Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 Circuit Theory I

Fall 2007-08

<u>Instructor:</u>

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Final EXAMINATION

Jan 15, 2008

Duration : 120 minutes

Number of Problems: 4

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	
GROUP NO	

Problem		**Points**
1		30
2		20
3		25
4		25
TOTAL		100

1. Find v_0 in the circuit in Fig. P1,
   a) using nodal analysis. (15 pts.)
   b) using mesh analysis. (15 pts.)

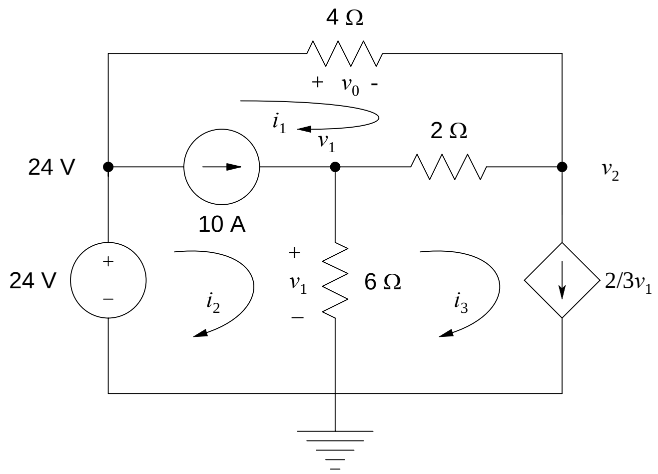


Figure P1

<u>Using Nodal Analysis</u>

KCL at v_1:

$$\left(\frac{1}{6}+\frac{1}{2}\right)v_1-\frac{1}{2}v_2=10$$

Multiply both sides by 6 yields:

$$4v_1-3v_2=60.................(1)$$

KCL at v_2:

$$-\frac{1}{2}v_1+\left(\frac{1}{2}+\frac{1}{4}\right)v_2-\frac{1}{4}(24)=-\frac{2}{3}v_1$$

Multiply both sides by 12 gives:

$$-6v_1+9v_2-72=-8v_1$$

or

$$2v_1+9v_2=72...................(2)$$

Muliply Eq.(2) by -2 and add to Eq.(1) gives:

$$-21v_2=-84 \quad \Rightarrow v_2=4\ V$$

$$v_0=24-v_2=20V$$

<u>Using Mesh Analysis</u>

$$i_3=\frac{2}{3}v_1=\frac{2}{3}(6)(i_2-i_3)=4i_2-4i_3$$

$$i_3=\frac{4}{5}i_2.............(1)$$

$$i_2-i_1=10$$

$$i_1=i_2-10...............(2)$$

Mesh 1 and mesh 2 constitute a SUPERMESH.
KVL around the SUPERMESH:

$$4i_1+2(i_1-i_3)+6(i_2-i_3)-24=0$$

$$4(i_2-10)+2\left(i_2-10-\frac{4}{5}i_2\right)+6\left(i_2-\frac{4}{5}i_2\right)=24$$

$$\left(4+2-\frac{8}{5}+6-\frac{24}{5}\right)i_2=24+40+20$$

$$(12-6.4)i_2=84$$

$$5.6i_2=84$$

$$i_2=\frac{84}{5.6}=15\,A$$

$$v_0=4(i_2-10)=20V$$

2. Find the value of R that will draw the maximum power from the rest of the circuit in Fig. P2. Also find the maximum power. (20 pts.)

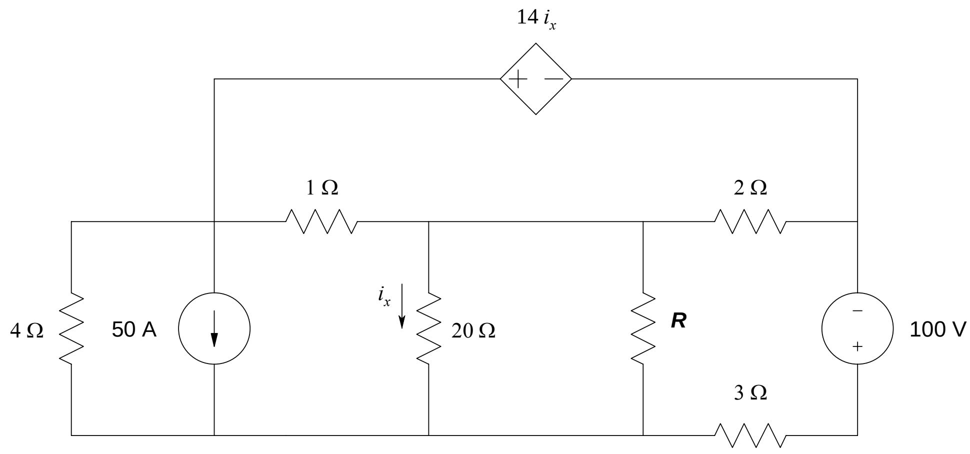
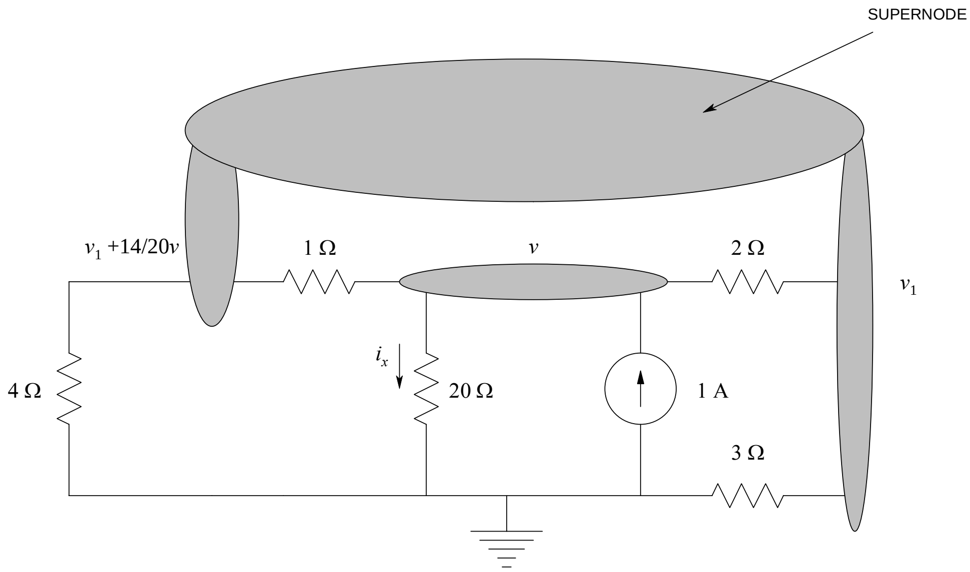


Figure P2

In order to find R_{TH} :



KCL at v :

$$\left(\frac{1}{2}+1+\frac{1}{20}\right)v-\frac{1}{2}v_1-\left(v_1+\frac{14}{20}v\right)=1$$

Multiply both sides by 20 yields:

$$(10+20+1)v-10v_1-20v_1-14v=20$$

$$17v-30v_1=20..............(1)$$

<u>KCL at the SUPERNODE:</u>

$$\left(\frac{1}{4}+1\right)\left(v_1+\frac{14}{20}v\right)+\left(\frac{1}{2}+\frac{1}{3}\right)v_1-\left(1+\frac{1}{2}\right)v=0$$

$$\left(\frac{5}{4}v_1+\frac{7}{8}v\right)+\left(\frac{1}{2}+\frac{1}{3}\right)v_1-\left(1+\frac{1}{2}\right)v=0$$

Multiply both sides by 24 yields:

$$30v_1+21v+20v_1-36v=0$$

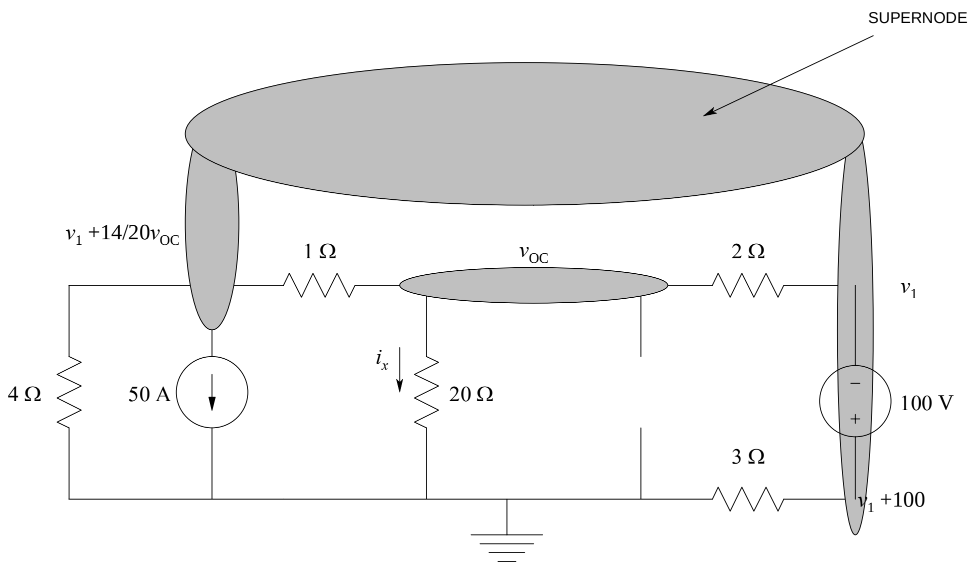
$$-15v+50v_1=0..............(2)$$

From Eq. (1) and (2) we can obtain v as:

$$\boxed{v=2.5V}$$

$$R_{TH}=\frac{v}{1}=2.5\Omega$$

For V_{TH}:



KCL at v_{oc}:

$$\left(\frac{1}{2}+1+\frac{1}{20}\right)v_{oc}-\frac{1}{2}v_1-\left(v_1+\frac{14}{20}v_{oc}\right)=0$$

Multiply both sides by 20 yields:

$$(10+20+1)v_{oc}-10v_1-20v_1-14v_{oc}=0$$

$$17v_{oc}-30v_1=0.............(1)$$

KCL at the SUPERNODE:

$$\left(\frac{1}{4}+1\right)\left(v_1+\frac{14}{20}v_{oc}\right)+\left(\frac{1}{2}\right)v_1+\left(\frac{1}{3}\right)(v_1+100)-\left(1+\frac{1}{2}\right)v_{oc}=-50$$

$$\left(\frac{5}{4}v_1+\frac{7}{8}v_{oc}\right)+\left(\frac{1}{2}\right)v_1+\left(\frac{1}{3}\right)(v_1+100)-\left(1+\frac{1}{2}\right)v_{oc}=-50$$

Multiply both sides by 24 yields:

$$30v_1+21v_{oc}+12v_1+8v_1+800-36v_{oc}=-1200$$

$$-15v_{oc}+50v_1=-2000...............(2)$$

From Eq. (1) and (2) v_{oc} is obtained as:

$$v_{oc}=V_{TH}=-150\,V$$

When the value of $R=R_{TH}$ then it draws maximum power and the maximum power is:

$$P_{\max}=\frac{V^2{}_{TH}}{4R_{TH}}=\frac{22500}{10}=2250\,W$$

3. In the circuit given in Fig. P3, label (insert with values) the voltages V_1, V_2, V_3, and resistors(in ohms), so that the output voltage $V_0 = 3V_1 - 2V_2 - V3$.

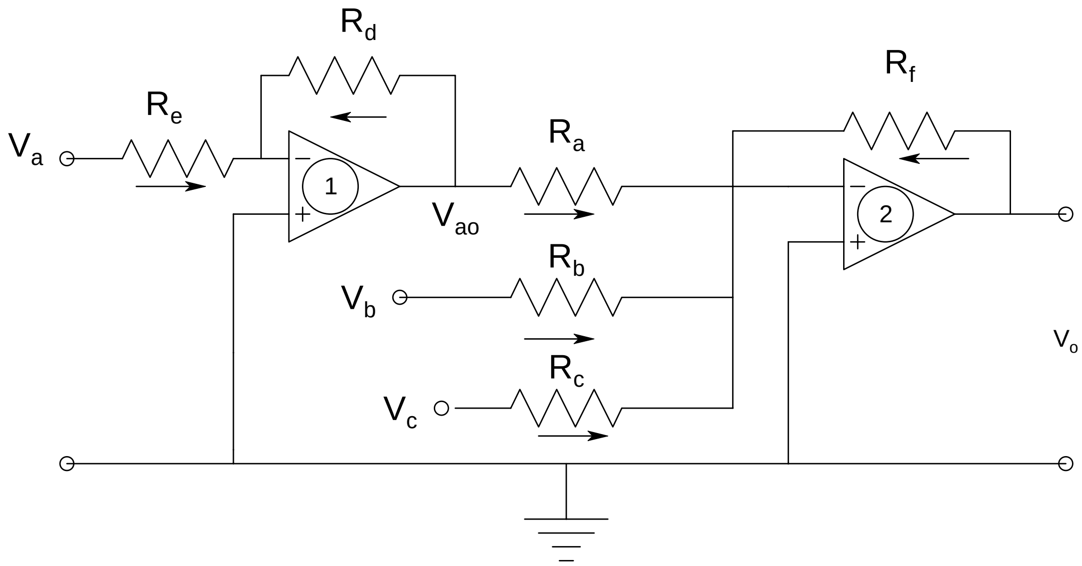


Figure P3

KCL at the inverting input terminal of OP-AMP 1

$$\frac{V_a}{R_e} + \frac{V_{ao}}{R_d} = 0 \Rightarrow V_{ao} = -\frac{R_d}{R_e} V_a$$

KCL at the inverting input terminal of OP-AMP 2

$$\frac{V_{ao}}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} + \frac{V_o}{R_f} = 0 \Rightarrow V_{ao} = -\frac{R_f}{R_a} V_{ao} - \frac{R_f}{R_b} V_b - \frac{R_f}{R_c} V_c = \frac{R_f}{R_a}\frac{R_d}{R_e} V_a - \frac{R_f}{R_b} V_b - \frac{R_f}{R_c} V_c$$

Therefore

$V_1 = V_a$

$V_2 = V_b$

$V_3 = V_c$

And

$$\frac{R_f}{R_a}\frac{R_d}{R_e} = 3$$

$$\frac{R_f}{R_b} = 2 = R_f = 2R_b$$

$$\frac{R_f}{R_c} = 1 \Rightarrow R_f = R_c$$

if we choose $R_f = 150\Omega$ then $R_c = 150\Omega$, $R_b = 75\Omega$ and
if we choose $R_d = R_e = 100\Omega$ then $R_a = 50\Omega$

4. In the circuit given in Fig. P4, switches S_1 and S_2 are open for a long time. When t=0 switch S_1 is closed while switch S_2 is kept open. After 10 seconds the switch S_2 is ALSO closed.
   a. Find i(t) flowing through the inductor for t>0
   b. The voltage across the 4-Ω resistor when t = 2 seconds
   c. The voltage across the 4-Ω resistor when t = 12 seconds.

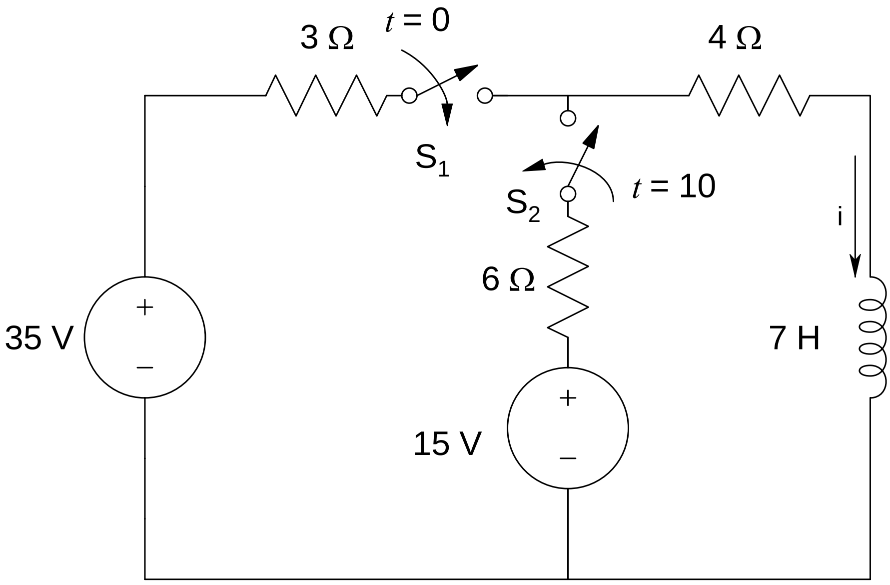
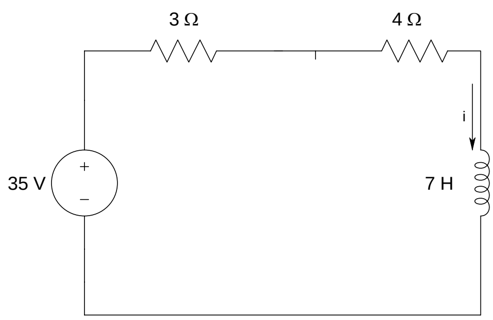


Figure P4

$t < 0$

$i(t) = 0\ A$

$0 < t < 10$s



$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-\frac{t}{\tau}}$$

Where $i(\infty) = \dfrac{35}{7} = 5\,A$ (since the inductor acts like a short circuit at $t = \infty$)

And

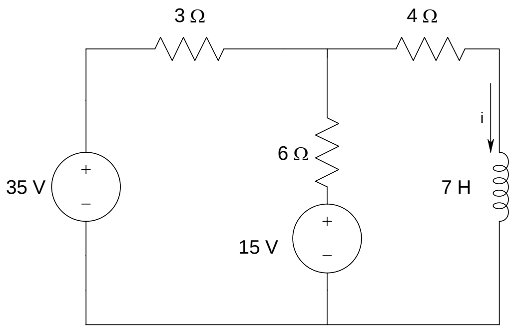
$$\tau = \frac{L}{R_{eq}} = \frac{7}{7} = 1s$$

Therefore

$i(t) = 5 + [0 - 5]e^{-t} = 5 - 5e^{-t} \quad A$

$$\boxed{v(2s) = 4i(2) = 20 - 20e^{-2}\ V}$$

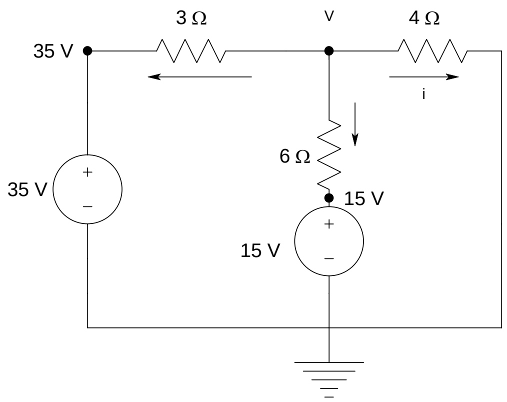
For $t > 10s$



$$i(t) = i(\infty) + [i(10) - i(\infty)]e^{-\frac{(t-10)}{\tau}}$$

where

$$i(10) = 5 - 5e^{-10} A$$



At $t = \infty$, the circuit is under dc conditions. In order to find $i(\infty)$:

KCL at v:

$$\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4}\right)v - \frac{1}{3}35 - \frac{1}{6}15 = 0$$

Multiply both sides by 12 yields:

$$(9)v = 140 + 30 = 170$$

$$v = \frac{170}{9} V$$

$$i(\infty) = \frac{v}{4} = \frac{170}{36} = \frac{85}{18} A$$

Time constant $\tau = \dfrac{L}{R_{eq}}$

$$R_{eq} = 3 // 6 + 4 = 6\Omega$$

$$\tau = \frac{7}{6}s$$

$$i(t) = \frac{85}{18} + \left[5 - 5e^{-10} - \frac{85}{18})\right]e^{-\frac{6}{7}(t-10)}$$

$$v(12) = 4i(12) = \frac{340}{18} + \left[20 - 20e^{-10} - \frac{340}{18}\right]e^{-\frac{12}{7}}\ V$$