Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 Circuit Theory I
INFE221 – Electrical Circuits

Spring 2010-11

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Final EXAMINATION

June 3, 2011

Duration : 150 minutes

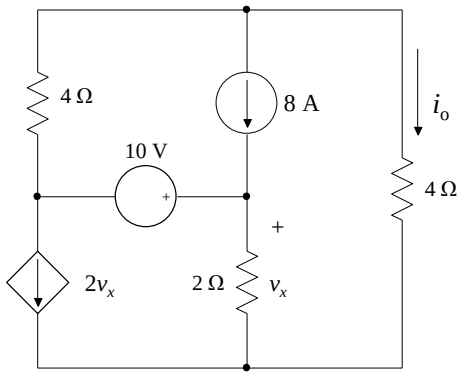
Number of Problems: 5

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	SOLUTIONS
GROUP NO	

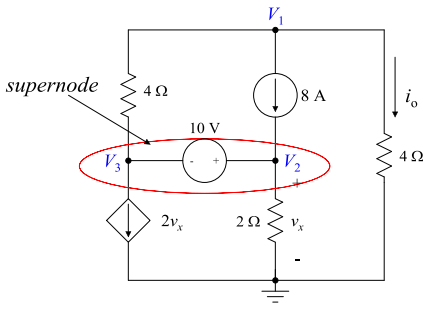
Problem		**Points**
1		30
2		20
3		15
4		20
5		15
TOTAL		100

1. Find the current i_0 in the circuit shown using
   (a) Nodal analysis. (15 pts.)
   (b) Mesh analysis. (15 pts.)



Solution:

(a) Nodal analysis



KVL for the 10-V source :

$$V_3 = V_2 - 10$$

$$v_x = V_2$$

KCL for the supernode:

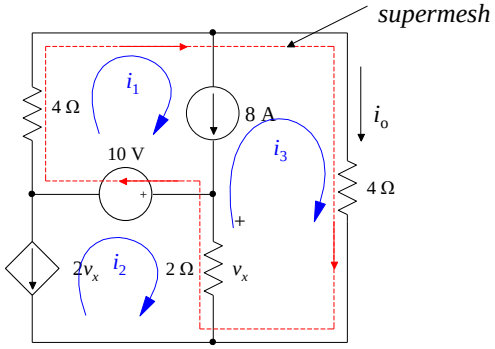
$$2V_2 + \frac{V_2}{2} + \frac{V_2 - 10 - V_1}{4} = 8 \quad \Rightarrow \quad 11V_2 - V_1 = 42 \qquad (1)$$

KCL for node 1:

$$\frac{V_1 - V_3}{4} + 8 + \frac{V_1}{4} = 0 \quad \Rightarrow \quad 2V_1 - (V_2 - 10) = -32 \quad \Rightarrow \quad 2V_1 - V_2 = -42 \qquad (2)$$

From Eqs. (1) & (2) $\Rightarrow$ $v_x = 2$ V $\Rightarrow$ $V_1 = -20$ V $\therefore$ $i_0 = -5$ A

(b) Mesh analysis



KVL for the supermesh:

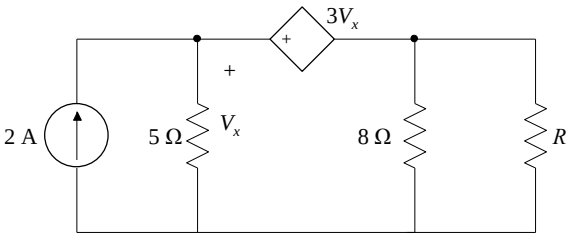
$$4i_3 + 2(i_3 - i_2) + 10 + 4i_1 = 0 \qquad (1)$$

For the 8-A current source: $i_1 - i_3 = 8$ A (2)

$$i_2 = -2v_x \quad v_x = 2(i_2 - i_3) \quad \Rightarrow \quad 5i_2 = 4i_3 \qquad (3)$$

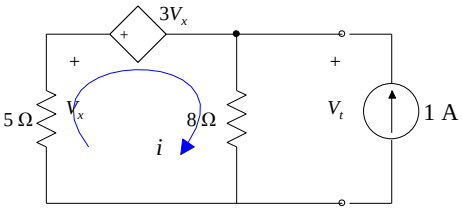
(1), (2) & (3) $\Rightarrow$ $i_3 = i_0 = -5$ A

2. In the circuit shown, find the value of R that will absorb maximum power from the circuit. Find also the value of this maximum power. (20 pts.)



Solution:

$R = R_{Th}$:

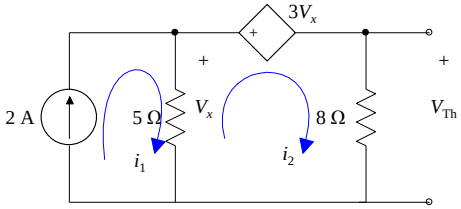


KVL:

$$-V_x + 3V_x + 8(i+1) = 0 \qquad V_x = -5i$$

$$\Rightarrow \quad i = 4 \text{ A} \quad V_t = 40 \text{ V} \quad \therefore \quad R_{Th} = \frac{V_t}{1 \text{ A}} = 40 \ \Omega$$

V_{Th}:



KVL: $-V_x + 3V_x + 8i_2 = 0 \quad \Rightarrow \quad i_2 = -\frac{1}{4}V_x$

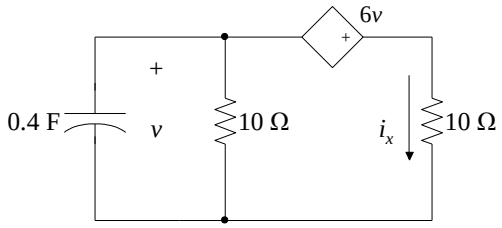
$$V_x = 5(i_1 - i_2) \quad i_1 = 2 \text{ A} \quad \Rightarrow \quad V_x = 10 + \frac{5}{4}V_x$$

$$\Rightarrow \quad V_x = -40 \text{ V}, \quad V_{Th} = -2V_x = 80 \text{ V}$$

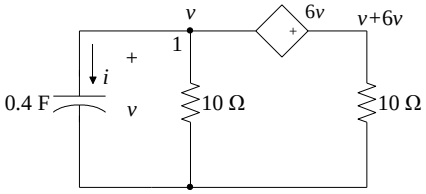
$$P_{\max} = \frac{V_{Th}^2}{4R} = \frac{6400}{4 \times 40} = 40 \text{ W}$$

3. The initial capacitor voltage in the circuit shown is $v(0) = 10$ V.

   (a) Find the capacitor voltage $v(t)$ for $t > 0$. (10 pts.)
   (b) Find the current i_x at $t = 0.5$ s. (5 pts.)



Solution:



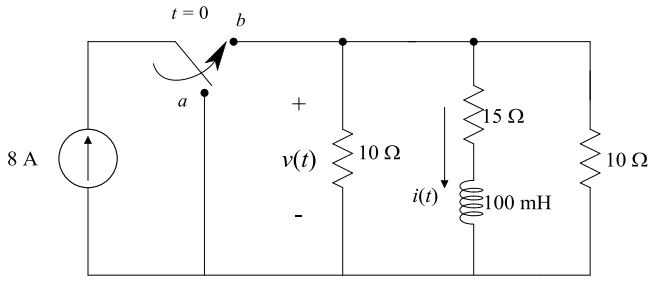
(a) KCL for node 1: $$i + \frac{v}{10} + \frac{v+6v}{10} = 0 \quad \Rightarrow \quad i = -0.8v$$

$$i = C\frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = \frac{i}{0.4} = -2v$$

$$v(t) = v(0)\, e^{-2t} = 10\, e^{-2t} \text{ V}$$

(b) $$i_x = \frac{7v}{10} = 0.7 \times 10\, e^{-2t} = 7e^{-2t} \text{ A} \quad i_x(t = 0.5) = 7\, e^{-1} \text{ A}$$

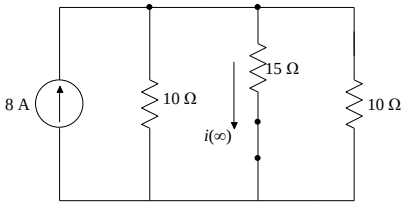
4. In the circuit shown, the switch has been in position *a* for a long time. At t = 0, the switch moves to *b.*
   (a) Find the current $i(t)$ for $t \geq 0$ s. (10 pts.)
   (b) Find the voltage $v(t)$ for $t \geq 0$ s. (10 pts.)



Solution:

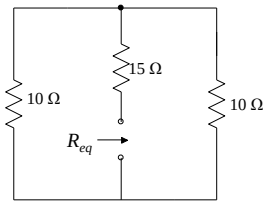
(a) $i(0) = 0$ A since for $t < 0$ the source current is zero.

To find $i(\infty)$:



$$i(\infty) = \frac{(10\,//\,10)}{15 + (10\,//\,10)} \times 8 = 2 \text{ A}$$

R_{eq}:



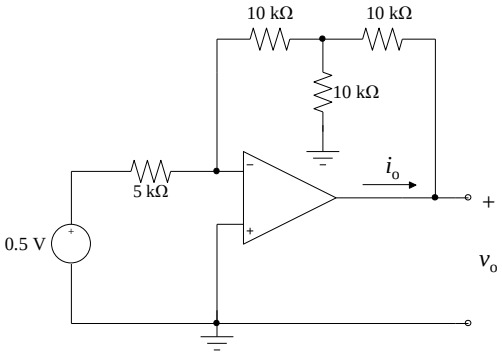
$$R_{eq} = 15 + (10\,//\,10) = 20\,\Omega$$

$$\therefore \quad \tau = \frac{L}{R_{eq}} = \frac{100 \text{ mH}}{20\,\Omega} = 5 \text{ ms}$$

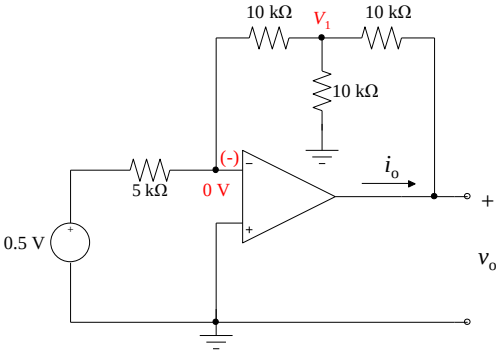
$$\Rightarrow \quad i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau} = 2 + [0 - 2]\, e^{-t/0.005} = 2(1 - e^{-200t})$$

$$v(t) = 15\, i(t) + 0.1 \frac{di(t)}{dt} = 30\,(1 - e^{-200t}) + 0.2 \times 200\, e^{-200t} = 30 + 10\, e^{-200t} \text{ V}$$

5. Find the output voltage v_0 and the output current i_0 of the op-amp circuit shown. (15 pts.)



Solution:



KCL at nodes (-) and V_1 :

$$\frac{0.5-0}{5\ \text{k}\Omega}=\frac{0-V_1}{10\ \text{k}\Omega} \quad \Rightarrow \quad V_1=-1\ \text{V}$$

$$\frac{V_1}{10\ \text{k}\Omega}+\frac{V_1}{10\ \text{k}\Omega}+\frac{V_1-v_0}{10\ \text{k}\Omega}=0 \quad \Rightarrow \quad v_0=3V_1=-3\ \text{V}$$

$$i_0=\frac{v_0-V_1}{10\ \text{k}\Omega}=-0.2\ \text{mA}$$