Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 Circuit Theory I
INFE221 – Electrical Circuits

Spring 2007-08

Instructor:

M. K. Uyguroğlu

Final EXAMINATION

May 27, 2008

Duration : 120 minutes

Number of Problems: 4

Good Luck

STUDENT'S	
NUMBER	
NAME	SOLUTIONS
SURNAME	
GROUP NO	

Problem		Points
1		25
2		25
3		25
4		25
TOTAL		100

1. Find v_0 in the circuit in Fig. P1 using
   a) Superposition. (15 pts.)
   b) Source transformation. (10 pts.)

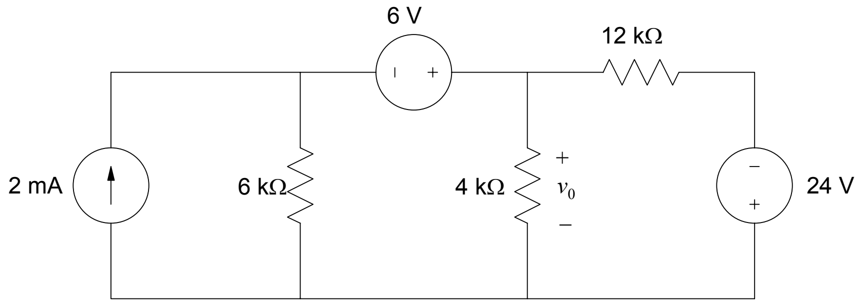
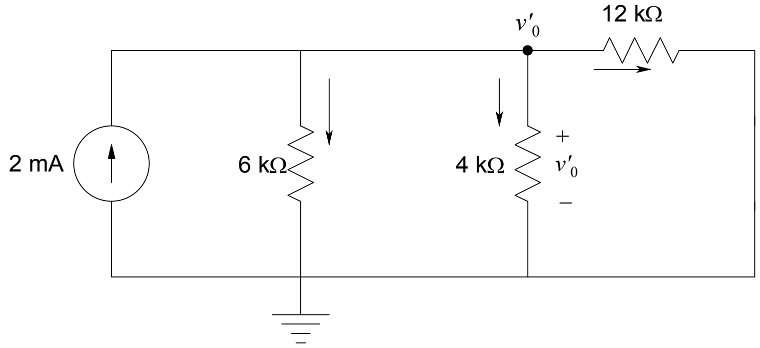


Figure P1

2 mA Current Source is active



KCL at v_0:

$$\frac{v_0'}{6k}+\frac{v_0'}{4k}+\frac{v_0'}{12k}=2m$$

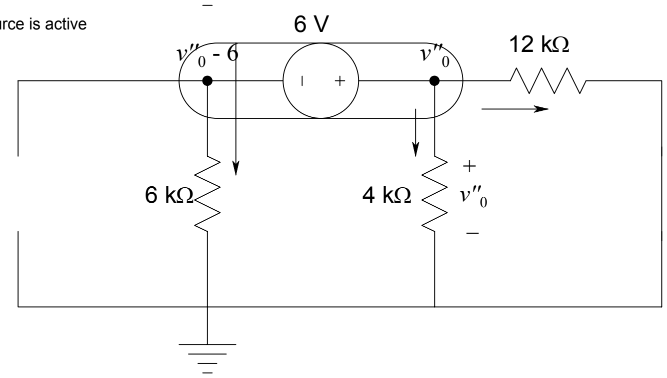
Multiply both sides by 12k yields:

$$2v_0'+3v_0'+v_0'=24$$

$$6v_0'=24$$

$$v_0'=4V$$

6 V Voltage Source is active



KCL at supernode:

$$\frac{v_0''-6}{6k}+\frac{v_0''}{4k}+\frac{v_0''}{12k}=0$$

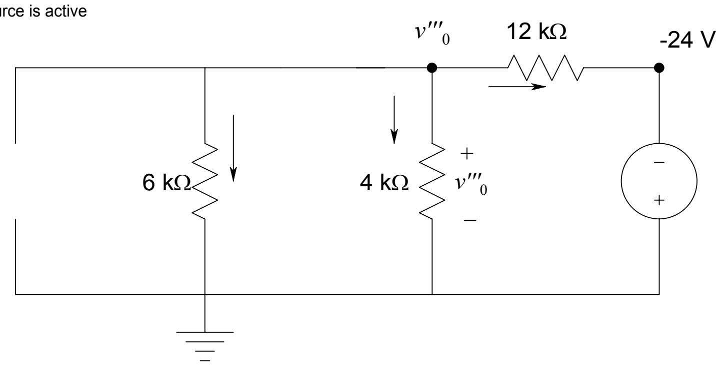
Multiply both sides by 12k yields:

$$2v_0''-12+3v_0''+v_0''=0$$

$$6v_0''=12$$

$$v_0''=2V$$

24V Voltage Source is active



KCL at v_0''':

$$\frac{v_0'''}{6k}+\frac{v_0'''}{4k}+\frac{v_0'''+24}{12k}=0$$

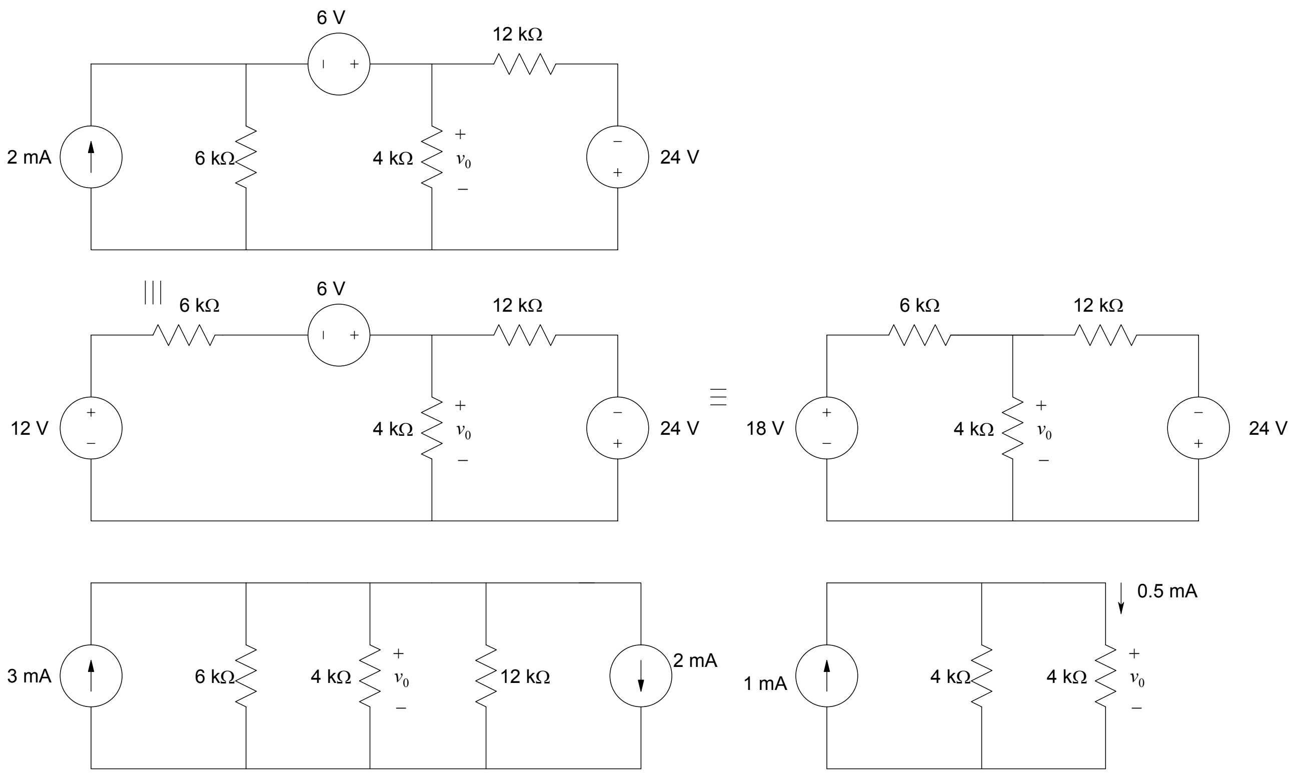
Multiply both sides by 12k yields:

$$2v_0'''+3v_0'''+v_0'''+24=0$$

$$6v_0'''=-24$$

$$v_0'''=-4V$$

$$v_0=v_0'+v_0''+v_0'''=4+2-4=2\ V$$



$$v_0 = 4k \times 0.5m = 2V$$

2. Find R_L for maximum power transfer and the maximum power that can be transferred to the load in Fig. P2. (20 pts.)

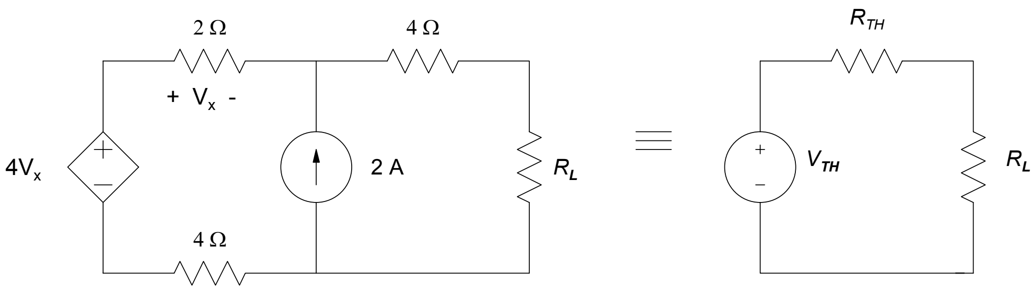
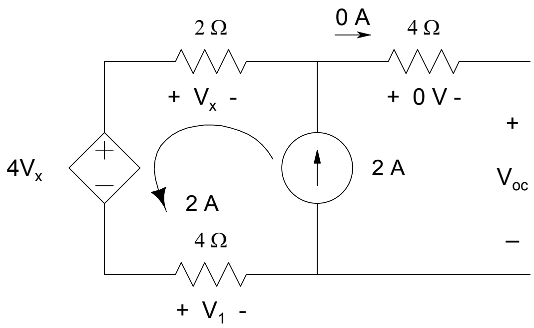


Figure P2

When $R_L = R_{TH}$ it absorbs maximum power.

$$P_{\max} = \frac{V_{TH}{}^2}{4R_{TH}}$$

In order to find V_{TH}, open circuit voltage V_{oc} is found.



KVL around the outer loop:

$$-4V_x + V_x + V_{OC} - V_1 = 0$$

$$V_{OC} = 3V_x + V_1$$

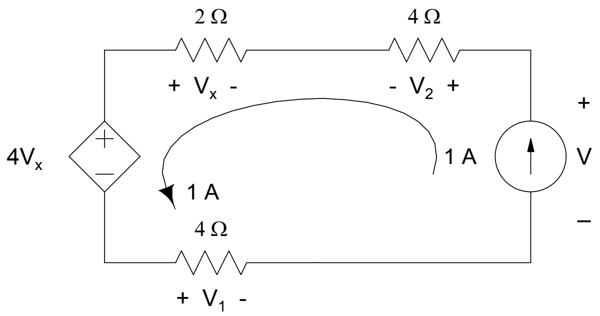
where

$$V_1 = 2 \times 4 = 8V$$

$$V_x = -2 \times 2 = -4V$$

$$\therefore V_{OC} = -12_x + 8 = -4V$$

In order to find R_{TH} :



KVL around the loop:

$$-4V_x + V_x - V_2 + V - V_1 = 0$$

$$V = 3V_x + V_2 + V_1$$

where

$$V_x = -1 \times 2 = -2V$$

$$V_1 = 4 \times 1 = 4V$$

$$V_2 = 4 \times 1 = 4V$$

$$\therefore V = -6 + 4 + 4 = 2V$$

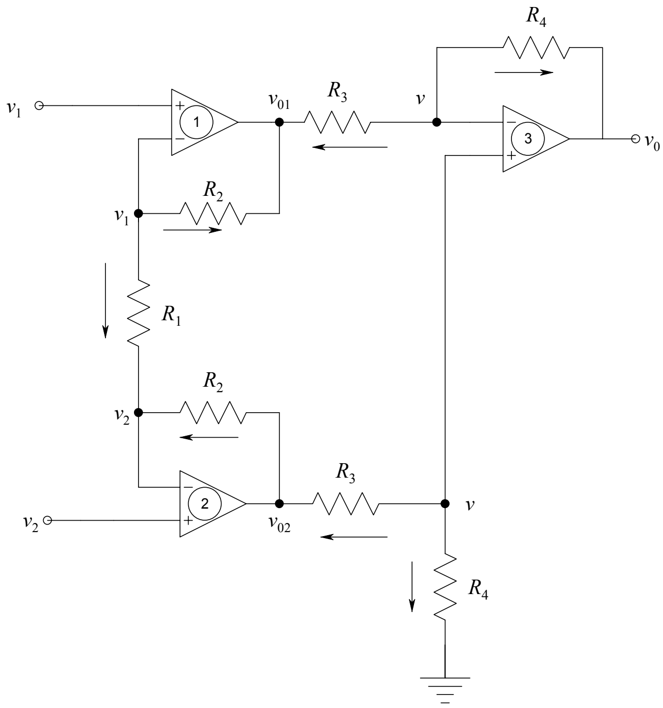
$$R_{TH} = \frac{V}{1} = 2\Omega$$

When $R_L = 2\Omega$ it absorbs maximum power.

$$P_{\max} = \frac{(-4)^2}{4 \times 2} = \frac{16}{8} = 2W$$

3. Show that the output of the circuit in Fig.P3 is

$$v_0 = \frac{R_4}{R_3}\left(1+2\frac{R_2}{R_1}\right)(v_2 - v_1)$$



KCL at the inverting input terminal of OP-AMP (3):

$$\frac{v - v_{01}}{R_3} + \frac{v - v_0}{R_4} = 0$$

$$\left(\frac{1}{R_3} + \frac{1}{R_4}\right)v - \frac{1}{R_3}v_{01} = \frac{v_0}{R_4}$$

$$v_0 = \left(\frac{R_4 + R_3}{R_3}\right)v - \frac{R_4}{R_3}v_{01}........(1)$$

KCL at the inverting input terminal of OP-AMP (3):

$$\frac{v - v_{02}}{R_3} + \frac{v}{R_4} = 0$$

$$\left(\frac{1}{R_3} + \frac{1}{R_4}\right)v = \frac{v_{02}}{R_4}$$

$$v = \left(\frac{R_4}{R_3 + R_4}\right)v_{02}...........(2)$$

Subst. of Eq.(2) into (1) gives:

$$v_0 = \frac{R_4}{R_3}(v_{02} - v_{01})........(3)$$

KCL at the inverting input terminal of OP-AMP (1):

$$\frac{v_1 - v_{01}}{R_2} + \frac{v_1 - v_2}{R_1} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_1 - \frac{1}{R_1}v_2 = \frac{v_{01}}{R_2}$$

$$v_{01} = \left(\frac{R_2 + R_1}{R_1}\right)v_1 - \frac{R_2}{R_1}v_2........(4)$$

KCL at the inverting input terminal of OP-AMP (2):

$$\frac{v_2 - v_{02}}{R_2} + \frac{v_2 - v_1}{R_1} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_2 - \frac{1}{R_1}v_1 = \frac{v_{02}}{R_2}$$

$$v_{02} = \left(\frac{R_2 + R_1}{R_1}\right)v_2 - \frac{R_2}{R_1}v_1........(5)$$

Subst. of Eqns.(4) and (5) into (3) yields:

$$v_{01} = \frac{R_4}{R_3}\left(\left(\frac{R_2 + R_1}{R_1}\right)v_2 - \frac{R_2}{R_1}v_1 - \left(\frac{R_2 + R_1}{R_1}\right)v_1 + \frac{R_2}{R_1}v_2\right)$$

$$v_{01} = \frac{R_4}{R_3}\left(\left(2\frac{R_2}{R_1} + 1\right)(v_2 - v_1)\right)$$

4. Suppose that the switch in Fig.P4 has been closed for a long time and is opened at $t = 0$. Find $i_0(t)$ for $t > 0$.

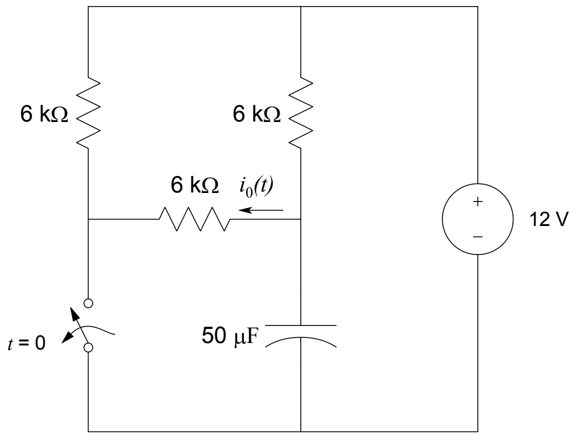
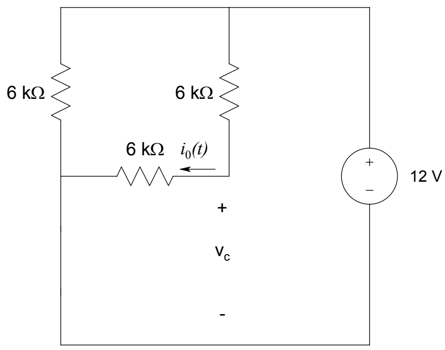


Figure P4

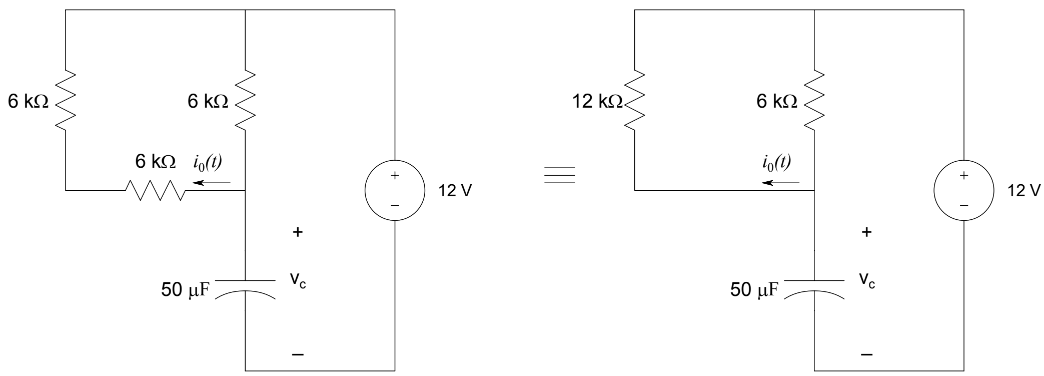
At $t = 0-$, the circuit is under dc conditions. (Capacitor acts like an open circuit.)



Using voltage division principle:

$$\boxed{v_c(0-) = 12\frac{6}{6+6} = 6\,\text{V}}$$

For $t \geq 0$



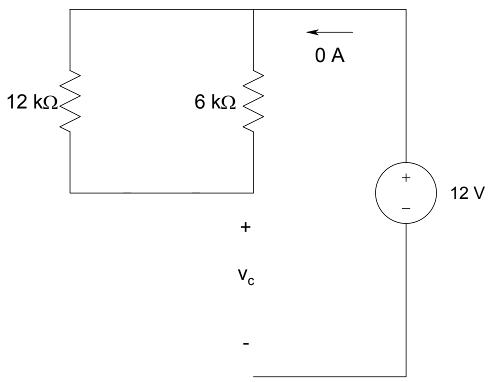
Since the circuit contains an independent source for $t \geq 0$, the voltage across the capacitor is:

$$v_c(t) = v_c(\infty) + \left[v_c(0) - v_c(\infty)\right]e^{-\frac{t}{\tau}}$$

Where $v_c(0) = v_c(0-) = 6 \text{ V}$.

In order to find $v_c(\infty)$, let us consider the circuit at $t=\infty$. At $t=\infty$ the circuit is under dc conditions.

$$v_c(\infty) = 12 \text{ V}$$

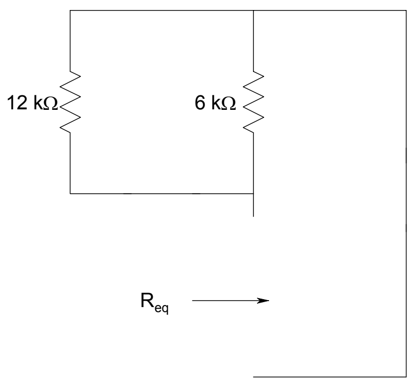


$\tau = R_{eq}C$

Where R_{eq} is the resistance seen by the capacitor.

$$R_{eq} = 12k // 6k = 4k\ \Omega$$

$$\tau = R_{eq}C = 4\times10^{3}\times50\times10^{-6} = 200\times10^{-3} = \frac{1}{5}s$$



$$v_c(t) = 12 + [6-12]e^{-5t} \text{ V}$$

$$i_0(t) = \frac{v_c(t)-12}{12} = \frac{12+[6-12]e^{-5t}-12}{12} = -0.5e^{-5t} \text{ A}$$