Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EEE 223 Circuit Theory I

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Final EXAMINATION

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Duration : 120 minutes

Number of Problems: 6

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	
GROUP NO	

Problem		Points
1		15
2		15
3		15
4		15
5		20
6		20
TOTAL		100

1. Find the power delivered by the 4 A current source in the circuit in Fig.P1.

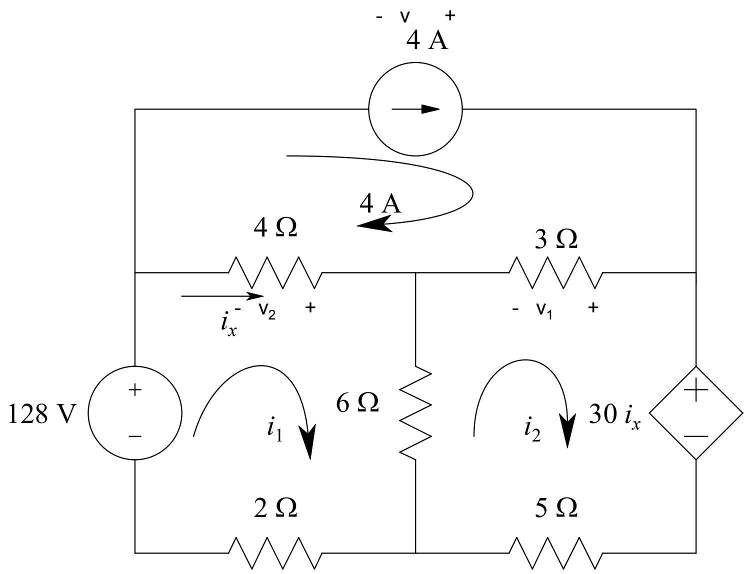


Figure P1

$i_x = i_1 - 4$

KVL around i_1 :

$12i_1 - 6i_2 - 4(4) = 128$

$12i_1 - 6i_2 = 144$...........(1)

KVL around i_2 :

$-6i_1 + 14i_2 - 3(4) = -30i_x = -30(i_1 - 4)$

$24i_1 + 14i_2 = 132$.........(2)

Multiplying Eq.(1) by (-2) and adding to Eq.(2) yields:

$26i_2 = -156$

$i_2 = -6$ A

$i_1 = 9$ A

KVL around the upper mesh:

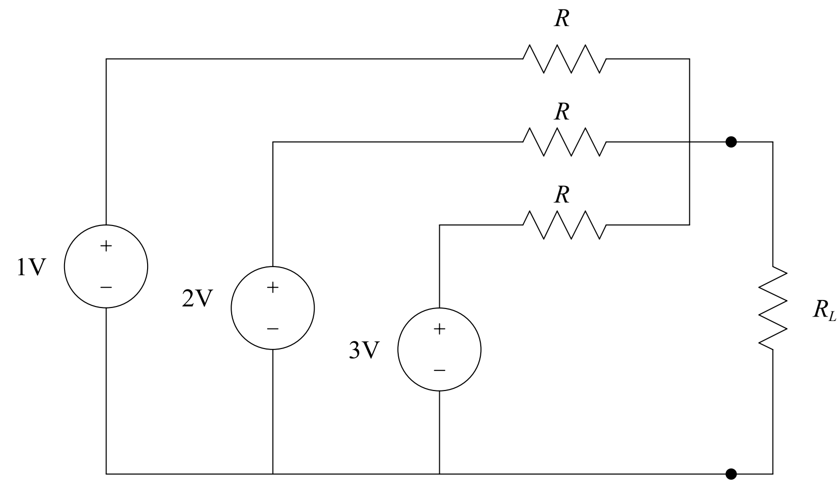
$-v + v_1 + v_2 = 0$

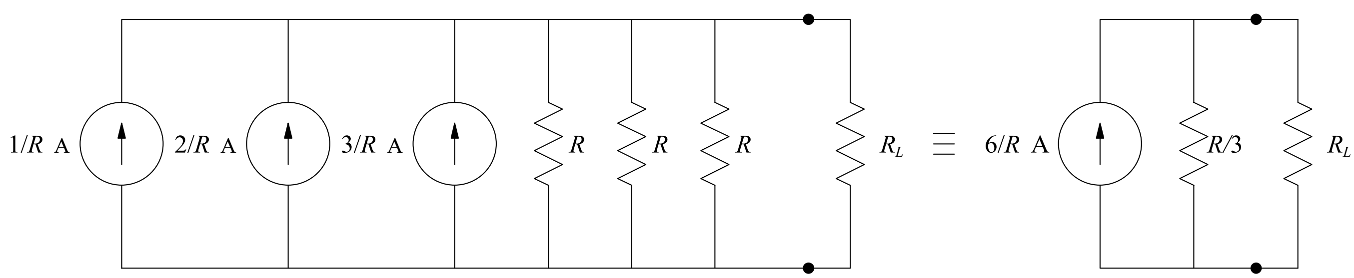
$v = v_1 + v_2 = (4 - i_2)3 + (4 - i_1)4 = 30 - 20 = 10$ V

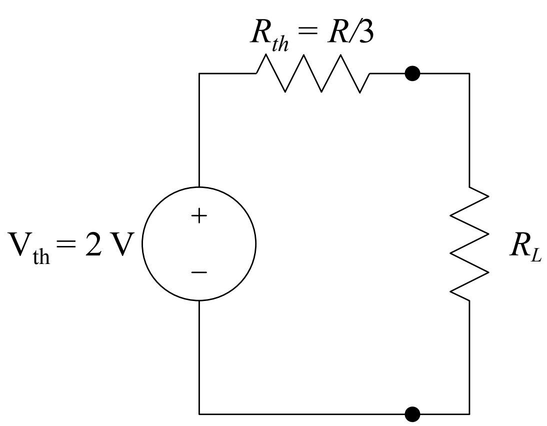
Power **delivered by** the 4 A current source

$p = 4 \times 10 = 40\,\text{W}$

2. For the circuit in Fig. P2, determine the value of R such that the maximum power delivered to the load, R_L, is 3 mW.







When $R_L = R_{th}$, it absorbs maximum power.

The maximum power absorbed by the R_L is:

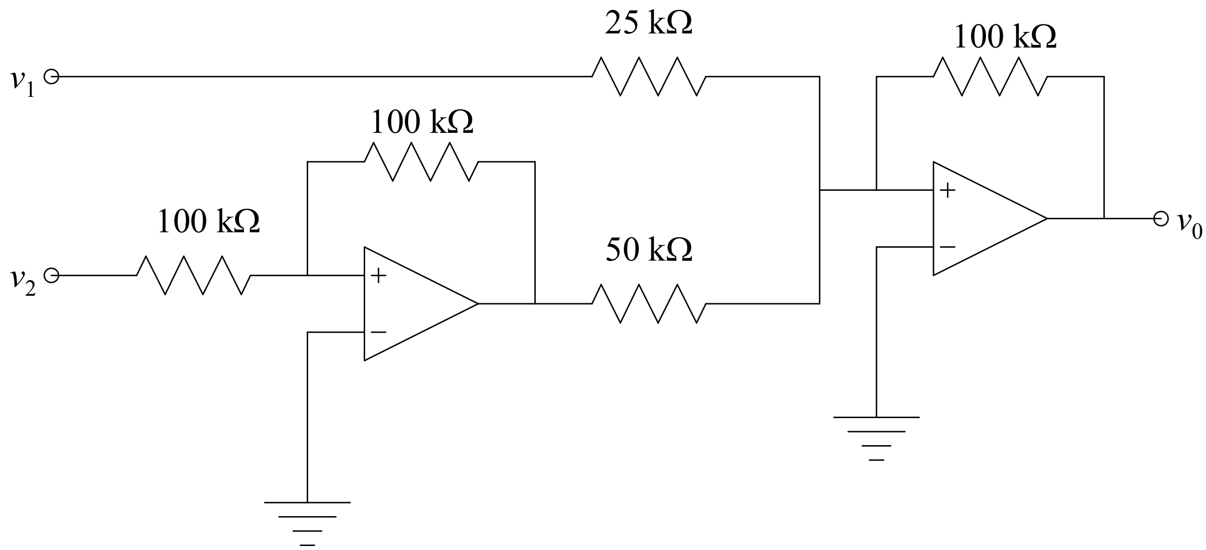
$$P_{L_{max}} = \frac{V_{th}^{2}}{4R_{th}}$$

Since the maximum power is 3mW,

$$3m = \frac{2^2}{4\frac{R}{3}} \Rightarrow \boxed{R = 1k\Omega}$$

Figure P2

3. Design an op-amp circuit with inputs v_1 and v_2 such that the output $v_0 = -4v_1 + 2v_2$. You may use one inverting and one inverting summer op amps to obtain the required output.



4. If the current waveform in Fig. P4 is applied to a $5\,\mu$F capacitor, find and draw the voltage $v(t)$ across the capacitor. Assume that $v(0)=12$ V.

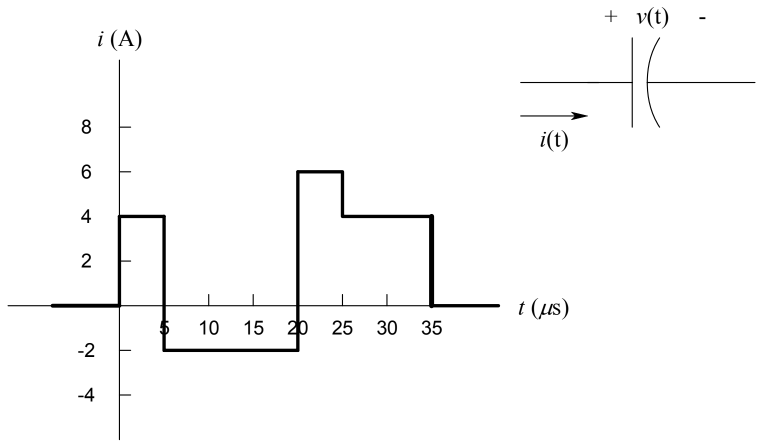
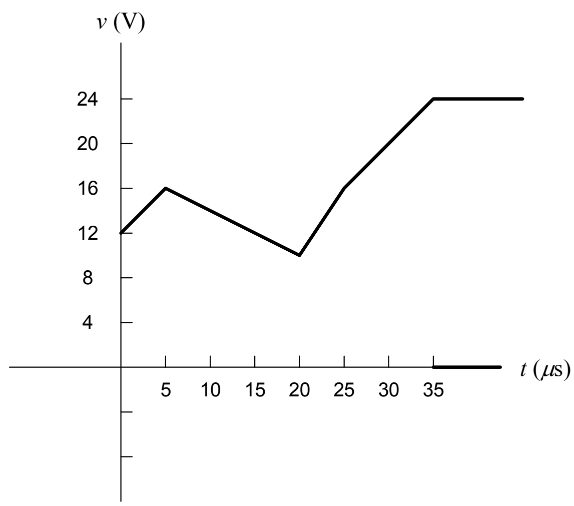


Figure P4



$$i(t)=\begin{cases}4\,\text{A} & 0<t<5\mu\text{s}\\ -2\,\text{A} & 5<t<20\mu\text{s}\\ 6\,\text{A} & 20<t<25\mu\text{s}\\ 4\,\text{A} & 25<t<35\mu\text{s}\\ 0 & t>35\mu s\end{cases}$$

$$v(t)=\frac{1}{C}\int_{t_0}^{t} i\,dt+v(t_0)$$

$0<t<5\mu s$

$$v(t)=\frac{1}{5\times10^{-6}}\int_0^t 4dt+12=0.8\times10^{6}t+12$$

$5\mu s < t < 20\mu s$

$$v(t) = \frac{1}{5\times10^{-6}}\int_{5\mu s}^{t} -2dt + 16 = -0.4\times10^{6}t + 2 + 16 = -0.4\times10^{6}t + 18$$

$20\mu s < t < 25\mu s$

$$v(t) = \frac{1}{5\times10^{-6}}\int_{20\mu s}^{t} 6dt + 10 = 1.2\times10^{6}t - 24 + 10 = 1.2\times10^{6}t - 14$$

$25\mu s < t < 35\mu s$

$$v(t) = \frac{1}{5\times10^{-6}}\int_{25\mu s}^{t} 4dt + 16 = 0.8\times10^{6}t - 20 + 16 = 0.8\times10^{6}t - 4$$

$t > 35\mu s$

$$v(t) = v(35\mu s) = 24\,\text{V}$$

5. In the circuit in Fig. P5, it is given that $i(0)=10$ A. Calculate $i(t)$ and $i_x(t)$ for $t \geq 0$.

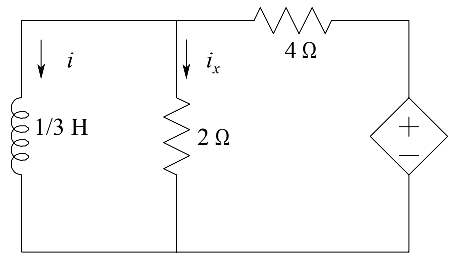
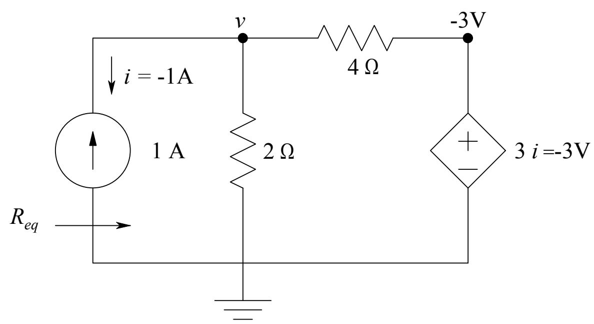


Figure P5

The circuit in Fig. P5 is a source-free RL circuit. Therefore the current flowing through the inductor is

$$i(t)=i(0)e^{-\frac{t}{\tau}}$$

where $\tau=\dfrac{L}{R_{eq}}$. In order to find R_{eq}, a test source is used.



It is obvious that $R_{eq}=\dfrac{v}{1}$.

KCL at v:

$$\left(\frac{1}{2}+\frac{1}{4}\right)v-\frac{1}{4}(-3)=1$$

Multiply both sides by (4):

$$3v=1 \Rightarrow \boxed{v=\frac{1}{3}V} \quad \therefore R_{eq}=\frac{1}{3}\Omega$$

$$\tau=\frac{L}{R_{eq}}=\frac{1/3}{1/3}=1s \Rightarrow i(t)=10e^{-t}\,V$$

Since the 2Ω resistor and the 1/3 H inductor are in parallel, they have the same voltage

$$v=\frac{1}{3}\frac{di}{dt}=2i_x \Rightarrow i_x=\frac{1}{6}(-10)e^{-t}=-\frac{10}{6}e^{-t}A$$

Consider the circuit shown in Fig. P3.

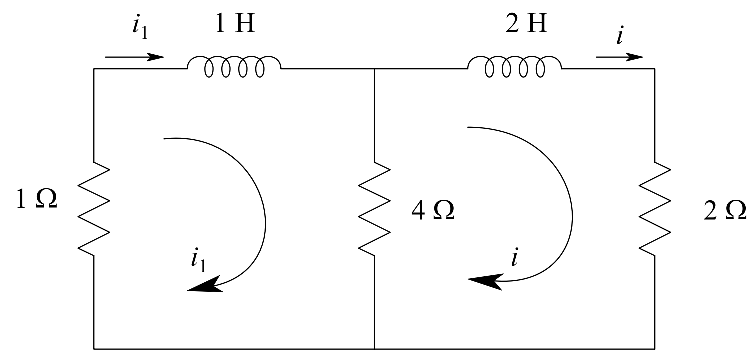


Figure P3

If $i_1(0) = 9\,\text{A}$ and $i(0) = 3\,\text{A}$ then find:

(a) $\dfrac{di(0)}{dt}$

(b) characteristic equation

(c) $i(t)$ for $t \geq 0$.

KVL around i_1:

$$5i_1 + \frac{di_1}{dt} - 4i = 0 \text{ ....(1)}$$

KVL around i_2:

$$-4i_1 + 6i + 2\frac{di}{dt} = 0 \Rightarrow i_1 = 1.5i + 0.5\frac{di}{dt} \text{ ....(2)}$$

At $t = 0$:

$$i_1(0) = 1.5i(0) + 0.5\frac{di(0)}{dt}$$

$$\boxed{\frac{di(0)}{dt} = 2i_1(0) - 3i(0) = 2(9) - 3(3) = 9\text{ A/s}}$$

Subst. Eq. (2) into (1) yields:

$$5\left(1.5i + 0.5\frac{di}{dt}\right) + \frac{d\left(1.5i + 0.5\frac{di}{dt}\right)}{dt} - 4i = 0$$

$$7.5i + 2.5\frac{di}{dt} + 1.5\frac{di}{dt} + 0.5\frac{d^2i}{dt^2} - 4i = 0$$

or

$$\boxed{\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 7i = 0}$$

Characteristic equation:
$s^2 + 8s + 7 = 0$

Therefore the natural frequencies are
$s_1 = -7$, and $s_2 = -1 \Rightarrow$ real and distinct (Overdamped case)
$i(t) = K_1 e^{-7t} + K_2 e^{-t}$
In order to find the constants K_1 and K_2, the initial conditions are used.
$i(0) = 3 = K_1 + K_2$ …………………...(3)

$$\frac{di(t)}{dt} = -7K_1 e^{-7t} - K_2 e^{-t}$$

$\frac{di(0)}{dt} = -7K_1 - K_2 = 9$....................(4)

By summing up Eqs.(3) and (4) K_1 can be obtained.
$-6K_1 = 12$

$\boxed{K_1 = -2}$

$\boxed{K_2 = 5}$

$\boxed{i(t) = -2e^{-7t} + 5e^{-t} \quad A}$