Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 Circuit Theory I

Fall 2006-07

<u>Instructor:</u>

M. K. Uyguroğlu

Final EXAMINATION

Jan 19, 2007

Duration : 150 minutes

Number of Problems: 4

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	SOLUTIONS
GROUP NO	

Problem		Points
1		25
2		25
3		25
4		25
TOTAL		100

1. Find i in the circuit in Fig.P1 using Superposition.

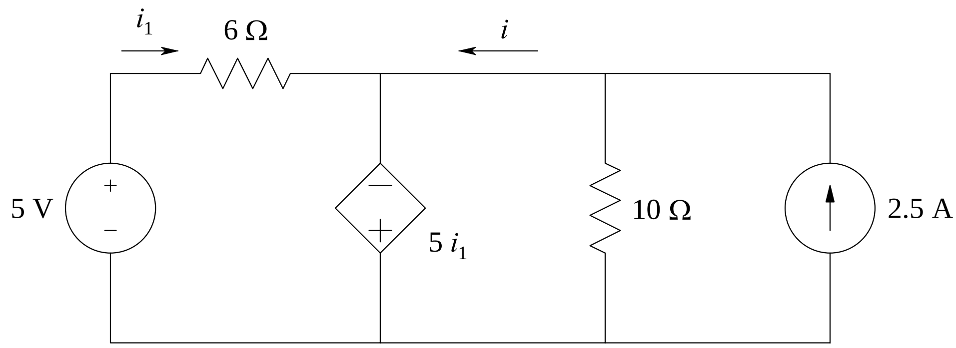
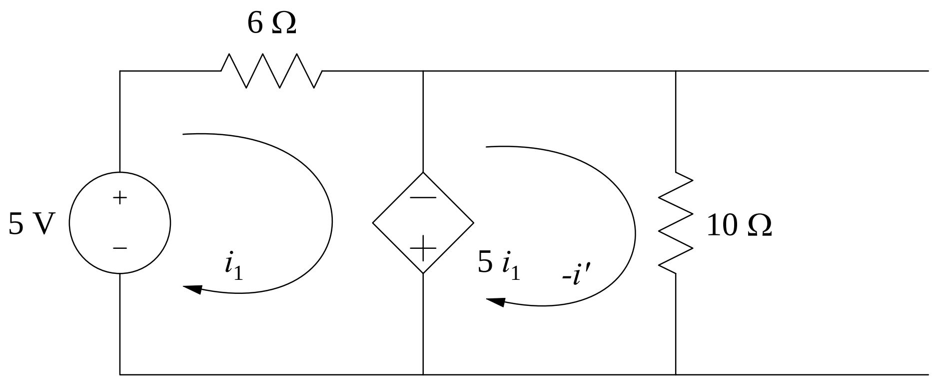


Figure P1

5 V Voltage source is active:



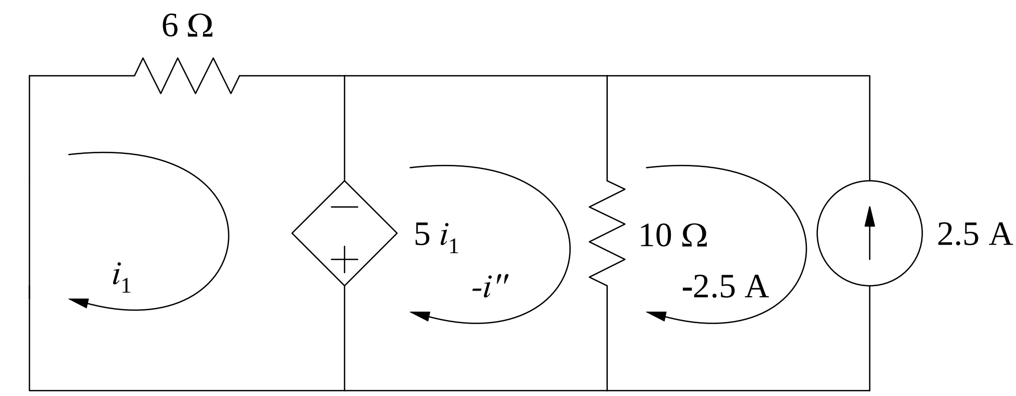
KVL around i_1:

$$6i_1 = 5 + 5i_1$$
$$i_1 = 5 \text{ A}$$

KVL around $-i'$:

$$10(-i') = -5i_1$$
$$i' = 0.5i_1 = 0.1(5) = 2.5 \text{ A}$$

2.5 A Current Source is active:



KVL around i_1:

$$6i_1 = 5i_1$$
$$i_1 = 0$$

KVL around $-i''$:

$$10(-i'') - 10(-2.5) = -5i_1 = 0$$
$$i'' = 2.5 \text{ A}$$

$$i = i' + i'' = 2.5 + 2.5 = 5 \text{ A}$$

2. Find the value of R that will draw the maximum power from the rest of the circuit in Fig. P2. Also find the maximum power.

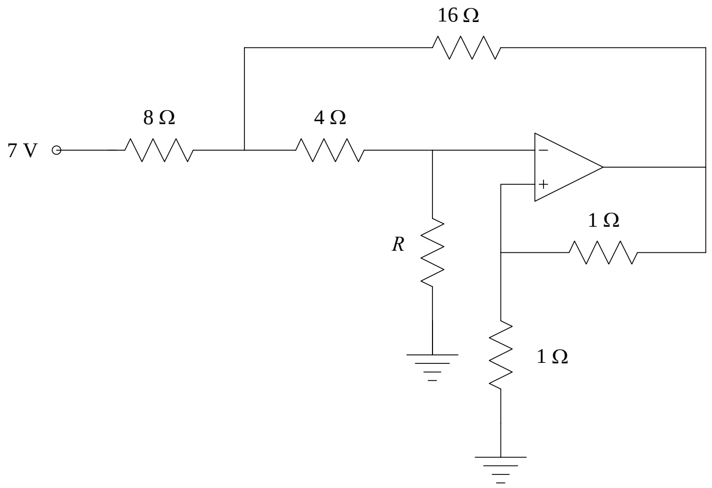
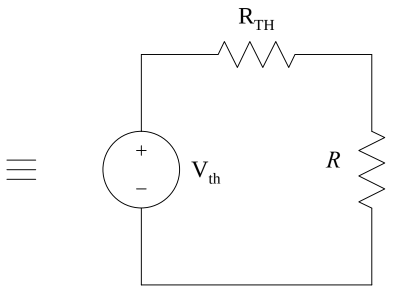


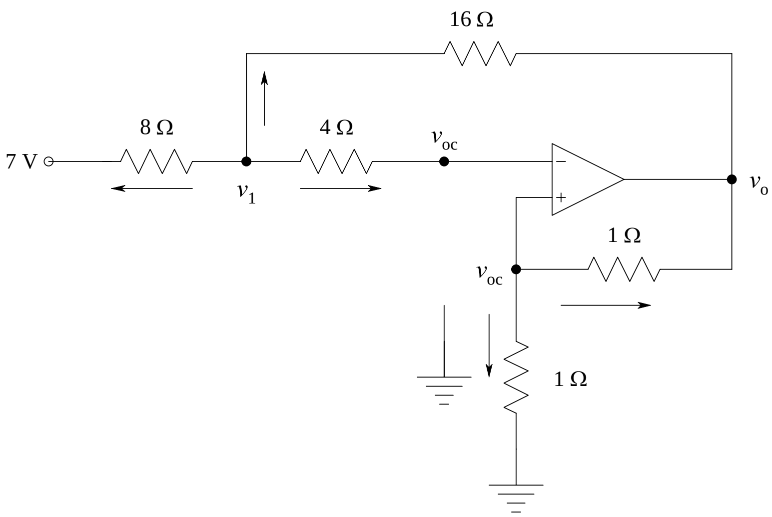
Figure P2



When $R = R_{TH}$ it will draw maximum power.

$$P_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

In order to find V_{TH} we will find open circuit voltage.



KCL at the non-inverting input terminal:

$$\frac{v_{oc}}{1} + \frac{v_{oc} - v_o}{1} = 0$$

$$v_o = 2v_{oc}$$

KCL at the inverting input terminal:

$$\frac{v_1 - v_{oc}}{4} = 0 \Rightarrow v_1 = v_{oc}$$

KCL at v_1:

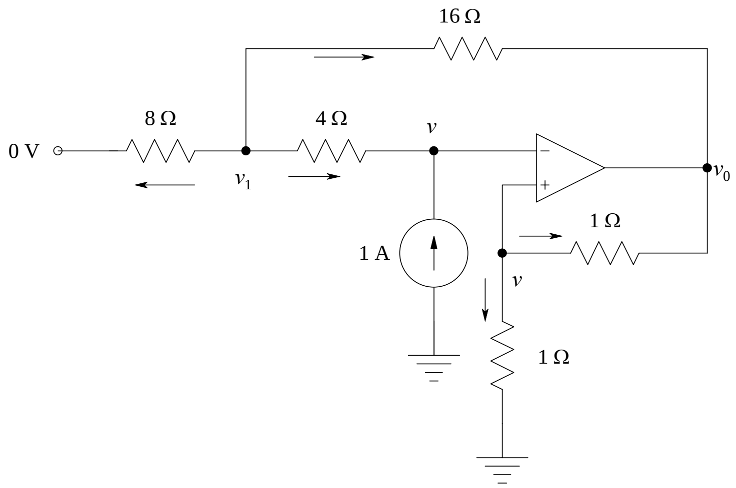
$$\frac{v_1 - 7}{8} + \frac{v_1 - v_{oc}}{4} + \frac{v_1 - v_o}{16} = 0$$

multiply both sides by 16 yields:

$$2v_1 - 14 + 4v_1 - 4v_{oc} + v_1 - v_o = 0$$

$$v_{oc} = 14 \text{ V}$$

In order to find R_{TH}:



KCL at the non-inverting input terminal:

$$\frac{v - v_0}{1} + \frac{v}{1} = 0$$

$$v_0 = 2v$$

KCL at the inverting input terminal:

$$\frac{v - v_1}{4} = 1 \Rightarrow v_1 = v - 4$$

KCL at v_1:

$$\frac{v_1}{8} + \frac{v_1 - v}{4} + \frac{v_1 - v_0}{16} = 0$$

multiply both sides by 16:

$$2v_1 + 4v_1 - 4v + v_1 - v_0 = 0$$

$$7(v - 4) - 4v - 2v = 0$$

$$v = 28 \text{ V}$$

$$R_{TH} = \frac{v}{1} = 28\Omega$$

When $R = 28\Omega$ it will draw maximum power.

$$P_{\max} = \frac{14^2}{4 \times 28} = 7/4W$$

3. Find $v(t)$ for $t \geq 0$ in the circuit in Fig.P3 if $i(0) = 1$ A . (25 pts.)

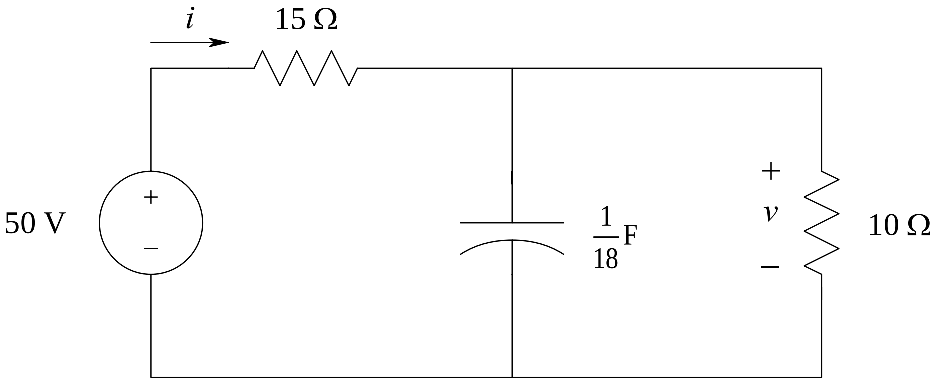


Figure P3

Since 10Ω resistor and the capacitor are in parallel, the same voltage is present across the capacitor as well. Therefore

$$v(t) = v(\infty) + \left[v(0) - v(\infty)\right]e^{-\frac{t}{\tau}} \text{ for } t \geq 0$$

In order to find $v(t)$ we need to determine $v(0)$, $v(\infty)$ and the time constant τ.

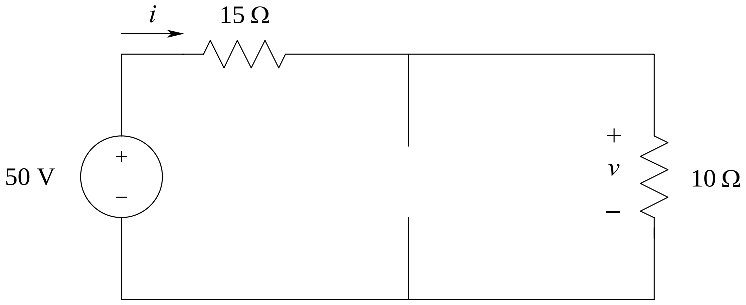
KVL around the outer loop gives:

$$50 = 15i + v$$

At $t = 0$:

$$50 = 15i(0) + v(0) \Rightarrow v(0) = 50 - 15(1) = 35 \text{ V}$$

At $t = \infty$, the circuit is under dc conditions:

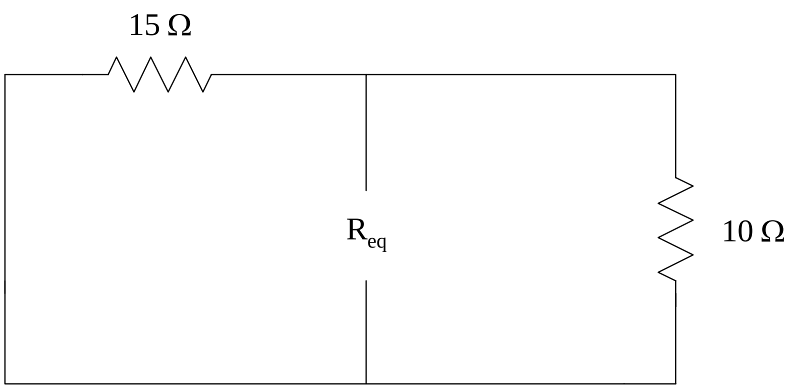


$$v(\infty) = 50\frac{10}{10+15} = 20\,\text{V}$$

For the time constant:

$$\tau = R_{eq}C$$

R_{eq} is the resistance seen by the capacitor.



$$R_{eq} = 10//15 = \frac{10\times15}{10+15} = 6\Omega$$

$$\tau = 6\times\frac{1}{18} = \frac{1}{3}s$$

$$v(t) = 20 + \left[35 - 20\right]e^{-3t}$$

$$v(t) = 20 + 15e^{-3t}\ V \text{ for } t \geq 0.$$

4. Find $i(t)$ for $t \geq 0$ in the circuit in Fig. P4 if $i(0) = 4$ A and $v(0) = 8$. (25 pts.)

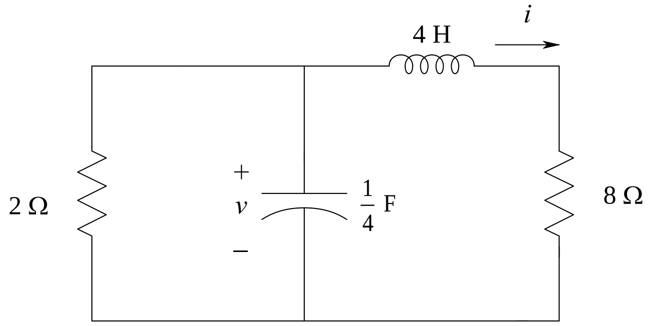
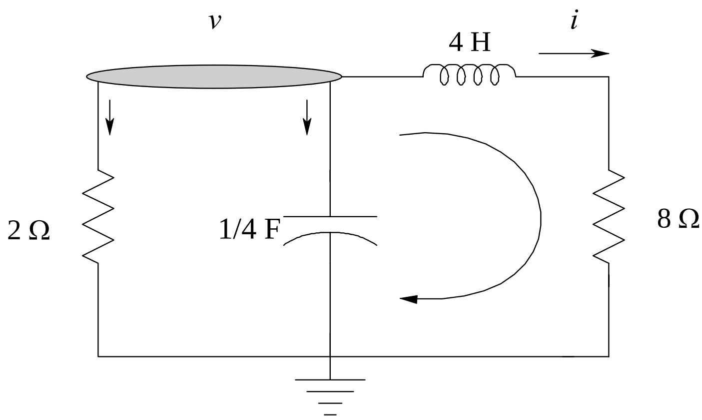


Figure P4



KCL at *v*:

$$\frac{v}{2} + \frac{1}{4}\frac{dv}{dt} + i = 0 ....(1)$$

KVL around the loop:

$$-v + 4\frac{di}{dt} + 8i = 0 ....(2)$$

$$v = 4\frac{di}{dt} + 8i ....(3)$$

Subst. Eq.(3) into (1) yields:

$$\frac{1}{2}\left(4\frac{di}{dt} + 8i\right) + \frac{1}{4}\frac{d}{dt}\left(4\frac{di}{dt} + 8i\right) + i = 0$$

$$2\frac{di}{dt} + 4i + \frac{d^2 i}{dt^2} + 2\frac{di}{dt} + i = 0$$

$$\boxed{\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 5i = 0}$$

If we write Eq.(2) at $t = 0$:

$$-v(0) + 4\frac{di(0)}{dt} + 8i(0) = 0$$

$$\frac{di(0)}{dt} = \frac{1}{4}\left(v(0) - 8i(0)\right) = \frac{1}{4}(8 - 32) = -6A/s$$

Characteristic equation:

$$s^2 + 4s + 5 = 0$$

$$s_{1,2} = \frac{-4 \mp \sqrt{16 - 20}}{2} = -2 \mp j$$

Therefore

$$i(t) = e^{-2t}\left(A\cos t + B\sin t\right)$$
$$i(0) = 4 = A$$
$$\frac{d}{dt}i(t) = -2e^{-2t}\left(A\cos t + B\sin t\right) + e^{-2t}\left(-A\sin t + B\cos t\right)$$
$$\frac{d}{dt}i(0) = -6 = -2\left(A\right) + \left(B\right) \Rightarrow B = 2$$
$$\therefore \ i(t) = e^{-2t}\left(4\cos t + 2\sin t\right) \ \text{ for } t \geq 0$$