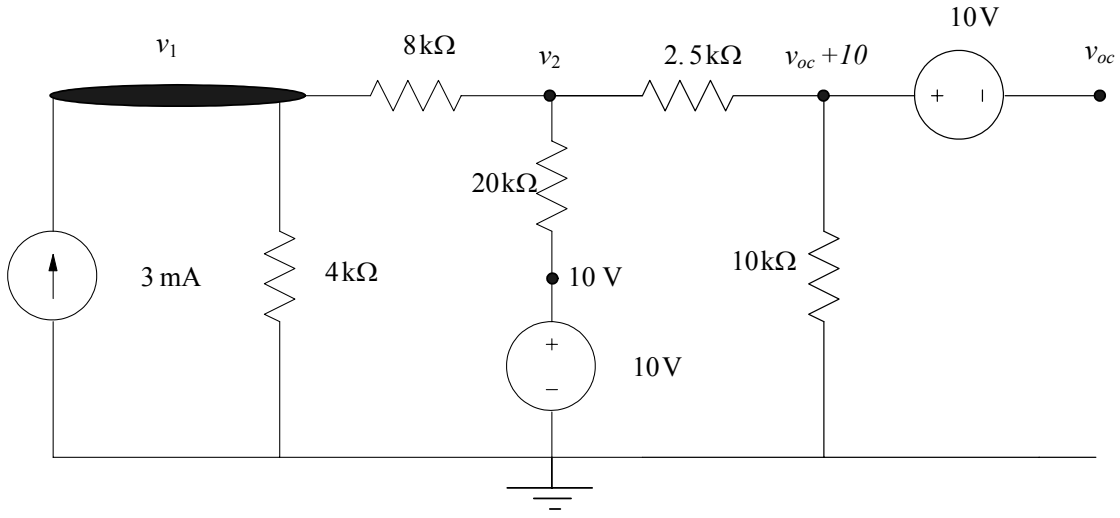
In order to find R_{th} all the independent source values are set to 0.



$$R_{th} = \left(\left(8k + 4k\right) // \, 20k + 2.5k\right) // 10k$$

$$R_{th} = \left(7.5k + 2.5k\right) // 10k$$

$$\boxed{R_{th} = 5k\Omega}$$



KCL at v_1 :

$$\left(\frac{1}{4k}+\frac{1}{8k}\right)v_1-\frac{1}{8k}v_2=3m$$

multiply both sides by 8k yields:

$$3v_1-v_2=24$$

$$v_1=8+\frac{1}{3}v_2 \text{.....(1)}$$

KCL at v_2:

$$-\frac{1}{8k}v_1+\left(\frac{1}{8k}+\frac{1}{20k}+\frac{1}{2.5k}\right)v_2-\frac{1}{20k}10-\frac{1}{2.5k}\left(v_{oc}+10\right)=0$$

multiply both sides by 40k yields:

$$-5v_1+23v_2-16v_{oc}=180 \text{.....(2)}$$

KCL at v_3:

$$-\frac{1}{2.5k}v_2+\left(\frac{1}{2.5k}+\frac{1}{10k}\right)\left(v_{oc}+10\right)=0$$

multiply both sides by 10:

$$-4v_2+5v_{oc}=-50$$

$$v_{oc}=-10+\frac{4}{5}v_2 \text{.....(3)}$$

Subst. Eqs. (1) and (3) into (2) gives:

$$-5\left(8+\frac{1}{3}v_2\right)+23v_2-16\left(-10+\frac{4}{5}v_2\right)=180$$

$$\left(-\frac{5}{3}+23-\frac{64}{5}\right)v_2=180+40-160=60$$

$$\left(-25+345-192\right)v_2=900$$

$$128v_2=900$$

$$v_2=7.03125V$$

$$v_{oc}=-10+0.8(7.03125)=-4.375V$$

When $R_0 = R_{\text{th}} = 5\text{k}\Omega$ it absorbs maximum power.

$$P_{\max}=\frac{V_{oc}^{\ 2}}{4R_{th}}=\frac{\left(-4.375\right)^2}{4(5k)}=0.957mW$$