Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 Circuit Theory I

Spring 2006-07

Instructor:

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Final EXAMINATION

June 08, 2007

Duration : 150 minutes

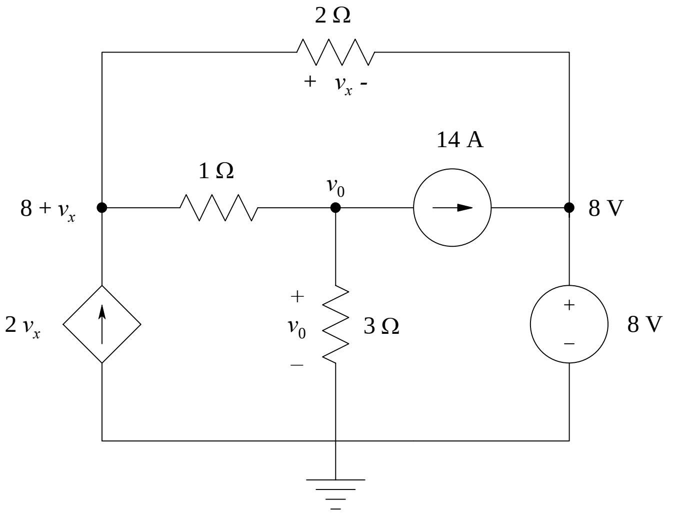
Number of Problems: 6

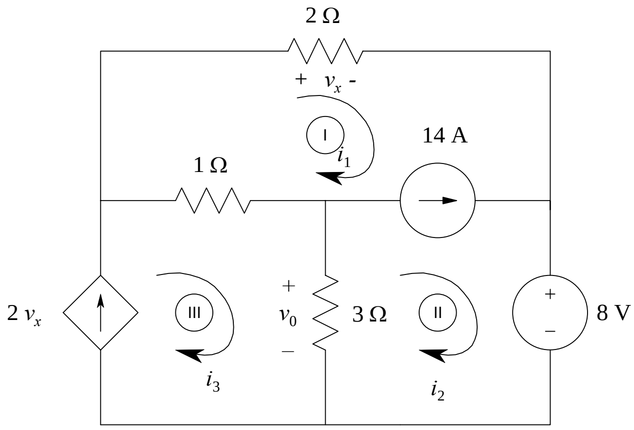
Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	
GROUP NO	

Problem		Points
1		20
2		15
3		15
4		15
5		15
6		20
TOTAL		100

1. For the circuit in Fig. P1, find the value of v_0 using
   a. mesh analysis. (10 pts.)
   b. nodal analysis. (10 pts.)





KCL at v_0:

$$\left(1+\frac{1}{3}\right)v_0-(1)(8+v_x)=-14$$

$$\frac{4}{3}v_0-v_x=-6 \quad or$$

$$4v_0-3v_x=-18.............(1)$$

KCL at $(8+v_x)$:

$$-1v_0+\left(1+\frac{1}{2}\right)(8+v_x)-\frac{8}{2}-2v_x=0$$

$$-v_0-\frac{1}{2}v_x=-8 \quad or$$

$$-2v_0-v_x=-16...........(2)$$

Multiply Eq.(2) by (-3) and add to (1) yields:

$$10v_0=30 \Rightarrow \boxed{v_0=3\text{V}}$$

$$i_3=2v_x=2(2i_1)=4i_1.............(1)$$

$$i_2-i_1=14$$

$$i_2=14+i_1...............................(2)$$

Mesh (I) and mesh (II) constitute a SUPERMESH.

KVL around the supermesh:

$$2i_1+8+3(i_2-i_3)+1(i_1-i_3)=0$$

$$2i_1+3(14+i_1-4i_1)+i_1-4i_1=-8$$

$$-10i_1=-50$$

$$i_1=5A$$

$$v_0=3(i_3-i_2)=3(4i_1-14-i_1)=3(15-14)$$

$$\boxed{v_0=3\text{V}}$$

2. Find v_x, i_a, and i_0 in the circuit in Fig. P2. (15 pts.)

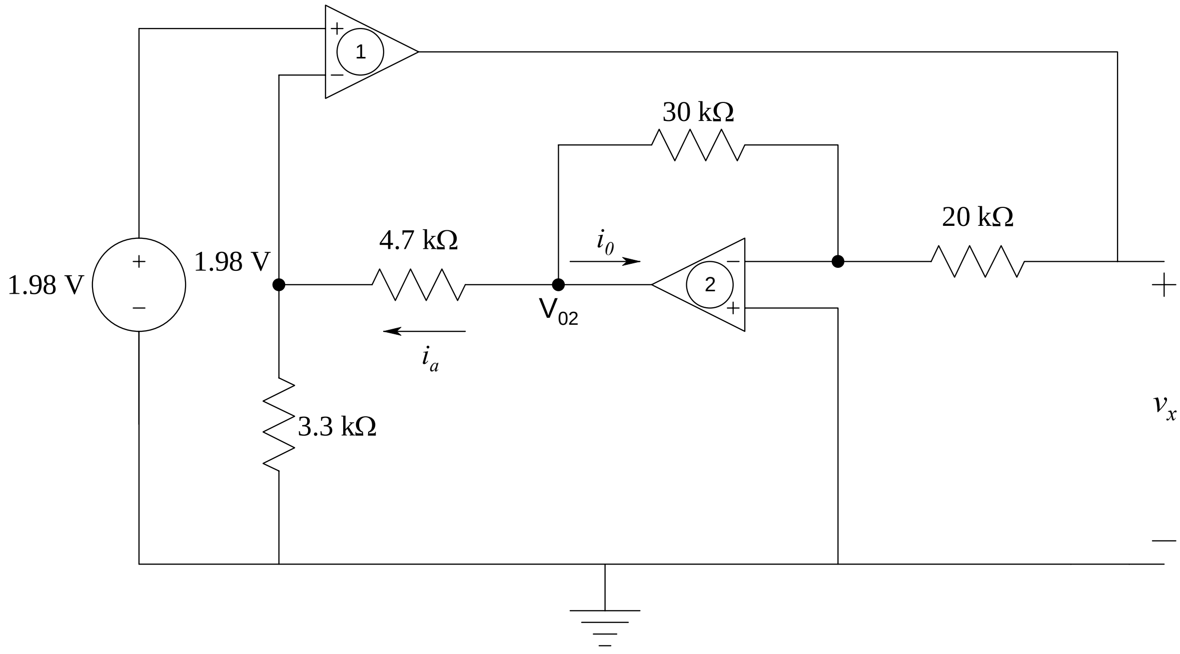


Figure P2

KCL at the inverting input terminal of OP AMP (1):

$$1.98\left(\frac{1}{3.3k}+\frac{1}{4.7k}\right)-\frac{1}{4.7k}v_{01}=0$$

$$v_{01}=\left(1+\frac{4.7}{3.3}\right)1.98=4.8\,\text{V}$$

KCL at the inverting input terminal of OP AMP (2):

$$\frac{v_{01}}{30k}+\frac{v_x}{20k}=0$$

$$v_x=-\frac{2}{3}v_{01}=-3.2\ \text{V}$$

$$i_a=\frac{4.8-1.98}{4.7k}=0.6\ \text{mA}$$

$$i_0=-\left(0.6m+\frac{4.8}{30k}\right)=-0.76\ \text{mA}$$

3. Use the principle of superposition to find i in the circuit in Fig. P3. (15 pts.)

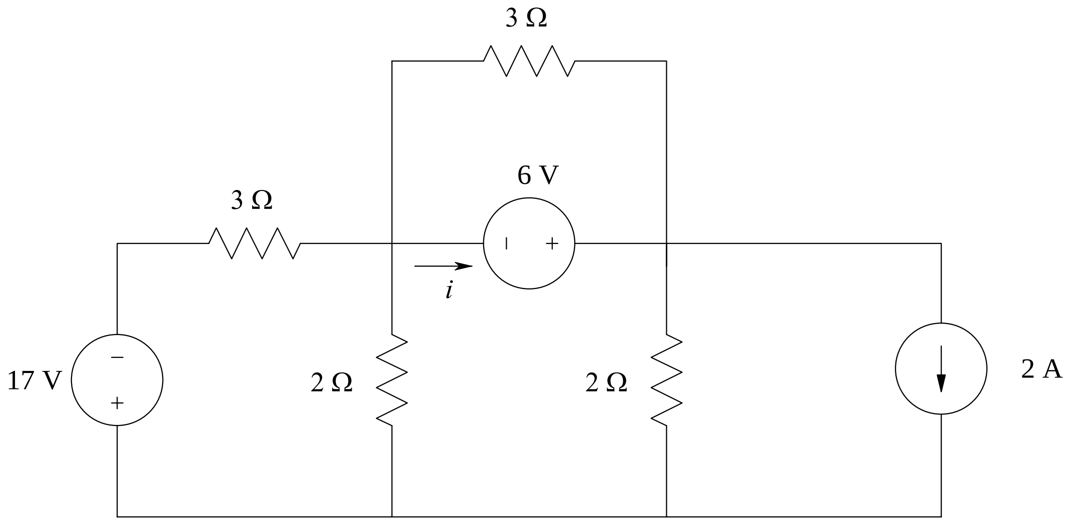


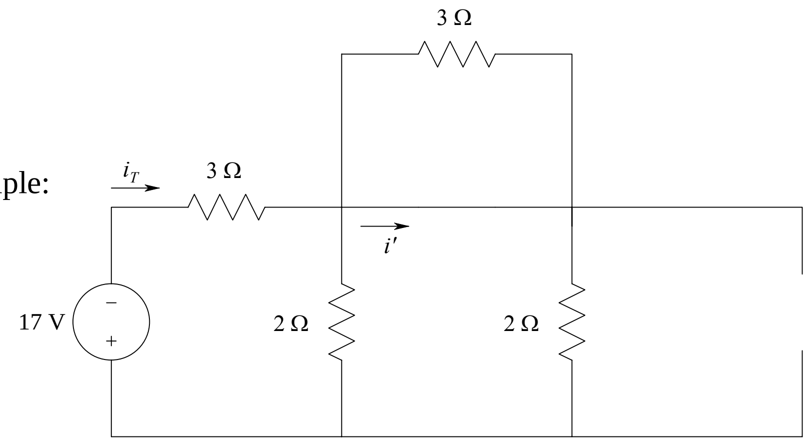
Figure P3

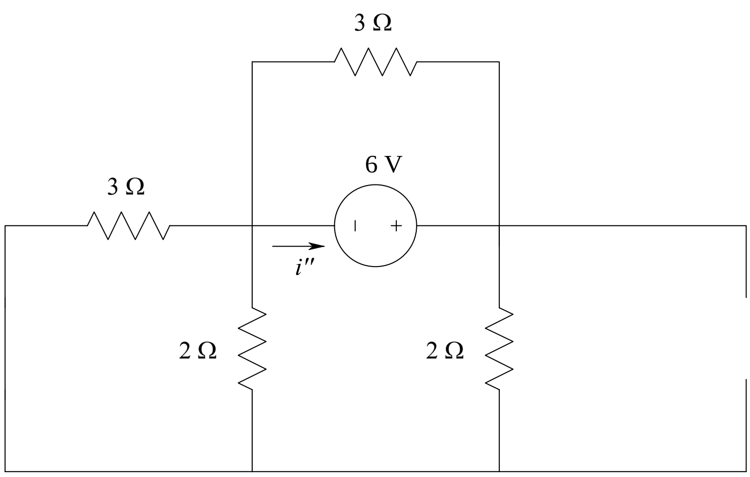
17 V is active:

$$i_T = -\frac{17}{R_T} = -\frac{17}{4}A$$

by using current division principle:

$$i' = i_T \frac{2}{4} = -\frac{17}{8} \text{ A}$$

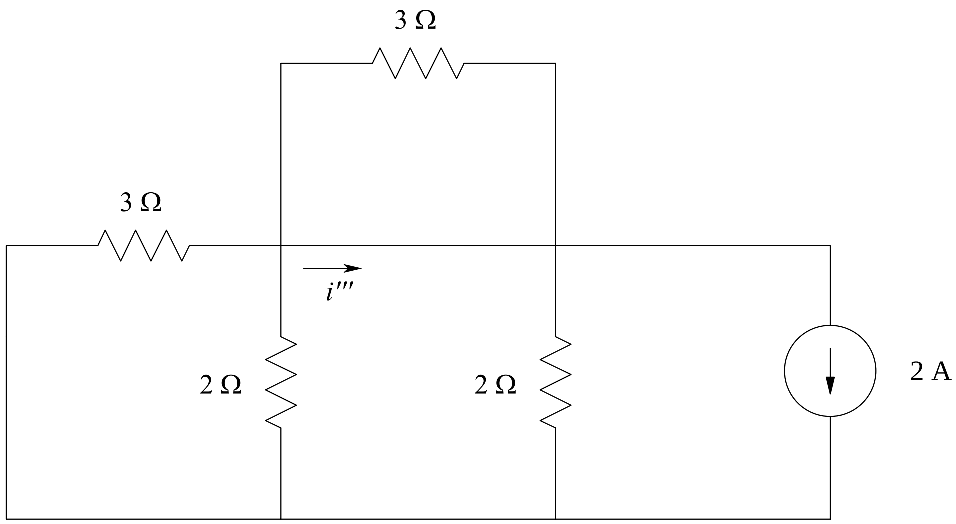




6 V is active:

$$i'' = \frac{6}{R_T} = \frac{6}{(3//2+2)//3} = \frac{6}{\frac{48}{31}} = \frac{31}{8} \text{ A}$$

2 A is active:



$$i''' = 2\frac{2}{2+\frac{6}{5}} = \frac{20}{16}\text{ A}$$

$$i = i' + i'' + i'''$$

$$i = -\frac{17}{8} + \frac{31}{8} + \frac{20}{16} = 3\text{ A}$$

4. The variable resistor in the circuit in Fig. P4 is adjusted for maximum power transfer to *R*.

   a. Find the value of *R*. (10 pts.)
   b. Find the maximum power that can be delivered to *R*. (5 pts.)

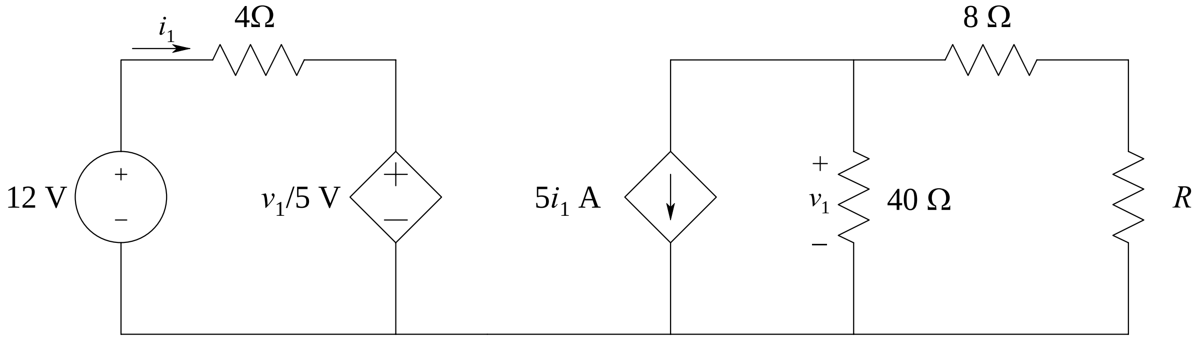
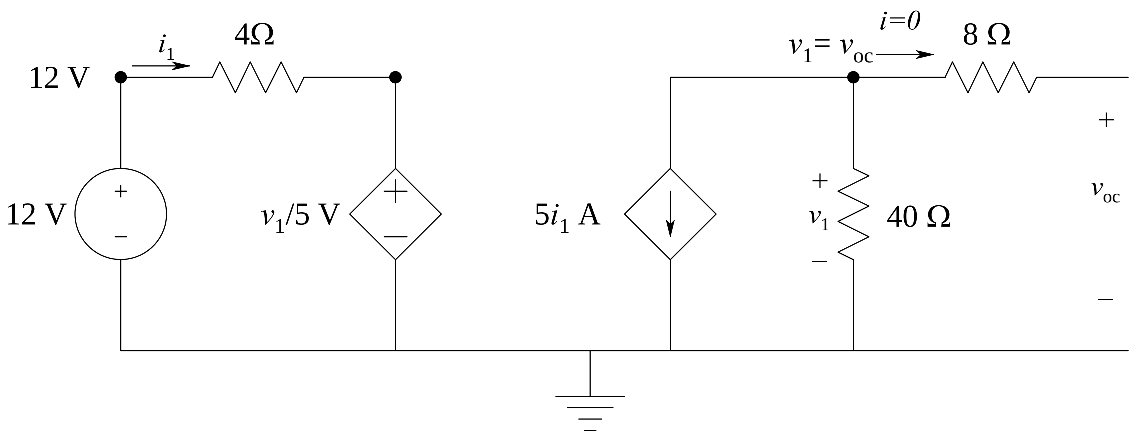


Figure P4

When the value of $R = R_{TH}$ then it will absorb maximum power. The maximum power is

$$P_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

In order to find V_{TH} we will find V_{oc}



$$v_{oc} = -40 \times 5i_1 = -200i_1$$

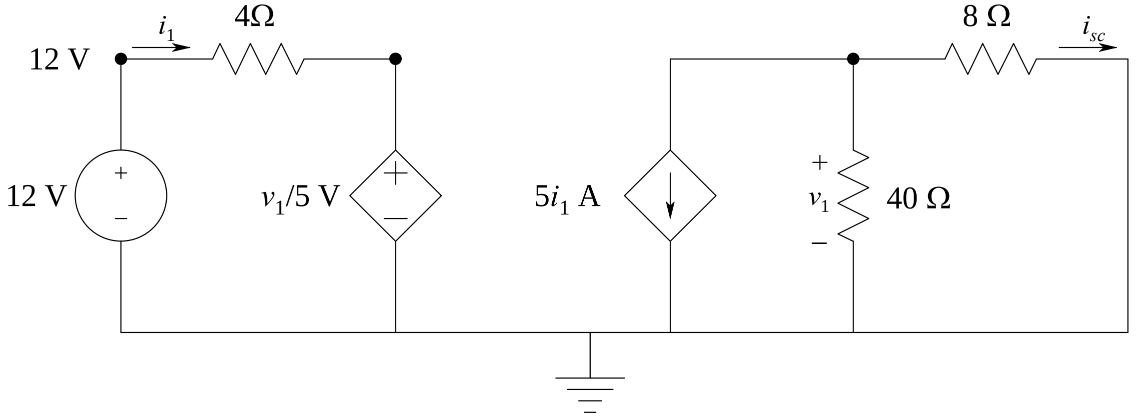
$$i_1 = \frac{12 - \frac{v_{oc}}{5}}{4} = \frac{60 - v_{oc}}{20}$$

Therefore

$$v_{oc} = -200i_1 = -10\left(60 - v_{oc}\right)$$

$$-9v_{oc} = -600$$

$$v_{oc} = \frac{600}{9}\ V$$



By using current division principle:

$$i_{sc} = -5i_1\frac{40}{48} = -\frac{25}{6}i_1$$

$$v_1 = -40\left(5i_1\frac{8}{48}\right) = -\frac{200}{6}i_1$$

And

$$i_1 = \frac{12-\frac{v_1}{5}}{4} = \frac{60-v_1}{20} = 3-\frac{1}{20}\left(-\frac{200}{6}i_1\right)$$

$$i_1 = -\frac{18}{4}A$$

Therefore

$$i_{sc} = -\frac{25}{6}\left(-\frac{18}{4}\right) = \frac{75}{4}A$$

$$R_{TH} = \frac{V_{oc}}{i_{sc}} = \frac{600/9}{75/4} = \frac{32}{9}\Omega$$

When $R = \frac{32}{9}\Omega$ it will absorb maximum power.

$$P_{\max} = \frac{\left(\frac{600}{9}\right)^2}{4\left(\frac{32}{9}\right)} = 312.5W$$

5. Find i for $t \geq 0$ if the circuit in Fig.P5 is under dc conditions at $t = 0^{-}$. (15 pts.)

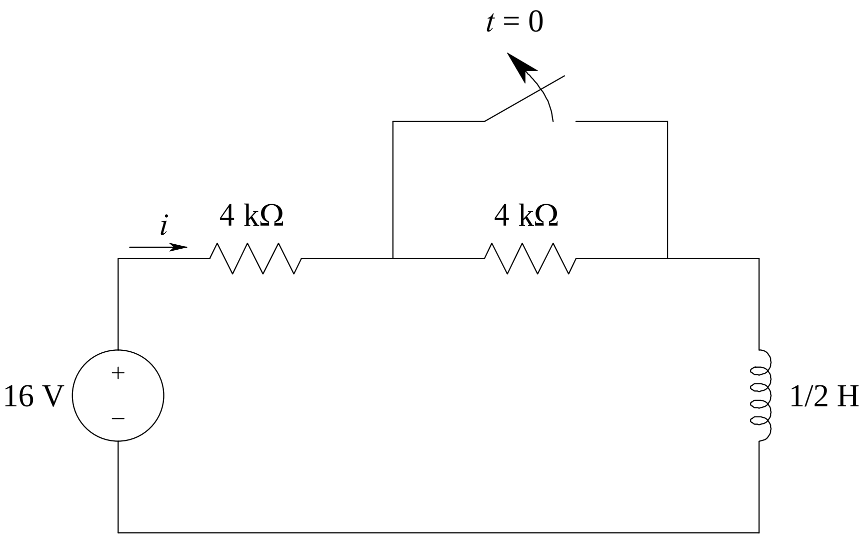
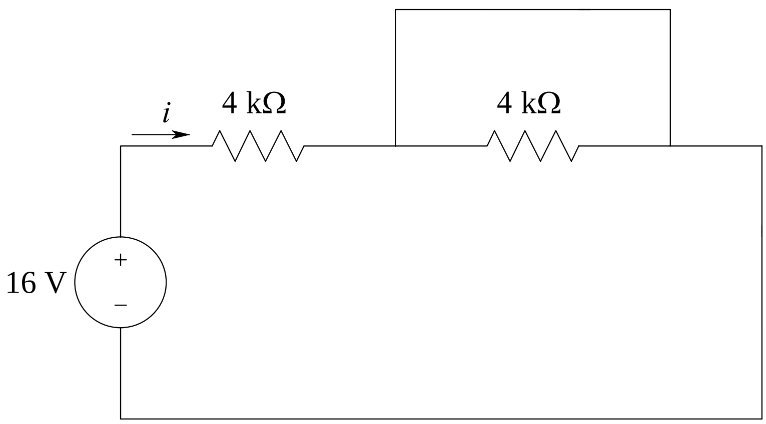


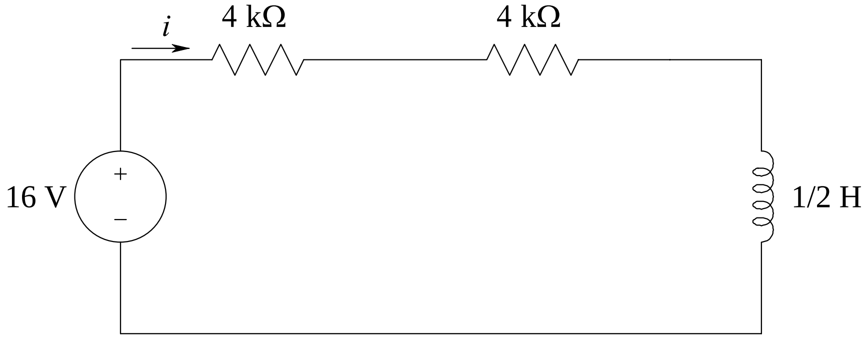
Figure P5

At $t = 0-$



$$i(0-) = \frac{16}{4k} = 4 \text{ mA} = i(0+)$$

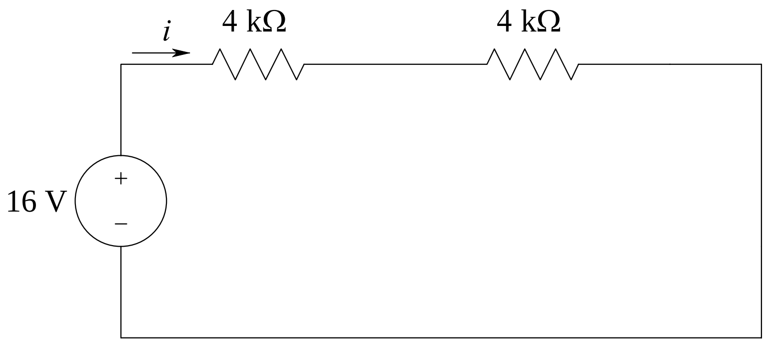
For $t \geq 0$



Since the circuit contains a source i will be:

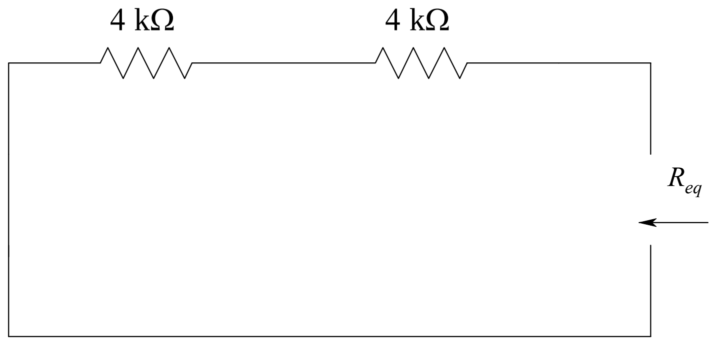
$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-\frac{t}{\tau}}$$

At $t = \infty$, the circuit is under dc conditions



$$i(\infty) = \frac{16}{8k} = 2 \text{ mA}$$

$\tau = \dfrac{L}{R_{eq}}$ where R_{eq} is the equivalent resistance seen by the inductor.



$$R_{eq} = 8k\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{16k}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-\frac{t}{\tau}} = 2 + (4-2)e^{-16000t}mA$$

$$i(t) = 2 - 2e^{-16000t}mA$$

6. Consider the circuit in Fig. P6. Find *v* for $t \geq 0$ if $v(0) = 4$ V and $i(0) = 3$ A. (20 pts.)

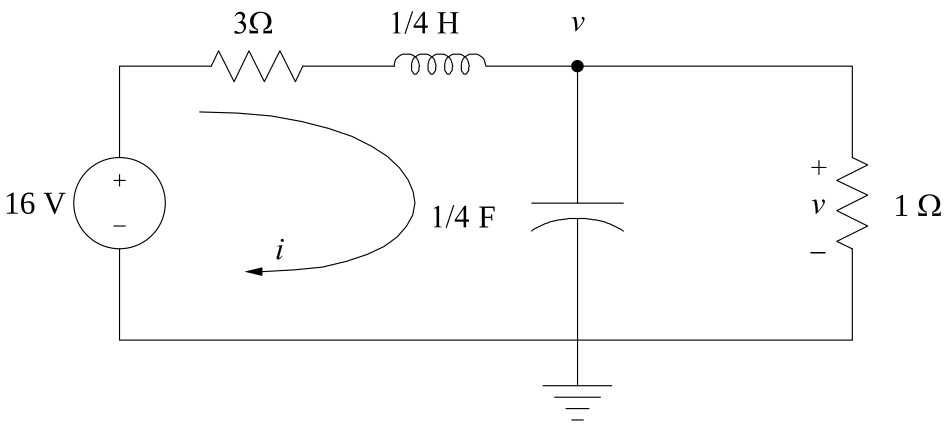


Figure P6

KCL at v:

$$i = \frac{1}{4}\frac{dv}{dt} + \frac{v}{1} \ldots\ldots\ldots(1)$$

KVL around the loop (i):

$$3i + \frac{1}{4}\frac{di}{dt} + v = 16 \ldots\ldots.(2)$$

Subst. Eq.(1) into (2) yields:

$$3\left(\frac{1}{4}\frac{dv}{dt} + \frac{v}{1}\right) + \frac{1}{4}\frac{d}{dt}\left(\frac{1}{4}\frac{dv}{dt} + \frac{v}{1}\right) + v = 16$$

$$\frac{3}{4}\frac{dv}{dt} + 3v + \frac{1}{16}\frac{d^2v}{dt^2} + \frac{1}{4}\frac{dv}{dt} + v = 16$$

Multiply both sides by 16:

$$\frac{d^2v}{dt^2} + 16\frac{dv}{dt} + 64v = 16^2$$

Characteristic equation:

$$s^2 + 16s + 64 = 0$$

$$\therefore s_{1,2} = -8$$

The natural response v_n:

$$v_n = (A + Bt)e^{-8t}$$

The force response v_f:

$v_f = K$ the trial forced response.

$64\text{K} = 16^2$

$K = 4$

$v_f = 4\,\text{V}$

The complete response

$v = v_n + v_f$

$v = (A + Bt)e^{-8t} + 4$

$v(0) = 4 = A + 4 \Rightarrow \boxed{A = 0}$

By writing Eq(1) at $t = 0$

$$i(0) = \frac{1}{4}\frac{dv(0)}{dt} + \frac{v(0)}{1}$$

$$\frac{dv(0)}{dt} = 4\big(i(0) - v(0)\big) = 4(3 - 4) = -4V/s$$

$$\frac{dv}{dt} = Be^{-8t} - 8(A + Bt)e^{-8t}$$

$$\frac{dv(0)}{dt} = -4 = B - 8(A) \Rightarrow \boxed{B = -4}$$

$$\therefore \boxed{v(t) = \left(-4te^{-8t} + 4\right)V}$$