



**Faculty of Engineering**

**ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT**

***EEE 223 Circuit Theory I***

**Fall 2005-06**

**Instructors:**

**M. K. Uygurođlu (Gr: 01)**

**E. Erdil (Gr: 02)**

***Final EXAMINATION***

**Jan 27, 2006**

***Duration : 150 minutes***

**Number of Problems: 9**

***Good Luck***

<b>STUDENT'S</b>	
NUMBER	
NAME	
SURNAME	
GROUP NO	

<b>Problem</b>		<b>Points</b>	<b>Problem</b>		<b>Points</b>
1		8	6		15
2		8	7		15
3		8	8		15
4		8	9		15
5		8			
<b>TOTAL</b>					<b>100</b>

1. Find the equivalent resistance  $R_{ab}$  for the circuit in Fig. P1.

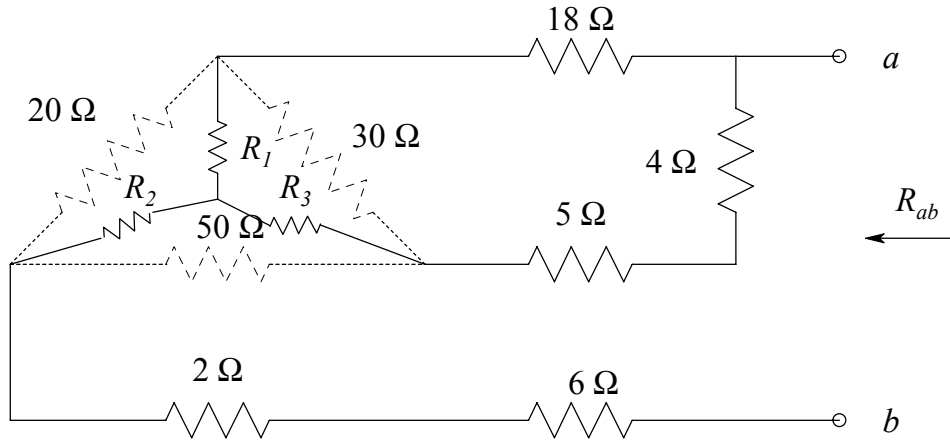


Figure P1

$$R_1 = \frac{20 \times 30}{20 + 30 + 50} = 6\Omega \quad R_2 = \frac{20 \times 50}{100} = 10\Omega \quad R_3 = \frac{50 \times 30}{100} = 15\Omega$$

$$R_{ab} = (18 + R_1) // (4 + 5 + R_3) + R_2 + 2 + 6$$

$$R_{ab} = \underbrace{24 // 24}_{12} + 10 + 2 + 6 = 30\Omega$$

2. Convert the circuit in Fig. P2 to current source representation.

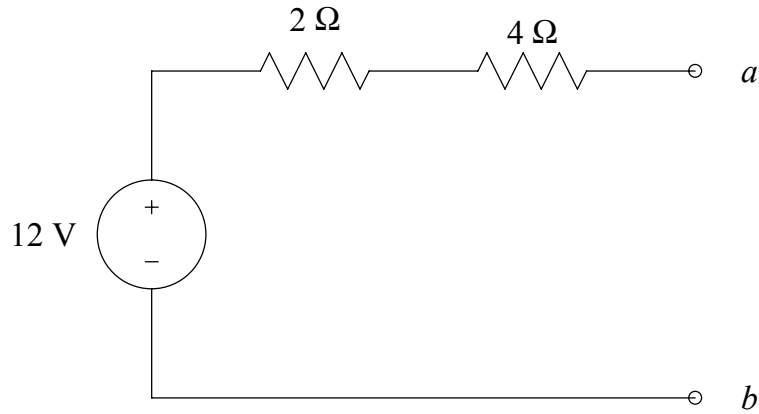
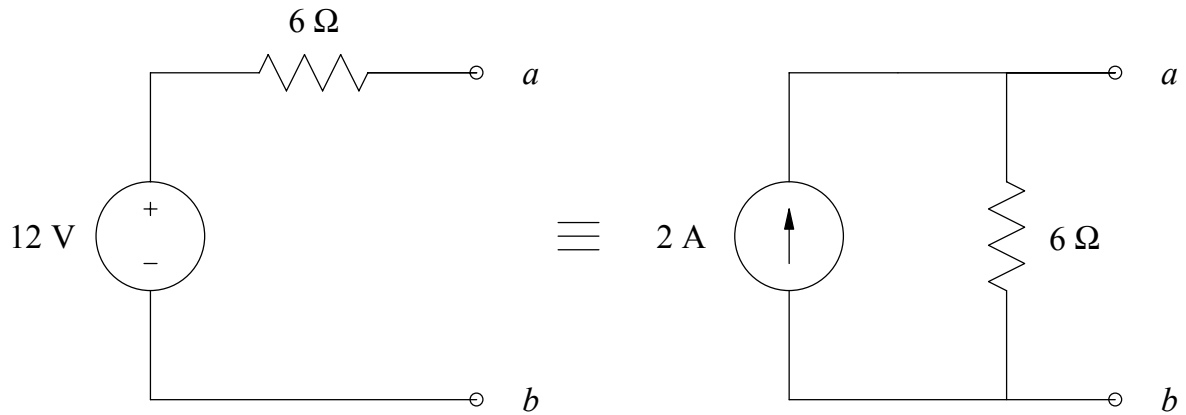


Figure P2



3. Calculate what value of  $R_L$  will absorb maximum power and find the maximum power for the circuit in Fig.P3.

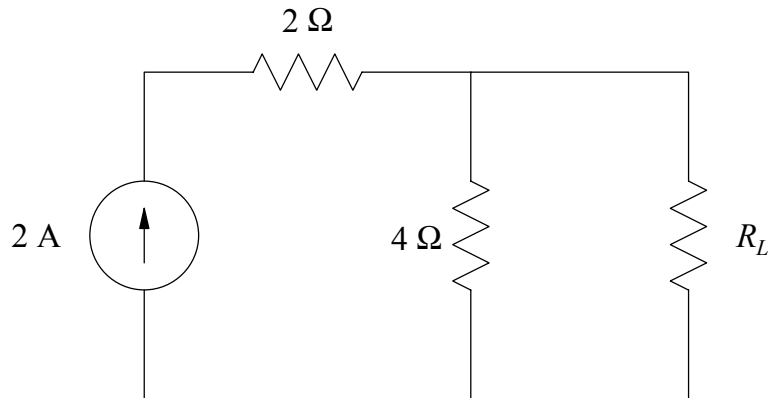
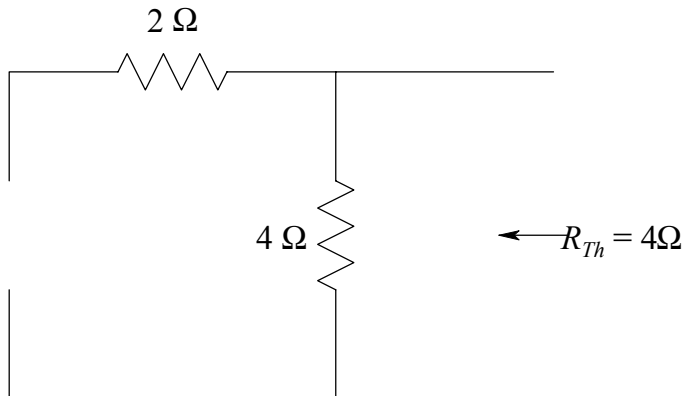


Figure P3

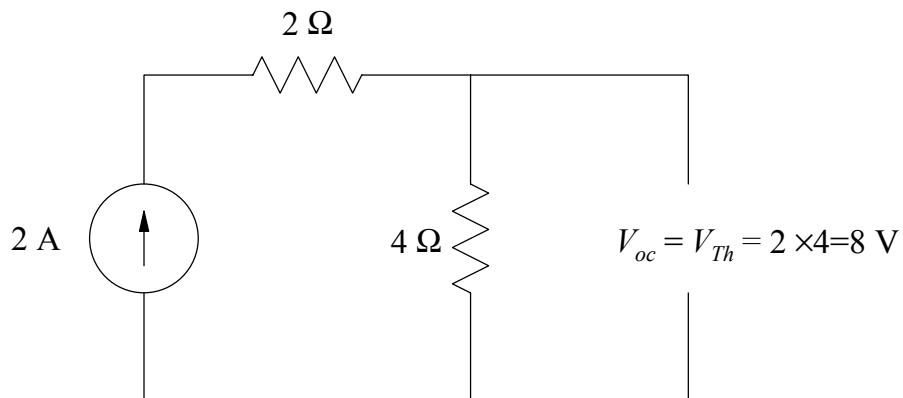
When  $R_L = R_{Th}$  it will absorb maximum power. The maximum power absorbed by  $R_L$  :

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{8^2}{4 \times 4} = \frac{64}{16} = 4W =$$

In order to find  $R_{Th}$  :



$V_{Th}$  :



4. Use superposition to find  $i$  for the circuit in Fig.P4.

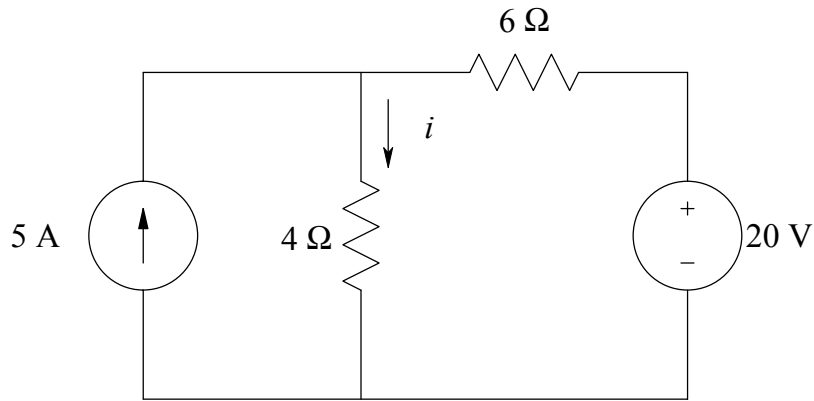
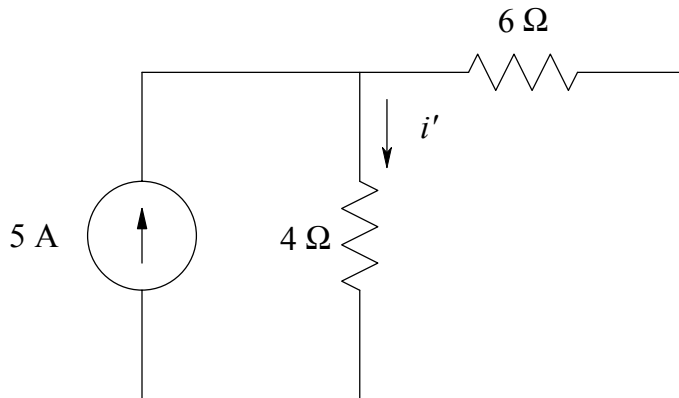
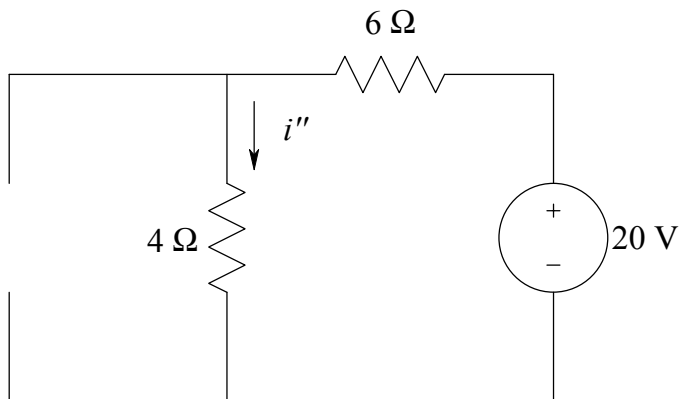


Figure P4



By using current division

$$i' = \frac{5 \times 6}{6 + 4} = 3 A$$



$$i'' = \frac{20}{6 + 4} = 2 A$$

$$i = i' + i'' = 3 + 2 = 5 A$$

5. (a) Use current division principle to find the current flowing through  $R_3$  for the circuit in Fig.P5(a).

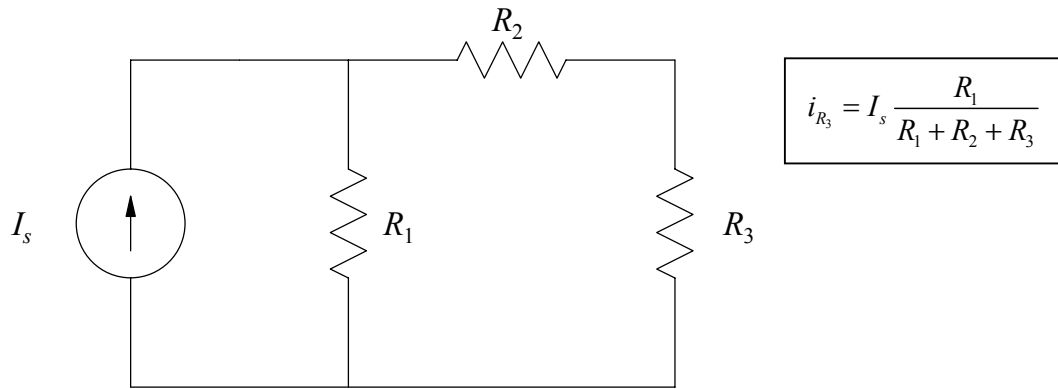


Figure 5(a)

- (b) Use voltage division principle to find the voltage across  $R_3$  for the circuit in Fig.P5(b).

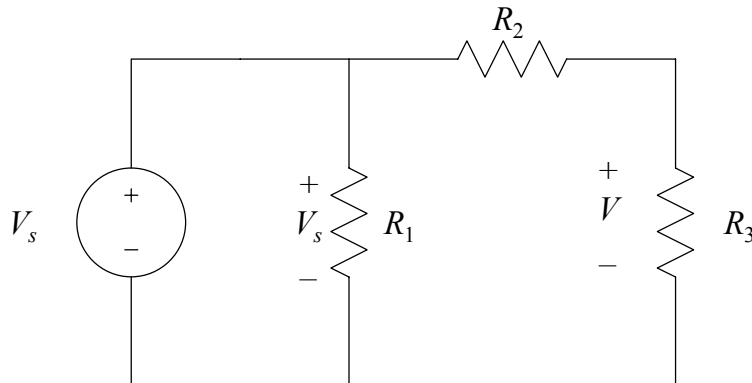
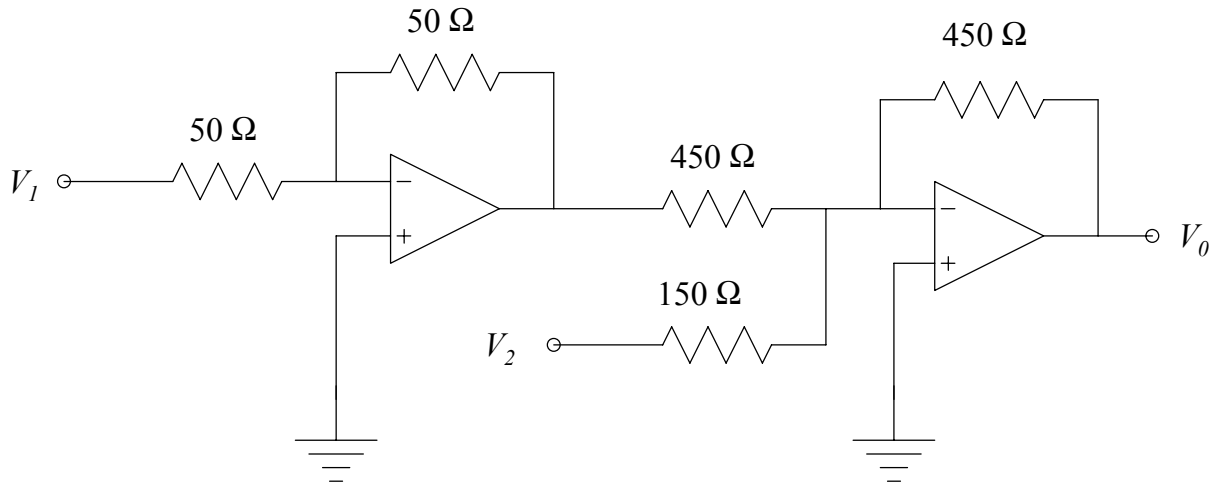


Figure 5(b)

$$V = V_s \frac{R_3}{R_2 + R_3}$$

6. Given signals  $V_1$  and  $V_2$ , cascade an inverting op-amp with an inverting summer to have an output

$$V_0 = V_1 - 3V_2$$



7. Find  $v_0$  in the circuit in Fig. P7.

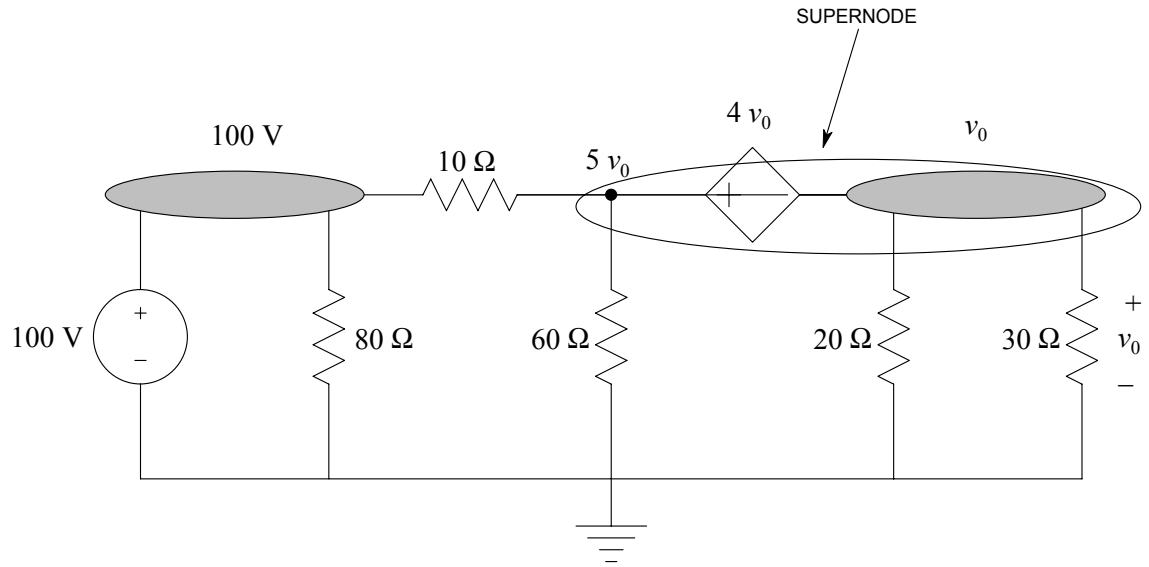


Figure P7

KCL at the SUPERNODE:

$$\frac{5v_0 - 100}{10} + \frac{5v_0}{60} + \frac{v_0}{20} + \frac{v_0}{30} = 0$$

multiply both sides by 60 yields:

$$30v_0 - 600 + 5v_0 + 3v_0 + 2v_0 = 0$$

$$40v_0 = 600$$

$$\boxed{v_0 = 15 \text{ V}}$$



8. The switch in the circuit in Fig. P8 has been in position 1 for a long time before moving to position 2 at  $t = 0$ . Find  $i(t)$  for  $t \geq 0$ .

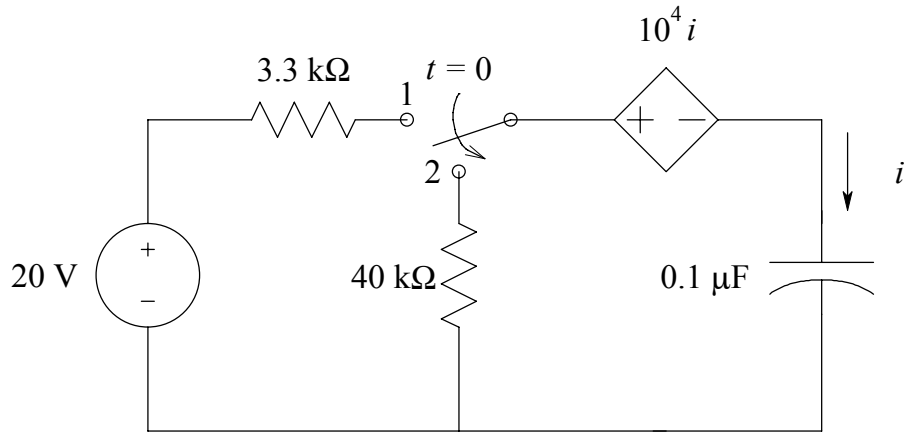
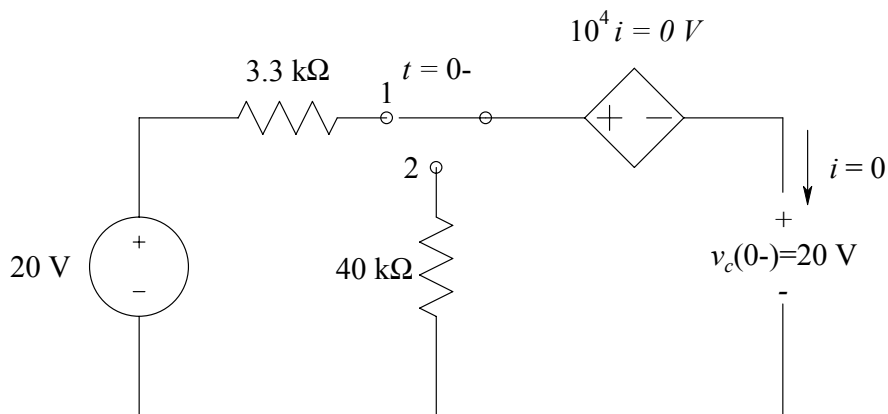


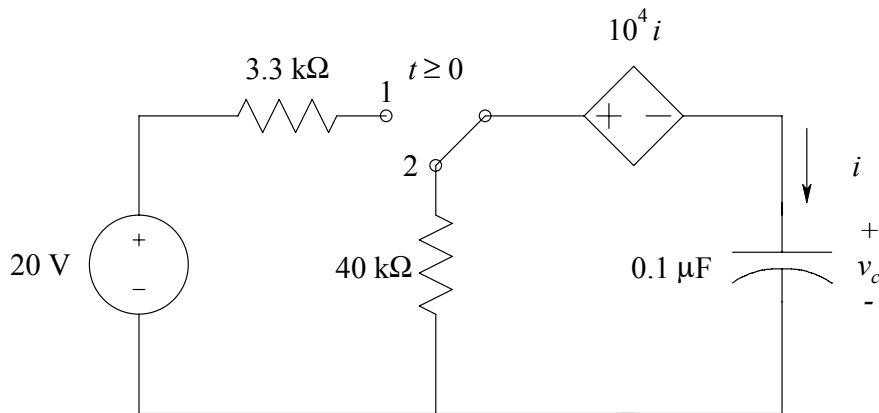
Figure P8

at  $t = 0^-$



for  $t \geq 0$

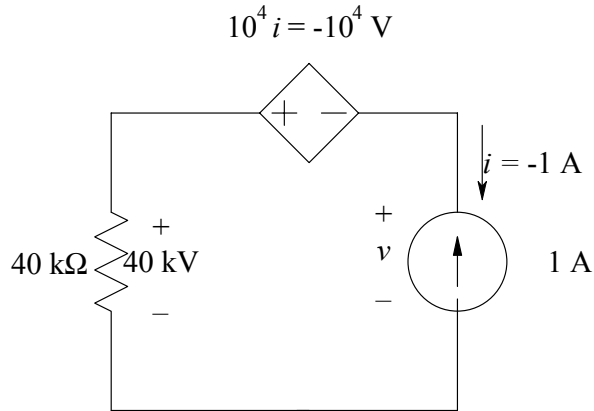
$$v_c(0^-) = v_c(0^+) = 12 \text{ V}$$



$$i = 0.1 \times 10^{-6} \frac{dv_c}{dt} \quad \text{and} \quad v_c = \underbrace{v_c(0)}_{20 \text{ V}} e^{-\frac{t}{\tau}}$$

$$\tau = R_{eq} C$$

In order to find the equivalent resistance  $R_{eq}$  seen by the capacitor, a test source is needed to connect .



KVL around the loop:

$$-40k + (-10^4) + v = 0$$

$$v = 50 \text{ kV}$$

$$R_{eq} = \frac{v}{1} = 50 \text{ k}\Omega$$

Therefore

$$\tau = 50 \times 10^3 \times 0.1 \times 10^{-6} = 5 \times 10^{-3}$$

$$v_c = 20 e^{-200t} \text{ V}$$

$$i = 0.1 \times 10^{-6} \frac{d}{dt} (20 e^{-200t}) = 0.1 \times 10^{-6} (-4000 e^{-200t}) = -0.4 e^{-200t} \text{ mA}$$

9. The switch in the circuit in Fig. P9 has been open for a long time before closing at  $t = 0$ . Find  $v(t)$  for  $t \geq 0$ .

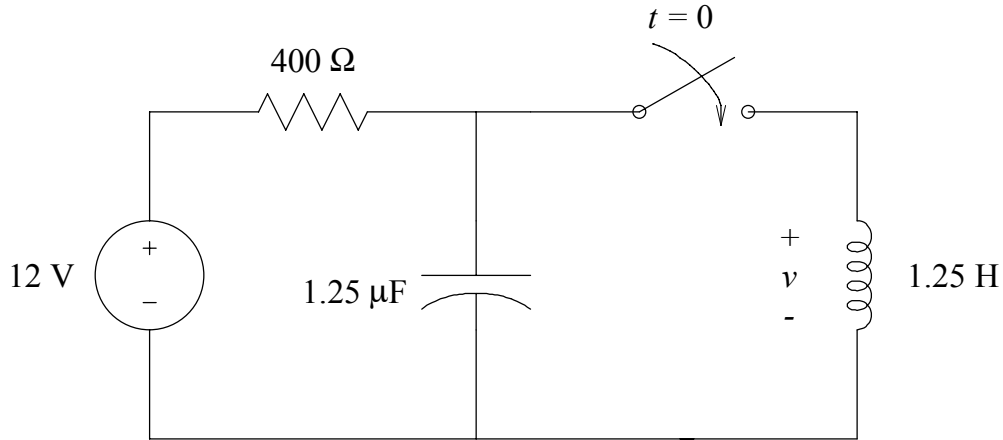
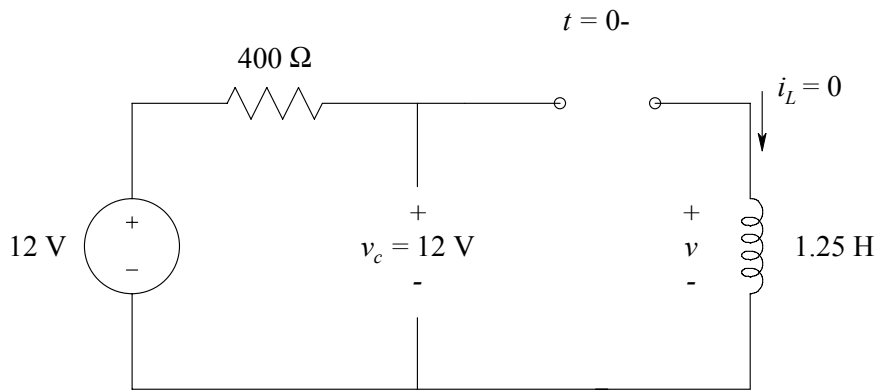


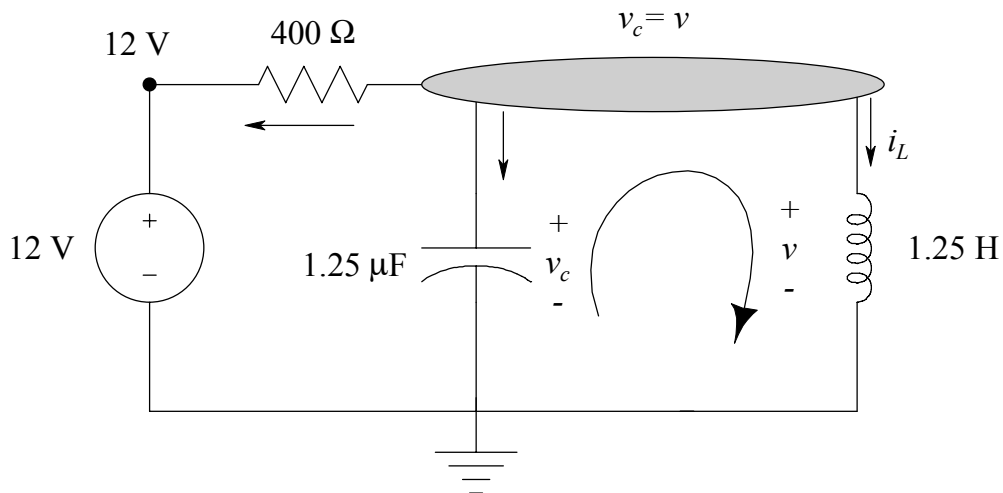
Figure P9

at  $t = 0^-$



$$v_c(0^-) = 12 \text{ V} \quad i_L(0^-) = 0 \text{ A}$$

for  $t \geq 0$



KCL at node  $v_c$ :

$$\frac{v_c - 12}{400} + 1.25 \times 10^{-6} \frac{dv_c}{dt} + i_L = 0 \quad (1)$$

KVL around the loop:

$$-v_c + 1.25 \frac{di_L}{dt} = 0 \quad (2)$$

If Eq. (1) is written at  $t = 0+$  then  $\frac{dv_c}{dt}(0+) = 0$ .

From Eq.(1)

$$i_L = -\frac{v_c - 12}{400} - 1.25 \times 10^{-6} \frac{dv_c}{dt} \quad (3)$$

Substitute Eq.3) into (2) yields:

$$-v_c + 1.25 \frac{d}{dt} \left( -\frac{v_c - 12}{400} - 1.25 \times 10^{-6} \frac{dv_c}{dt} \right) = 0$$

$$1.25^2 \times 10^{-6} \frac{d^2 v_c}{dt^2} + \frac{1.25}{400} \frac{dv_c}{dt} + v_c = 0$$

or multiply both sides by  $\frac{10^6}{1.25^2}$

$$\frac{d^2 v_c}{dt^2} + \frac{10^6}{500} \frac{dv_c}{dt} + \frac{10^6}{1.25^2} v_c = 0 \Rightarrow \boxed{\frac{d^2 v_c}{dt^2} + 2000 \frac{dv_c}{dt} + 640000 v_c = 0}$$

characteristic equation:

$$s^2 + 2000s + 640000 = 0$$

$$s_{1,2} = \frac{-2000 \mp \sqrt{2000^2 - 4 \times 640000}}{2} = -1000 \mp 600$$

The natural frequencies are

$$\left. \begin{matrix} s_1 = -400 \\ s_2 = -1600 \end{matrix} \right\} \text{real and distinct natural frequencies} \Rightarrow \text{OVERDAMPED case}$$

$$v_c(t) = A_1 e^{-400t} + A_2 e^{-1600t}$$

$$v_c(0) = A_1 + A_2 = 12 \dots \dots \dots (4)$$

$$\frac{d}{dt} v_c(t) = -400 A_1 e^{-400t} - 1600 A_2 e^{-1600t}$$

$$\frac{d}{dt} v_c(0) = -400 A_1 - 1600 A_2 = 0 \dots \dots \dots (5)$$

Multiply Eq.(4) by 400 and add to Eq.(5) gives:

$$-1200 A_2 = 4800$$

$$\boxed{A_2 = -4}$$

Substitute the value of  $A_2$  into Eq.(4) yields:

$$\boxed{A_1 = 16}$$

Therefore

$$v(t) = v_c(t) = 16e^{-400t} - 4e^{-1600t} \text{ V}$$