Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EEE 223 Circuit Theory I

<u>Instructor:</u>

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Final EXAMINATION

January 10, 2005

Duration : 90 minutes

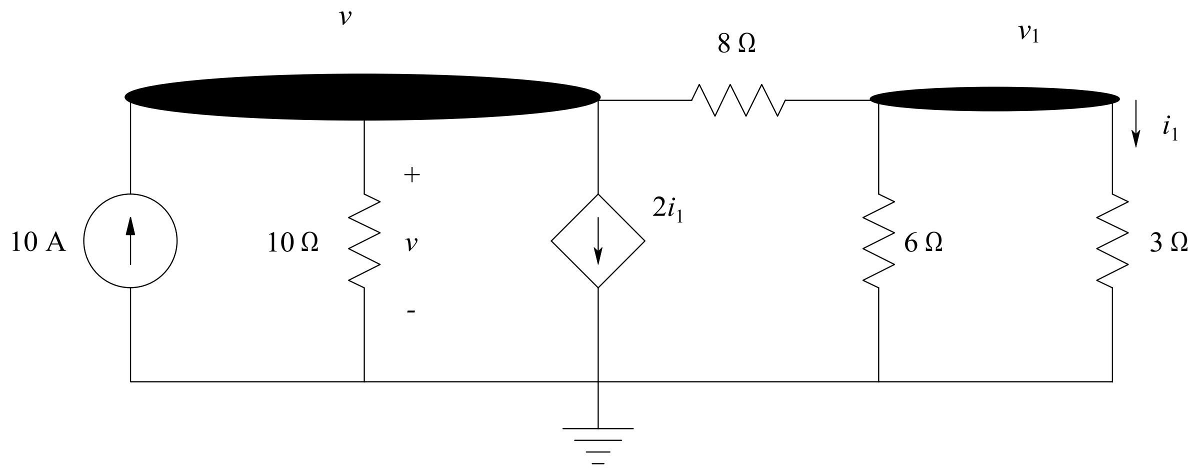
Number of Problems: 6

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	

Problem		**Points**
1		20
2		20
3		20
4		20
5		20
TOTAL		100

1. Find i_1 and v.



KCL at v:

$$\left(\frac{1}{10}+\frac{1}{8}\right)v-\frac{1}{8}v_1=10-2i_1$$

where $i_1=\frac{v_1}{3}$.

Multiply both sides by 240 yields:

$$54v+130v_1=2400 \qquad (1.1)$$

KCL at v_1:

$$-\frac{1}{8}v+\left(\frac{1}{8}+\frac{1}{6}+\frac{1}{3}\right)v_1=0$$

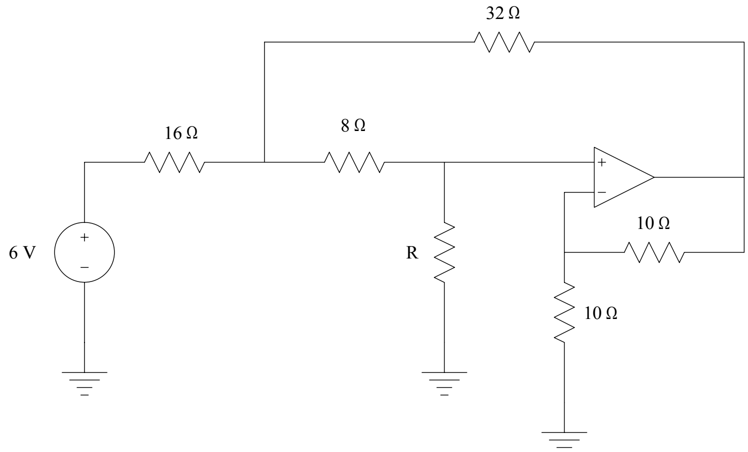
Multiply both sides by 24:

$$-3v+15v_1=0 \Rightarrow v=5v_1 \qquad (1.2)$$

Subst. Eqn.(1.2) into (1.1) yields:

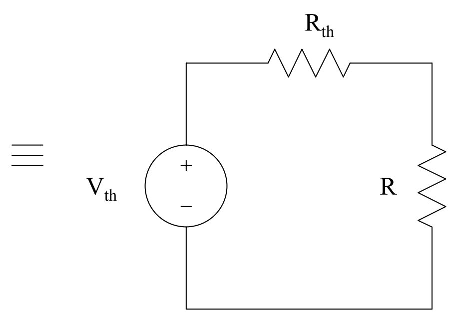
$$400v_1=2400$$
$$v_1=6\text{ V}$$
$$v=5v_1=30\text{ V} \qquad (1.3)$$
$$i_1=\frac{v_1}{3}=2\text{ A}$$

2. Find the maximum power that can be delivered to resistor R and the value of R for maximum power in the circuit shown below.



When $R = R_{th}$, it absorbs maximum power.

And $P_{\max} = \dfrac{(V_{th})^2}{4R_{th}}$

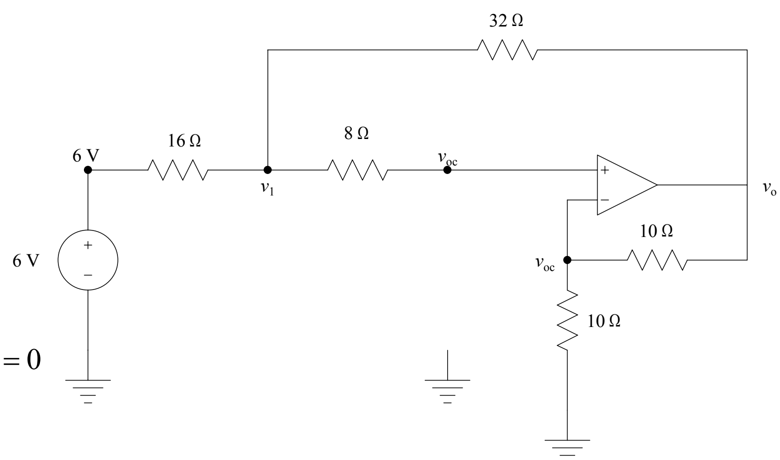


In order to find $V_{th,}$ open circuit voltage is found.

KCL at the inverting terminal:

$$\left(\frac{1}{10}+\frac{1}{10}\right)v_{oc} - \frac{1}{10}v_o = 0$$

$$v_o = 2v_{oc} \text{..................(1)}$$



KCL at the non-inverting terminal:

$$\frac{v_{oc}-v_1}{8} = 0 \Rightarrow v_1 = v_{oc} \text{......(2)}$$

KCL at v_1:

$$\left(\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\right)v_1 - \frac{1}{8}v_{oc} - \frac{1}{32}v_o - \frac{1}{16}6 = 0$$

or

$$7v_1 - 4v_{oc} - v_o = 12 \text{............(3)}$$

Subst. Eqns. (1) and (2) into (3) yields:

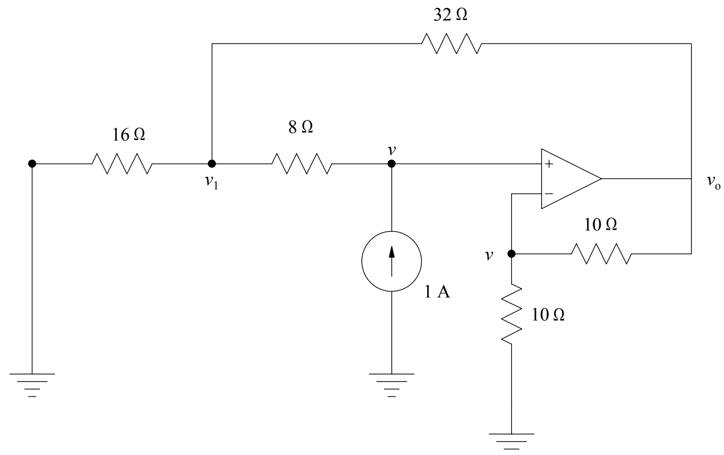
$v_{oc} = 12\ \text{V}$

In order to find $R_{th,}$ 1 A test source can be used.

KCL at the inverting terminal:

$$\left(\frac{1}{10}+\frac{1}{10}\right)v-\frac{1}{10}v_o=0$$

$$v_o=2v..................(4)$$



KCL at the non-inverting terminal:

$$\frac{v-v_1}{8}=1\Rightarrow v_1=v-8........(5)$$

KCL at v_1:

$$\left(\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\right)v_1-\frac{1}{8}v-\frac{1}{32}v_o=0$$

or

$$7v_1-4v-v_o=0............(6)$$

Subst. Eqns. (4) and (5) into (6) yields:

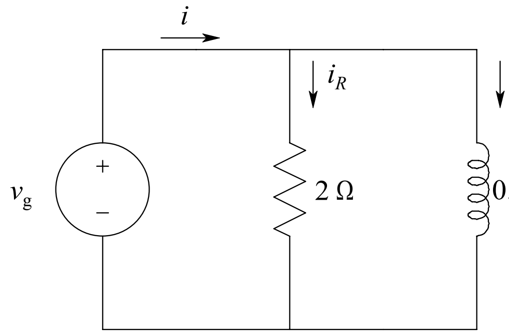
$$v=56\text{ V}$$

$$R_{th}=\frac{v}{1}=56\Omega$$

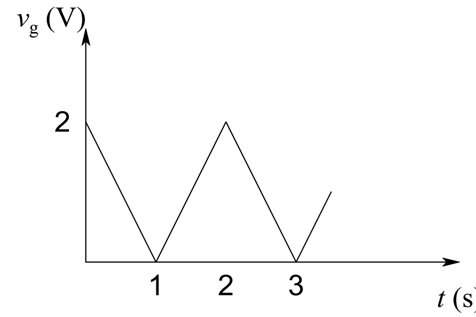
Therefore when $R = 56\ \Omega$ it absorbs maximum power and the maximum power is:

$$P_{\max}=\frac{12^2}{4(56)}=\frac{144}{224}=\frac{9}{14}W$$

3. For the circuit shown in (a), the source voltage is given in (b). Find the current i if $i_L(0) = -1$ A for (a) $0 < t < 1$ s and (b) $1 < t < 2$ s.



(a)



(b)

$$i = i_R + i_L = \frac{v_g}{2} + \frac{1}{0.5}\int_0^t v_g\,dt + i_L(0)$$

where

$$v_g(t) = \begin{cases} -2t+2 & 0<t<1 \\ 2t-2 & 1<t<2 \\ -2t+6 & 2<t<3 \end{cases}$$

$0 < t < 1$:

$$i = i_R + i_L = \frac{-2t+2}{2} + \frac{1}{0.5}\int_0^t (-2t+2)\,dt - 1 = -t + 1 + 2\left[-t^2 + 2t\right] - 1$$

$$i = -2t^2 + 3t \ \ A$$

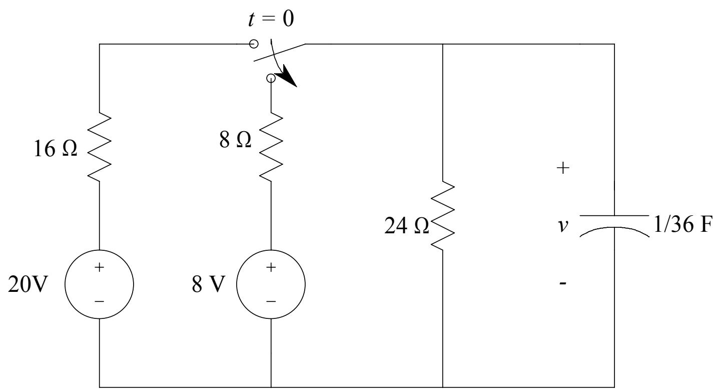
$1 < t < 2$:

$$i = i_R + i_L = \frac{2t-2}{2} + \frac{1}{0.5}\int_1^t (2t-2)\,dt + i_L(1) = t - 1 + 2\left[t^2 - 2t - 1 + 2\right] + 1$$

$$\textit{where}\ \ i_L(1) = -2(1)^2 + 3(1) = 1$$

$$i = 2t^2 - 3t + 2\ A$$

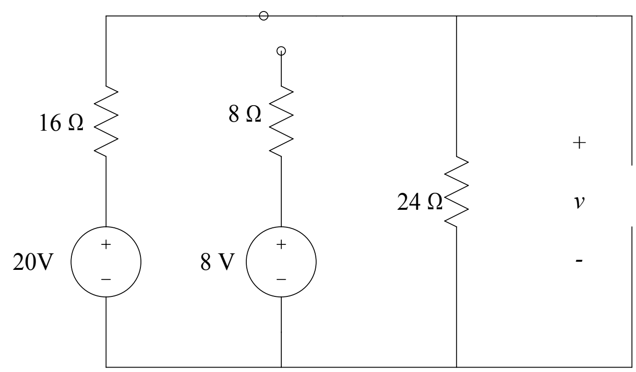
4. Find v for $t > 0$ if the circuit is under dc condition at $t = 0$-.



At $t = 0$-

Using voltage division principle:

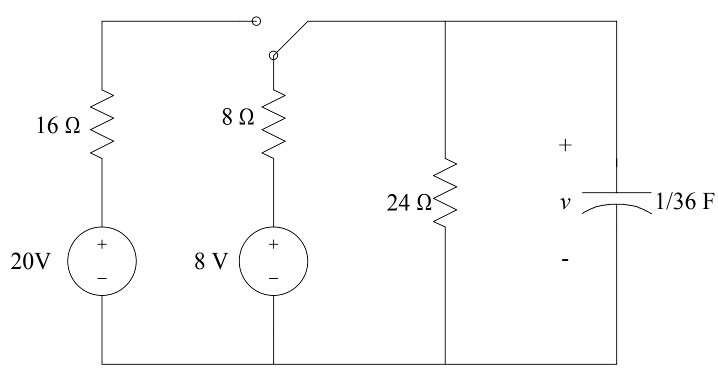
$$v(0) = 20\frac{24}{24+16} = 12V$$



For $t > 0$:

It is known that

$$v(t) = v(\infty) + \left[v(0) - v(\infty)\right]e^{-\frac{t}{\tau}}$$



$$\tau = R_{eq}C = 6\frac{1}{36} = \frac{1}{6}s$$

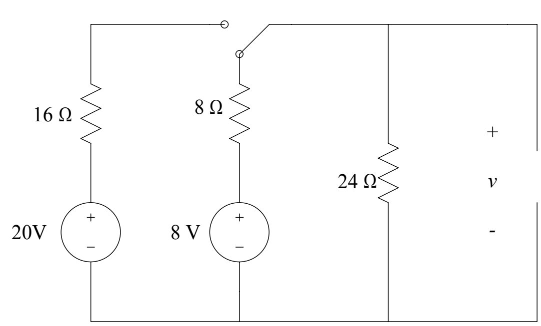
$$R_{eq} = 8 // 24 = \frac{8\times 24}{8+24} = 6\Omega$$

At $t = \infty$ the circuit is under dc conditions.

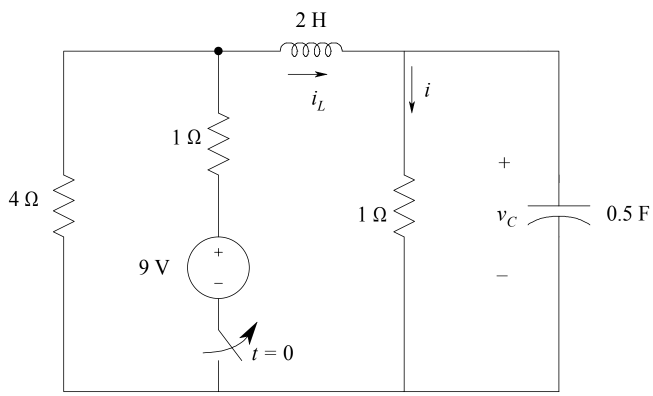
$$v(\infty) = 8\times\frac{24}{32} = 6V$$

Therefore

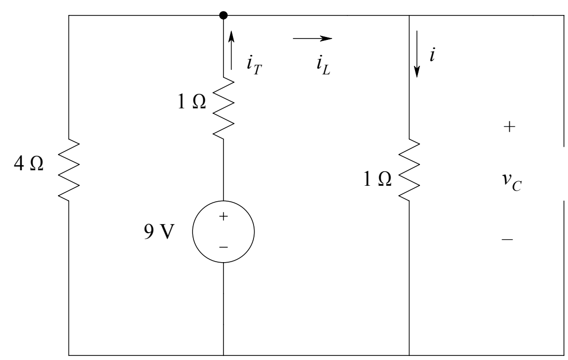
$$v(t) = 6 + \left[12 - 6\right]e^{-6t} = 6\left(1 + e^{-6t}\right)V$$



5. Find i for $t > 0$ if the circuit is under dc conditions at $t = 0^-$.



At $t = 0$-.



$$i_T = \frac{9}{R_T} = 5\,A$$

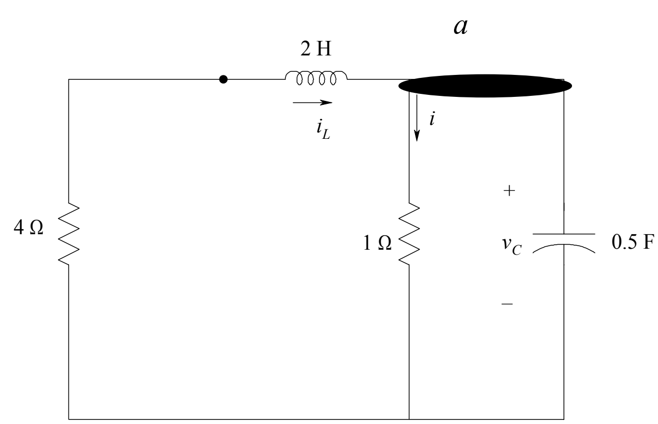
$$R_T = 1 + 1 // 4 = \frac{9}{5}\Omega$$

Using current division principle:

$$i_L(0) = i(0) = 5 \times \frac{4}{4+1} = 4\,A$$

$$v_C(0) = 4(1) = 4V$$

For $t > 0$:



$$i = \frac{v_C}{1}$$

KCL at node a:

$$i_L = v_c + 0.5\frac{dv_c}{dt} \text{ ....(1)}$$

At t=0

$$i_L(0) = v_c(0) + 0.5\frac{dv_c}{dt}(0) \Rightarrow \frac{dv_c}{dt}(0) = 0$$

KVL around the outer loop:

$$4i_L + 2\frac{di_L}{dt} + v_C = 0 \text{ ......(2)}$$

Subst. Eqn(1) into (2) yields:

$$4\left(v_C + 0.5\frac{dv_C}{dt}\right) + 2\frac{d}{dt}\left(v_C + 0.5\frac{dv_C}{dt}\right) + v_C = 0$$

$$\frac{d^2v_C}{dt^2} + 4\frac{dv_C}{dt} + 5v_C = 0$$

Characteristic equation

$s^2 + 4s + 5 = 0$

$s_{1,2} = \dfrac{-4 \mp \sqrt{16-20}}{2} = -2 \mp j1$ Complex conjugate natural frequencies

Therefore

$$v_C(t) = e^{-2t}\left(A_1 \cos t + A_2 \sin t\right)$$

In order to find A_1 and A_2 initial conditions will be used.

$$v_C(0) = 4 = A_1$$

$$\frac{dv_C}{dt} = -2e^{-2t}\left(A_1 \cos t + A_2 \sin t\right) + e^{-2t}\left(-A_1 \sin t + A_2 \cos t\right)$$

$$\frac{dv_C}{dt}(0) = -2A_1 + A_2 = 0$$

$$A_2 = 8$$

$\therefore \quad v_C(t) = e^{-2t}\left(4\cos t + 8\sin t\right)\ V$