Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EEE 223 Circuit Theory I

Spring 2005-06

Instructor:

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Final EXAMINATION

June 15, 2006

Duration : 120 minutes

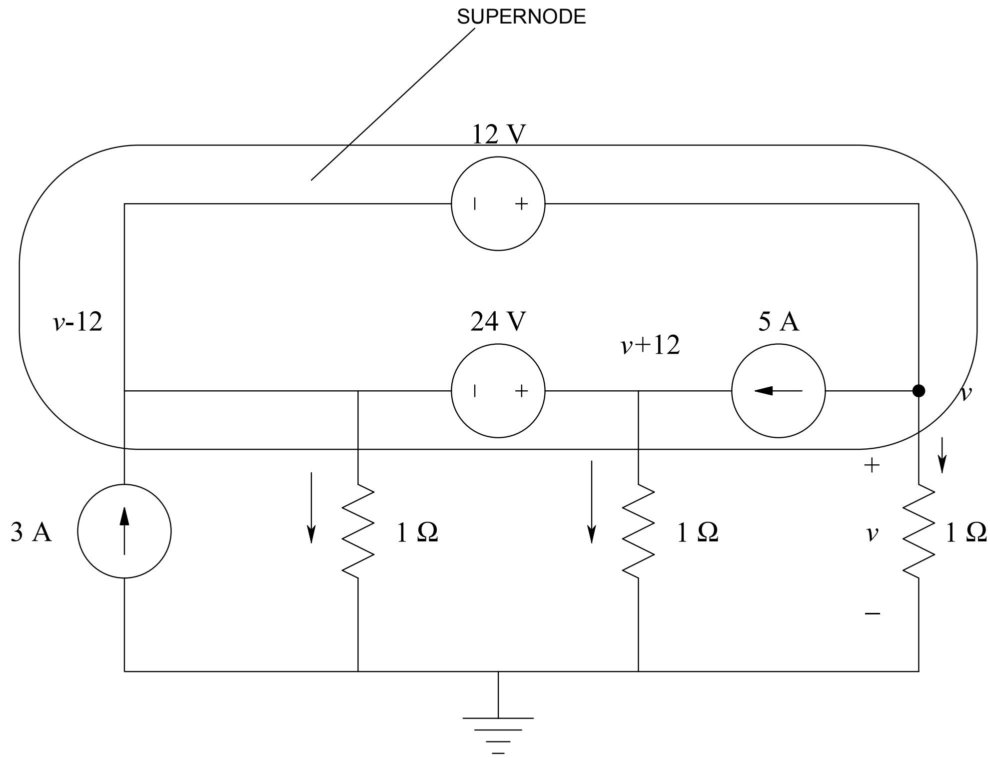
Number of Problems: 6

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	
GROUP NO	

Problem		Points
1		15
2		15
3		15
4		15
5		20
6		20
TOTAL		100

1. Use nodal analysis to find v.



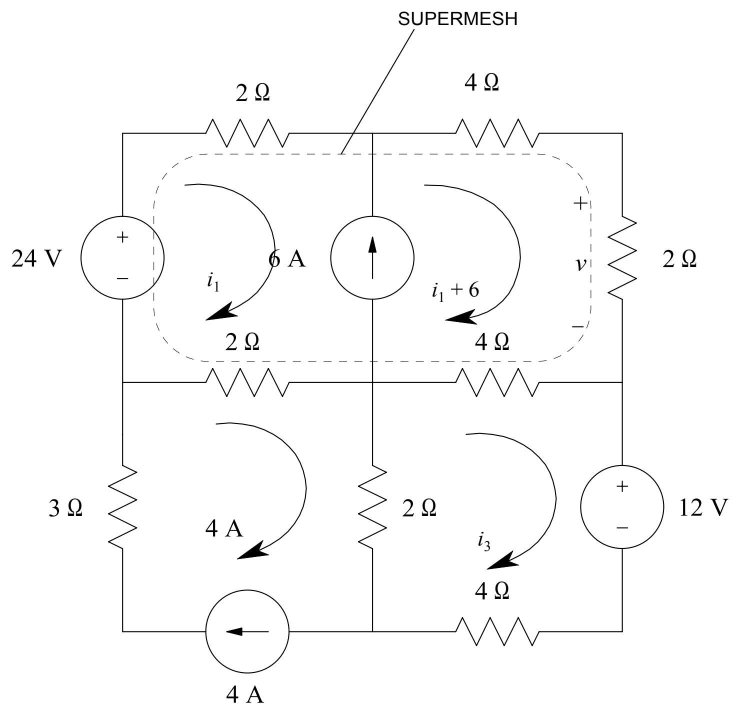
KCL at the supernode:

$$\frac{v-12}{1}+\frac{v+12}{1}+\frac{v}{1}=3$$

$$3v=3$$

$$\boxed{v=1\text{ V}}$$

2. Use mesh analysis to find v.



KVL around the supermesh:

$$2i_1 + 4(i_1 + 6) + 2(i_1 + 6) + 4(i_1 + 6 - i_3) + 2(i_1 - 4) = 24$$

$$14i_1 - 4i_3 = 24 - 60 + 8 = -28.........(1)$$

KVL around i_3:

$$2(i_3 - 4) + 4(i_3 - i_1 - 6) + 4i_3 = -12$$

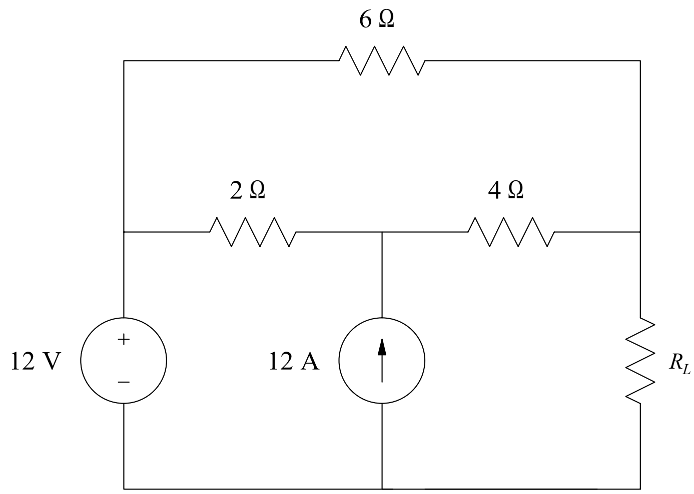
$$10i_3 - 4i_1 = 20....(2)$$

By using Eqns(1) and (2), i_1 is obtained as

$$i_1 = -1.613\ A$$

$$v = 2(i_1 + 6) = 8.774\ \text{V}.$$

3. Find R_L for maximum power transfer and the maximum power absorbed by the load.

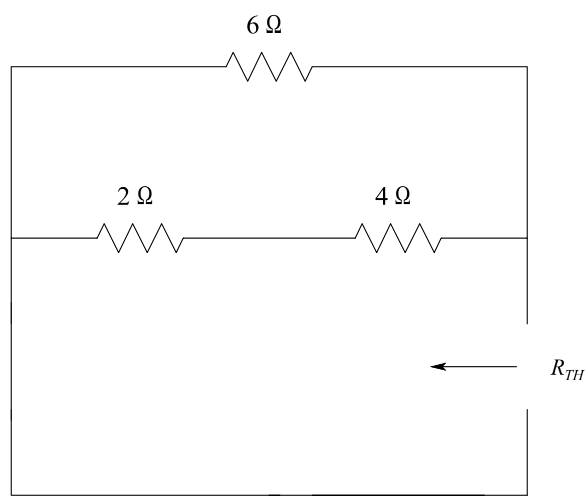


When $R_L = R_{TH}$ it will absorb maximum power. The maximum power;

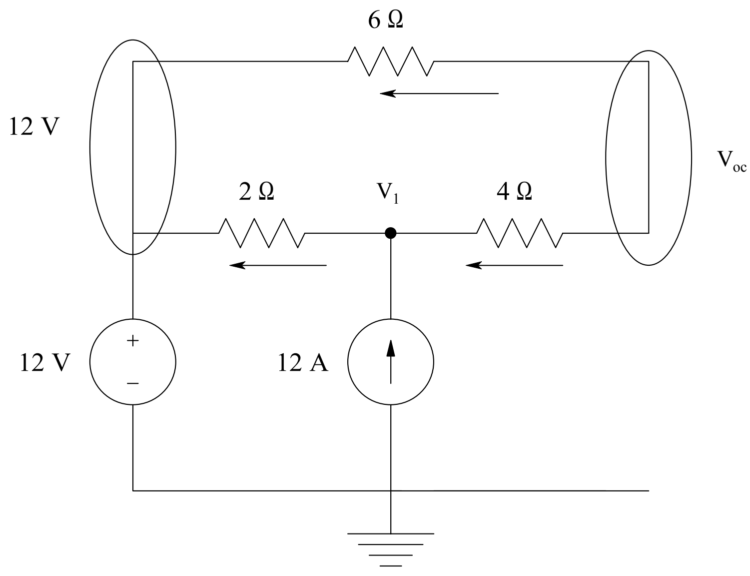
$$P_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

In order to find R_{TH} :

$$R_{TH} = 6//(4+2) = 3\Omega$$



For V_{TH}:



KCL at V_{oc}:

$$\frac{V_{oc}-12}{6}+\frac{V_{oc}-V_1}{4}=0$$

$$5V_{oc}-3V_1=24....(1)$$

KCL at V_1:

$$\frac{V_1-V_{oc}}{4}+\frac{V_1-12}{2}=12$$

$$-V_{oc}+3V_1=72.....(2)$$

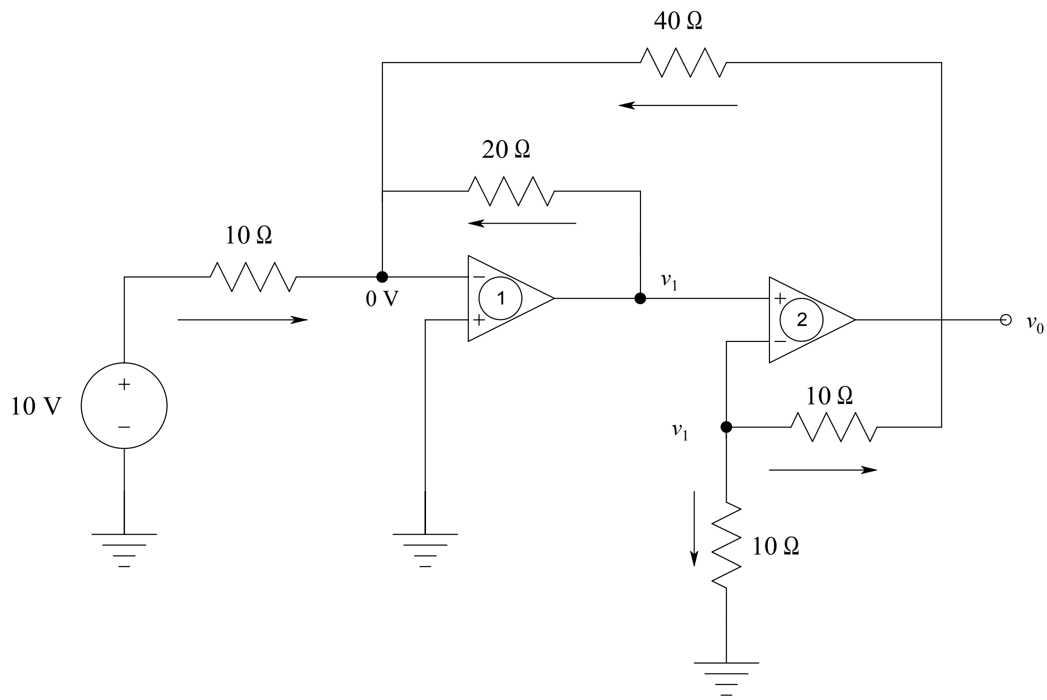
From Eqns (1) and (2),

$$\boxed{V_{oc}=24\text{ V}}$$

Therefore, when $R_L=3\Omega$ then it will absorb maximum power.

$$P_{\max}=\frac{24^2}{4\times 3}=48W$$

4. Find v_0 in the circuit.



KCL at the inverting input terminal of OPAMP (2):

$$\frac{v_1}{10}+\frac{v_1-v_0}{10}=0$$

$$\boxed{v_1=\frac{v_0}{2}}\;.....(1)$$

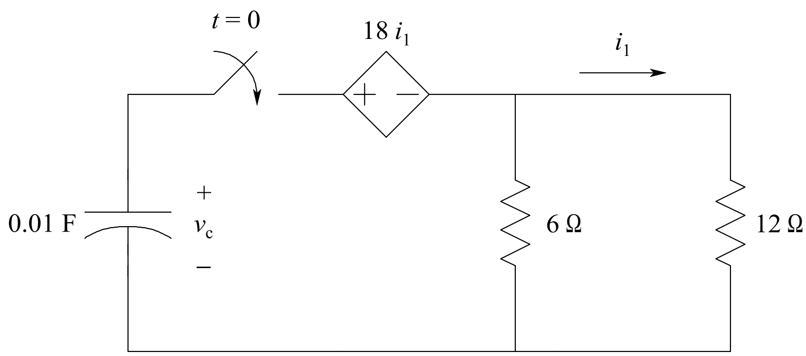
KCL at the inverting input terminal of OPAMP (1)

$$\frac{10}{10}+\frac{v_0/2}{20}+\frac{v_0}{40}=0$$

$$2v_0=-40$$

$$\boxed{v_0=-20\text{ V}}$$

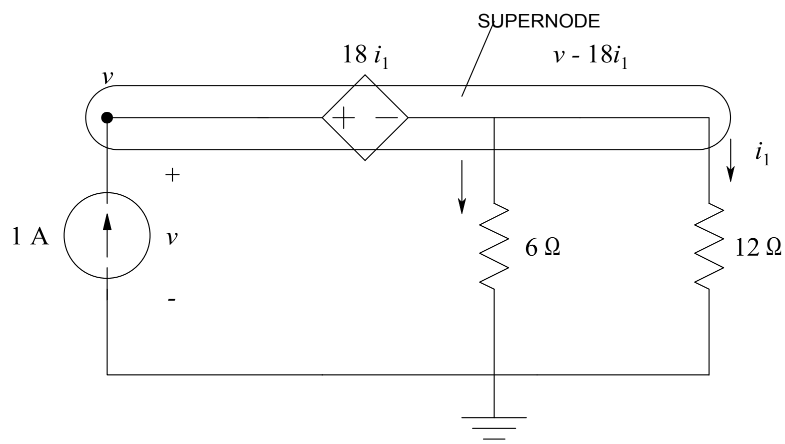
5. If $v_c(0) = 100$ V, find $v_c(t)$ for $t \geq 0$.



Since the circuit is a source-free RC circuit, the voltage across the capacitor is

$$v_c(t) = v_c(0)e^{-\frac{t}{\tau}}$$

Where $\tau = R_{eq}C$ and R_{eq} is the resistance seen by the capacitor.



$$i_1 = \frac{v - 18i_1}{12} \Rightarrow \frac{5}{2}i_1 = \frac{v}{12} \Rightarrow \boxed{i_1 = \frac{2}{60}v}$$

$$v - 18i_1 = v - 18\left(\frac{2}{60}v\right) = v - \frac{36}{60}v = \frac{24}{60}v = 0.4v$$

KCL at the supernode:

$$\frac{0.4v}{6} + \frac{0.4v}{12} = 1$$

$$1.2v = 12$$

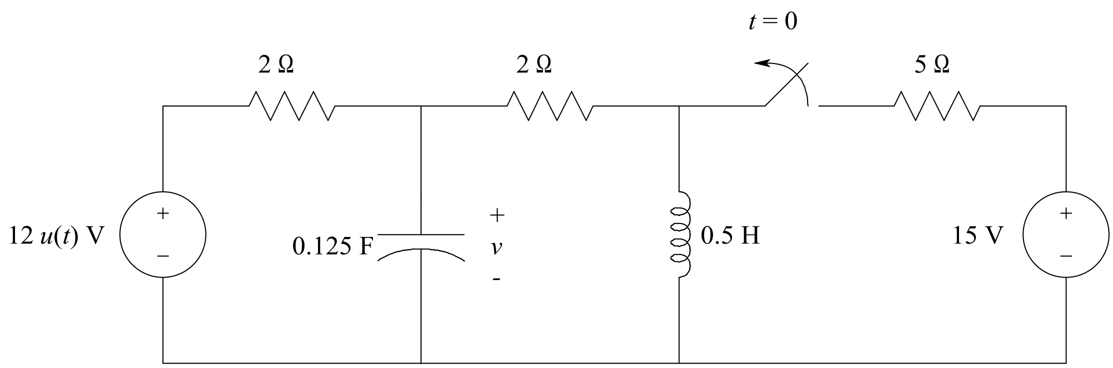
$$v = 10 \text{ V}$$

$$R_{eq} = \frac{v}{1} = 10\Omega$$

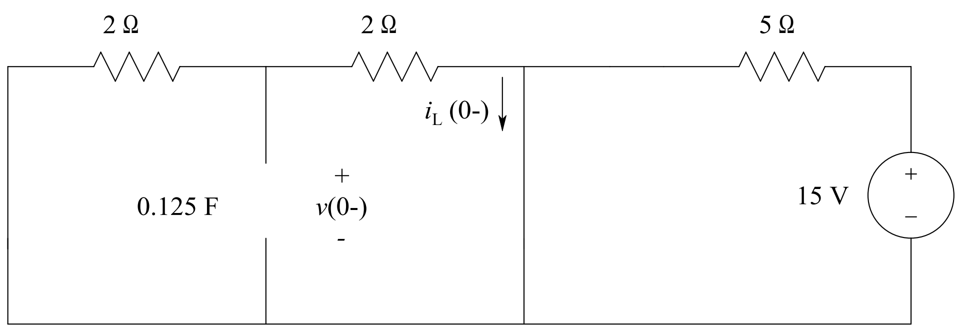
$$\tau = R_{eq}C = 10 \times 0.01 = 0.1s$$

$$v_c\left(t\right) = 100e^{-10t} \ \ V \ \ for \ \ t \geq 0$$

6. Suppose that the switch has been closed for a long time and is opened at $t = 0$, determine $v(t)$ for $t \geq 0$.



At $t = 0-$ (The circuit is under dc conditions)



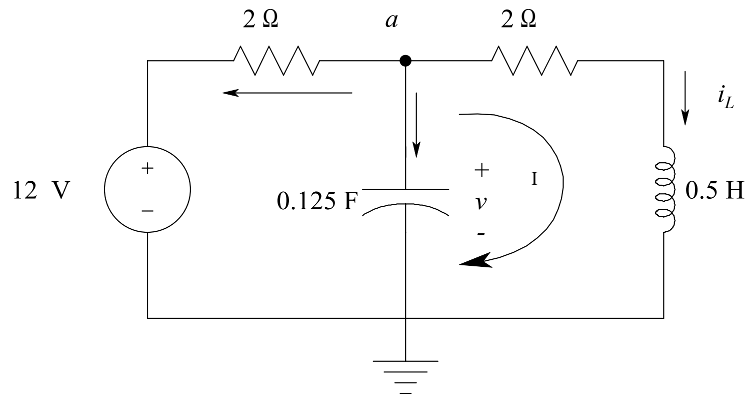
$$i_L(0-) = \frac{15}{5} = 3A$$

$$v_c(0-) = 0V$$

Since the inductor current and capacitor voltage cannot change instantaneously,

$$i_L(0-) = i_L(0+) = 3A$$

$$v_c(0-) = v_c(0+) = 0V$$



For $t \geq 0$

KCL at node *a*:

$$\frac{v-12}{2}+0.125\frac{dv}{dt}+i_L=0....(1)$$

KVL around the loop I:

$$-v+2i_L+0.5\frac{di_L}{dt}=0...(2)$$

From Eqn(1):

$$i_L=-\frac{v-12}{2}-0.125\frac{dv}{dt}...(3)$$

At t = 0

$$i_L(0)=-\frac{v(0)-12}{2}-0.125\frac{dv(0)}{dt}$$

$$3=6-0.125\frac{dv(0)}{dt}$$

$$\frac{dv(0)}{dt}=24V/s$$

Subst. Eqn. (3) into (2) gives:

$$-v+2\left(-\frac{v-12}{2}-0.125\frac{dv}{dt}\right)+0.5\frac{d}{dt}\left(-\frac{v-12}{2}-0.125\frac{dv}{dt}\right)=0$$

$$-2v-0.5\frac{dv}{dt}-0.0625\frac{d^2v}{dt^2}=-12$$

multiply both sides $\frac{-1}{0.0625}$ yields:

$$\boxed{\frac{d^2v}{dt^2}+8\frac{dv}{dt}+32v=192}$$

The characteristic equation

$$s^2+8s+32=0$$

The natural frequencies:

$$s_{1,2}=-4\mp j4$$

The natural response

$$v_n=e^{-4t}\left(K_1\cos 4t+K_2\sin 4t\right)$$

The forced response

$$v_f=\frac{192}{32}=6$$

The complete response

$$v=v_n+v_f$$

$$v=e^{-4t}\left(K_1\cos 4t+K_2\sin 4t\right)+6$$

$v(0) = 0 = e^{-0}\left(K_1 \cos 0 + K_2 \sin 0\right) + 6$

$K_1 + 6 = 0 \Rightarrow K_1 = -6$

$$\frac{dv}{dt} = -4e^{-4t}\left(K_1 \cos 4t + K_2 \sin 4t\right) + e^{-4t}\left(-4K_1 \sin 4t + 4K_2 \cos 4t\right)$$

$$\frac{dv(0)}{dt} = 24 = -4K_1 + 4K_2$$

$\boxed{K_2 = 0}$

Therefore

$v(t) = -6e^{-4t}\cos 4t + 6 \quad \text{for } t \geq 0$