Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 Circuit Theory I
INFE221 – Electrical Circuits

Spring 2010-11

Instructors:

M. K. Uyguroğlu

Midterm EXAMINATION

March 30, 2011

Duration : 100 minutes

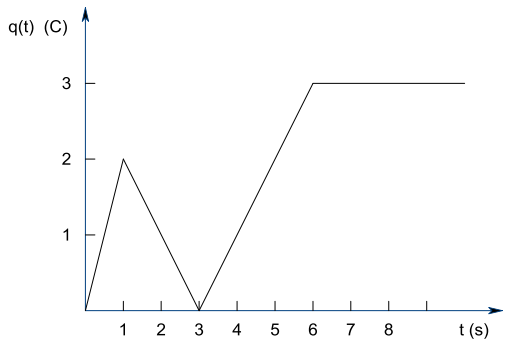
Number of Problems: 6

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	
GROUP NO	

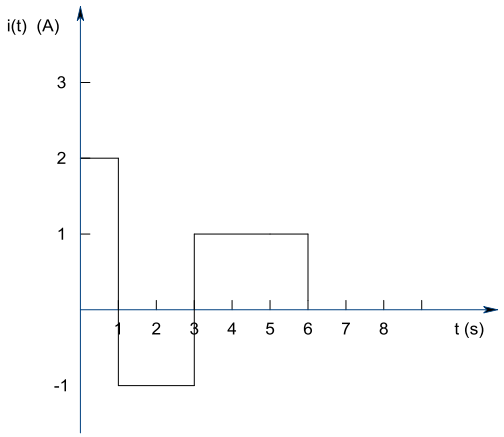
Problem		**Points**
1		15
2		15
3		15
4		15
5		20
6		20
TOTAL		100

1. The charge flowing in a wire is plotted in the following figure. Sketch the corresponding current.

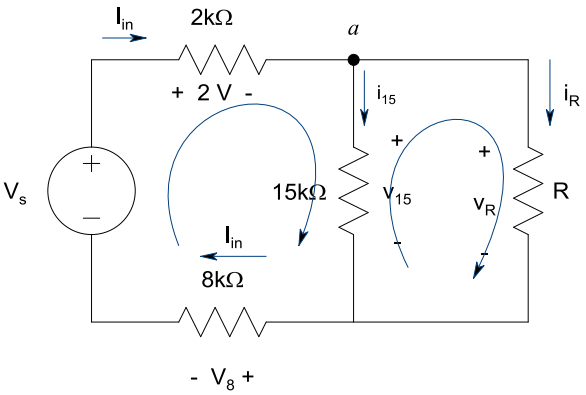


$$q = \begin{cases} 2tC & 0 < t < 1 \\ (-t+3)C & 1 < t < 3 \\ (t-3)\,C & 3 < t < 6 \\ 3C & t > 6 \end{cases}$$

$$i = \frac{dq}{dt} = \begin{cases} 2A & 0 < t < 1s \\ -1A & 1s < t < 3s \\ 1A & 3s < t < 6s \\ 0 & t > 6s \end{cases}$$



2. Consider the following circuit. If the power delivered by the source is 20 mW, find R and V_s.



$$I_{in} = \frac{2}{2k} = 1mA$$

$$P_{delivered\ by\ the\ source} = V_s I_{in} = 20\,\text{mW}$$

$$V_s = \frac{20m}{1m} = 20\,\text{V}$$

$$v_8 = I_{in} 8k = 1m \times 8k = 8\,\text{V}$$

KVL around the left loop:

$$-V_s + 2 + v_{15} + v_8 = 0$$

$$v_{15} = 20 - 2 - 8 = 10\,\text{V}$$

Using Ohm's Law:

$$i_{15} = \frac{v_{15}}{15k} = \frac{10}{15}\,m\text{A}$$

KCL at node *a* yields:

$$I_{in} = i_{15} + i_R \quad \Rightarrow i_R = I_{in} - i_{15} = 1 - \frac{10}{15} = \frac{5}{15} = \frac{1}{3}\,mA$$

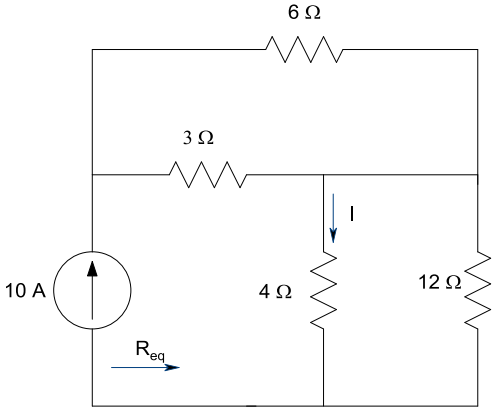
KVL around the right loop:

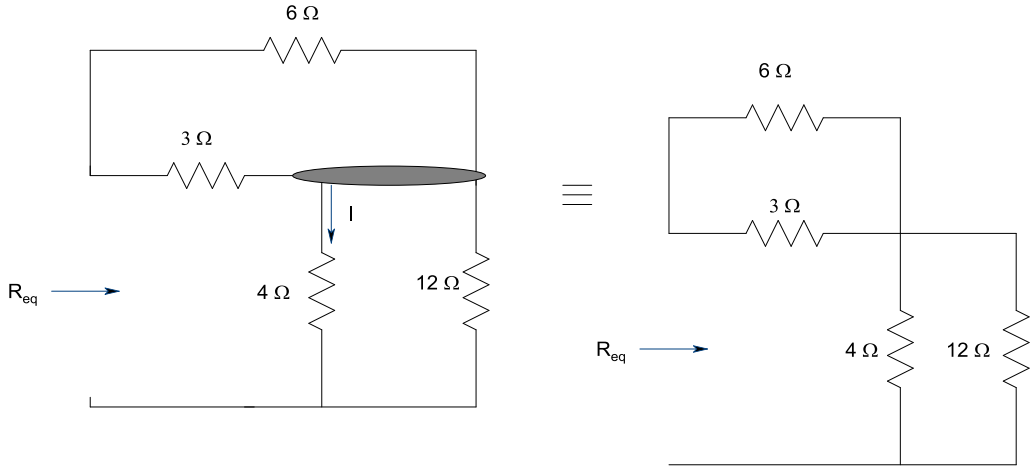
$$-v_{15} + v_R = 0 \Rightarrow v_R = v_{15} = 10V$$

Therefore

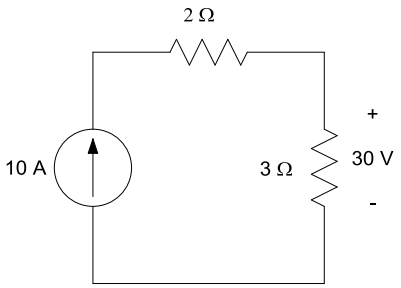
$$R = \frac{v_R}{i_R} = \frac{10}{\frac{1}{3}m} = 30k\,\Omega$$

3. Find the equivalent resistance (R_{eq}) seen by the current source and the current I shown in the following circuit.





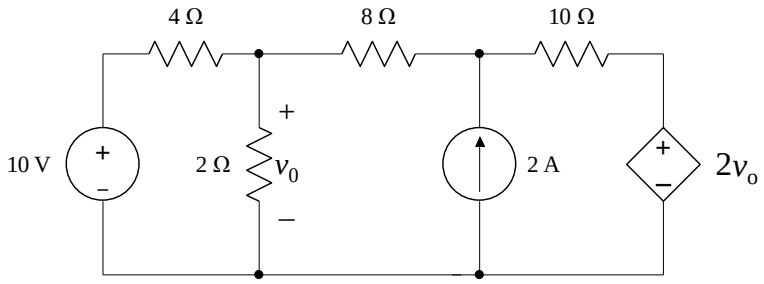
$$R_{eq} = \underbrace{6//3 + 4//12}_{\text{series}} = \underbrace{\frac{6\times 3}{6+3}}_{\text{parallel}} + \underbrace{\frac{4\times 12}{4+12}}_{\text{parallel}} = 2+3 = 5\Omega$$



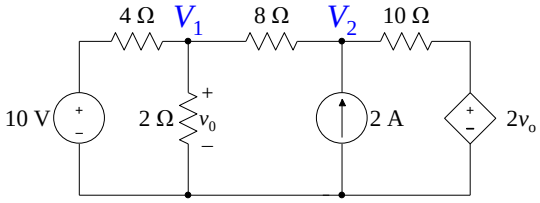
Since the voltage across the 4Ω is 30 V as well, then

$$I = \frac{30}{4} = 7.5A$$

4. In the following circuit, find the voltage v_o using nodal analysis.



SOLUTION:



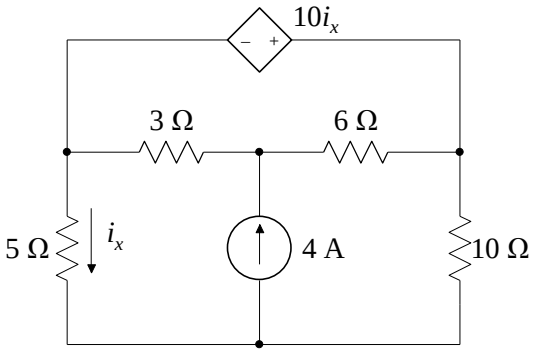
KCL equations:

Node 1: $$\frac{V_1-10}{4}+\frac{V_1}{2}+\frac{V_1-V_2}{8}=0 \quad \Rightarrow \quad 7V_1-V_2=20 \quad (1)$$

Node 2: $$\frac{V_2-V_1}{8}-2+\frac{V_2-2v_0}{4}=0, \quad v_0=V_1 \quad \Rightarrow \quad -13V_1+9V_2=80 \quad (2)$$

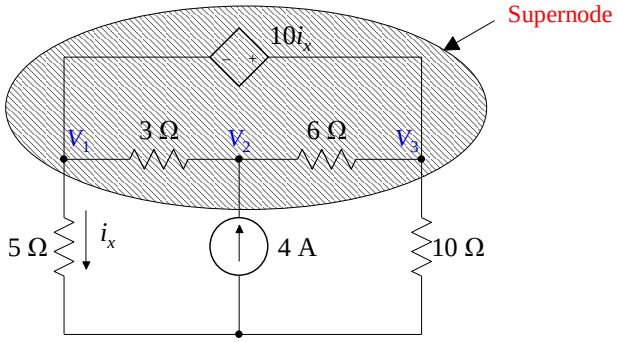
Multiply (1) by 9 and add it to (2): $50V_1=260 \Rightarrow V_1=5.2\text{ V} \Rightarrow v_0=5.2\text{ V}$

5. In the circuit shown below,
   (a) Find the current i_x using nodal analysis. (15 pts)
   (b) Find the power supplied by the 4-A current source. (5 pts)



SOLUTION:

(a)



KCL eqn. for the supernode: $\dfrac{V_1}{5}-4+\dfrac{V_3}{10}=0 \quad\Rightarrow\quad 2V_1+V_3=40$ (1)

KCL eqn. for node 2: $\dfrac{V_2-V_1}{3}-4+\dfrac{V_2-V_3}{6}=0 \quad\Rightarrow\quad -2V_1+3V_2-V_3=24$ (2)

KVL equation for the $10i_x$ source:

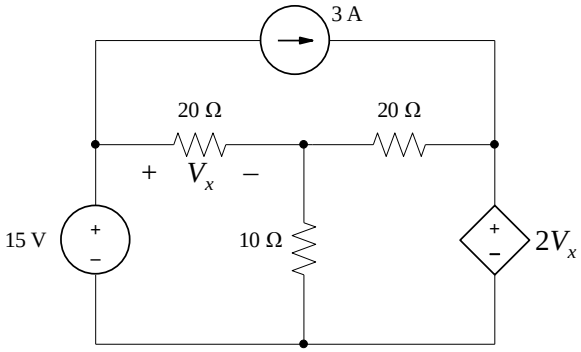
$$V_3-V_1=10i_x=2V_1 \quad\Rightarrow\quad V_3=3V_1$$

$$(1) \quad\Rightarrow\quad V_1=8\text{ V} \quad\Rightarrow\quad i_x=1.6\text{ A}$$

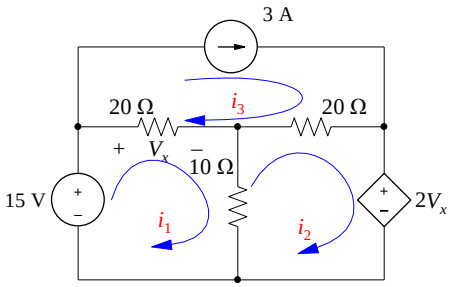
(b) Power supplied by the 4-A source:

$$P=4V_2 \qquad \text{From (2)} \quad V_2=\frac{24+2V_1+V_3}{3}=21.33\text{ V} \qquad\Rightarrow\qquad P=85.33\text{ W}$$

6. In the circuit below, find the voltage V_x using mesh analysis.



SOLUTION:



KVL equations:

Mesh 1:

$$-15+20(i_1-i_3)+10(i_1-i_2)=0,$$
$$i_3=3\text{ A} \quad\Rightarrow\quad 30i_1-10i_2=75 \qquad (1)$$

Mesh 2:

$$10(i_2-i_1)+20(i_2-i_3)+2V_x=0$$
$$V_x=20(i_1-i_3)\Rightarrow \quad 30i_1+30i_2=180 \qquad (2)$$

$$(2)-(1) \quad\Rightarrow\quad 40i_2=105 \quad i_2=2.625\text{ A} \quad\Rightarrow\quad i_1=\frac{75+26.25}{30}=3.375\text{ A}$$

$$\Rightarrow \quad V_x=7.5\text{ V}$$

6.