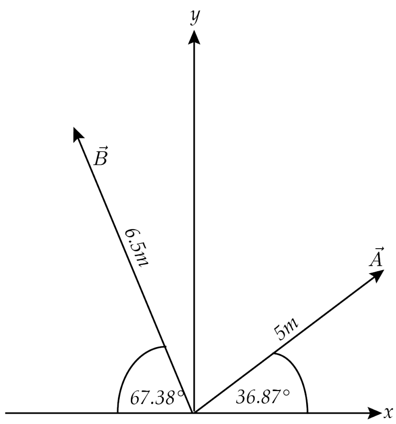
EASTERN MEDITERRANEAN UNIVERSITY
FACULTY OF ARTS AND SCIENCES
DEPARTMENT OF PHYSICS

PHYS 101 – MIDTERM EXAM Solution Set
2019-2020 Fall (13 November 2019)

Duration: 90 minutes

Problem 1 (4P):

Two vectors $\vec{A}$ and $\vec{B}$are given as shown in the figure. The magnitude of vector $\vec{A}$ is $A = 5m$, and the magnitude of vector $\vec{B}$ is $B = 6.5m$. The directions of the vectors are as stated in the figure.



a) Write the vectors $\vec{A}$ and $\vec{B}$ in unit vector notation. (2P)

$$\vec{A} = 5m\cos(36.87°)\ \hat{\imath} + 5m\sin(36.87°)\ \hat{\jmath} = (4\,\hat{\imath} + 3\,\hat{\jmath})m$$
$$\vec{B} = -6.5m\cos(67.38°)\ \hat{\imath} + 6.5m\sin(67.38°)\ \hat{\jmath} = (-2.5\,\hat{\imath} + 6\,\hat{\jmath})m$$

b) Calculate $\vec{C}$, $\vec{C} = \frac{1}{2}\vec{A} + 2\,\vec{B}$, in unit vector notation (2P)

$$\vec{C} = \frac{1}{2}\vec{A} + 2\,\vec{B} = \frac{1}{2}(4\,\hat{\imath} + 3\,\hat{\jmath})m + 2\,(-2.5\,\hat{\imath} + 6\,\hat{\jmath})m = (-3\,\hat{\imath} + 13.5\,\hat{\jmath})m$$

Problem 2 (8P):

Given are the x and y components of the position vector of a particle as $x(t) = 3t^2 + 2t - 3$ and $y(t) = \frac{1}{2}t^2 - 2t + 1$, where x and y are in meters and t is in seconds.

a) Write the position vector at any time t, $\vec{r}(t)$, and determine the positions of the particle at the time $t = 0$ and $t = 2s$. (3P)

$$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} = (3t^2 + 2t - 3)\,\hat{\imath} + \left(\frac{1}{2}t^2 - 2t + 1\right)\hat{\jmath}$$
$$\vec{r}(0) = (-3\,\hat{\imath} + \hat{\jmath})m$$
$$\vec{r}(2s) = (13\,\hat{\imath} - \hat{\jmath})m$$

b) Calculate the displacement the particle in the time interval $t = 0$ to $t = 2s$. (1P)

$$\Delta\vec{r} = \vec{r}(2s) - \vec{r}(0) = (-13\,\hat{\imath} - \hat{\jmath})m - (-3\,\hat{\imath} + \hat{\jmath})m = (16\,\hat{\imath} - 2\,\hat{\jmath})m$$

c) Calculate the average velocity of the particle in the time interval $t = 0$ to $t = 2s$. Calculate the velocity of the particle at any time t, $\vec{v}(t)$.(2P)

$$\langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} = \frac{(16\,\hat{\imath} - 2\,\hat{\jmath})m}{2s - 0} = (8\,\hat{\imath} - \hat{\jmath})\frac{m}{s}$$
$$\vec{v}(t) = \frac{d\,\vec{r}(t)}{dt} = (6t + 2)\,\hat{\imath} + (t - 2)\,\hat{\jmath}$$

d) Calculate the average acceleration in the time interval $t = 0$ to $t = 1s$ and the instantaneous acceleration of the particle at any instant of time t, $\vec{a}(t)$. (2P)

$$\langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(1s) - \vec{v}(0)}{1s} = \frac{(8\,\hat{\imath} - \hat{\jmath})\frac{m}{s^2} - (2\,\hat{\imath} - 2\hat{\jmath})\frac{m}{s^2}}{1s} = (6\,\hat{\imath} + \hat{\jmath})\frac{m}{s^2}$$
$$\vec{a} = (6\,\hat{\imath} + \hat{\jmath})\frac{m}{s^2}$$

Problem 3 (8P):
In a local coffee shop, a customer slides an empty coffee mug down the counter for a refill. The height of the counter is $1.60m$. The mug slides off the counter and strikes the floor $2.50\ m$ from the base of the counter.

a) Calculate the time the coffee mug flies from the edge of the counter to the floor. (2P)

$$\vec{v}_i = v_i\,\hat{\imath}, \vec{a} = -g\,\hat{\jmath}, \vec{r}_i = 1.6\,\hat{\jmath}\,m, \vec{r}_f = 2.5\,\hat{\imath}\,m$$

$$\vec{r}(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 = y_i\,\hat{\jmath} + v_i t\,\hat{\imath} - \frac{1}{2}g\,t^2\hat{\jmath} = \vec{r}_f = x_f\hat{\imath}$$

$$y_i - \frac{1}{2}gt^2 = 0 \Rightarrow t = \sqrt{\frac{2y_i}{g}} = \sqrt{\frac{2\cdot 1.6m}{9.8\frac{m}{s^2}}} = 0.57s$$

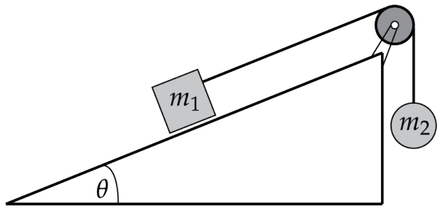
b) Calculate the initial velocity of the coffee mug in unit vector notation - in the instant just before it slides off the counter. (3P)

$$x_f = v_i t \Rightarrow v_i = \frac{x_f}{t} = \frac{2.5m}{0.57s} = 4.39\frac{m}{s}$$

c) What is the velocity of the coffee mug just before striking the ground? Write your answer in unit vector notation. (3P)

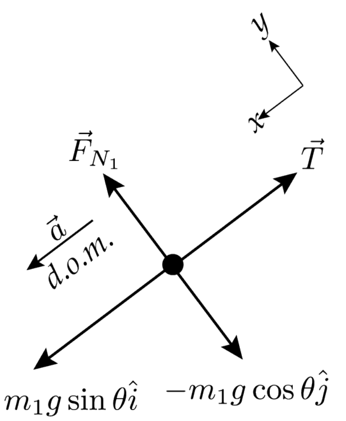
$$\vec{v}(t) = \vec{v}_i + \vec{a}t = 4.39\frac{m}{s}\,\hat{\imath} - 9.8\frac{m}{s^2}\,0.57s\,\hat{\jmath} = (4.39\,\hat{\imath} - 5.59\,\hat{\jmath})\frac{m}{s}$$

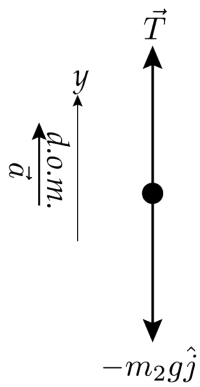
Problem 4 (10P):
A box of mass $m_1 = 4kg$ placed on a rough incline of angle $\theta = 37°$ is connected via a lightweight cord over a massless and frictionless pulley to a ball of mass $m_2 = 2\ kg$.



a) If the surface is frictionless draw the free body diagrams for m_1 and m_2 and state the directions of motion for both masses. (4P)

FBD for m_1 FBD for m_2





From FBD for m_1

$$-T + m_1 g\sin\theta = m_1 a\ (1)$$

From FBD for m_2

$$T - m_2 g = m_2 a\ (2)$$

(1)+(2)

$$m_1 g\sin\theta - m_2 g = m_1 a + m_2 a$$

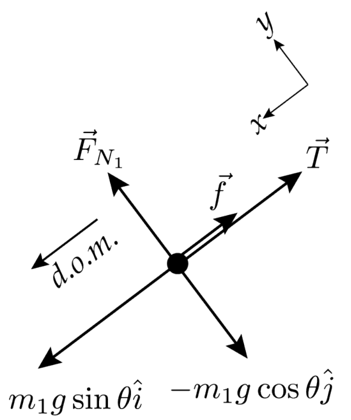
$$a = \frac{m_1\sin\theta - m_2}{m_1 + m_2}g = \frac{4kg\sin 37° - 2kg}{4kg + 2kg}9.8\frac{m}{s^2} = +0.67\frac{m}{s^2}$$

So, we can conclude that we selected the direction correctly, and the direction of the motion for the mass m_1 is downward and consequently for $m_2$upward.

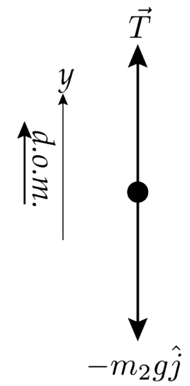
b) If the coefficients of static and kinetic frictions are $\mu_s = 0.2$ and $\mu_k = 0.1$ respectively; draw the free body diagrams for m_1 and m_2, determine if the friction acting on m_1 is static or kinetic, and find the acceleration of the system, if there is any. (6P)

In order to determine the if static of kinetic friction is applying, we consider the case of that the mass is not sliding. If it is sliding it will slide downwards. So we will determine the static friction force, if this static friction force is greater as the maximum static friction force with the coefficient of static friction μ_s, then the objects will move, if the static friction force is less than the maximum static friction force, then the objects will remain at rest.

FBD for m_1



FBD for m_2



From the FBD for m_1

$$\Sigma \vec{F}_1 = F_{N_1}\hat{\jmath} + m_1 g \sin\theta\,\hat{\imath} - m_1 g \cos\theta\,\hat{\jmath} - T\hat{\imath} - f\hat{\imath} = 0$$

$$m_1 g \sin\theta - T - f = 0 \ (1)$$

From the FBD for m_2

$$T - m_2 g = 0 \ (2)$$

(1)+(2)

$$m_1 g \sin\theta - f - m_2 g = 0$$

$$\Rightarrow f = m_1 g \sin\theta - m_2 g = (m_1 \sin\theta - m_2) g$$

$$= (4kg \sin 37° - 2kg)\, 9.8\frac{m}{s^2}$$

$$= 3.99\ N$$

On the other hand, if we calculate the maximum static friction force using the normal force and coefficient of static friction. We get.

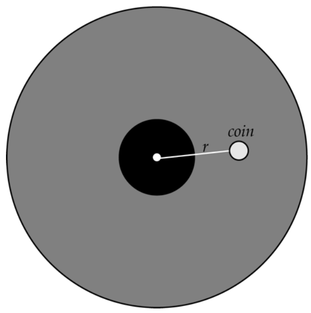
$$F_{N_1} = m_1 g \cos\theta = 31.31N$$

$$\Rightarrow f_{s,max} = \mu_s F_{N_1} = 0.2 \cdot 31.31N = 6.62N$$

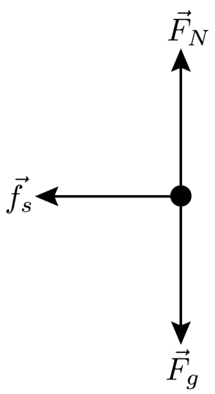
As $f_{s,max} > f$, we can conclude that the objects will remain at rest. Therefore there will be no acceleration and static friction force of $f = 3.99N$ applies.

Problem 5 (5P):

A coin placed on a record of diameter 0.4m, 0.1m from the rotation axis, revolves with a speed of $0.35\ m/s$ about the rotation axis without sliding.



a) What is the coefficient of static friction between the coin and the record? (3P)



The static frictional force takes over the role of the centripetal force, so we get

$$\mu_s m\,g = m\frac{v^2}{R} \Rightarrow \mu_s = \frac{v^2}{R\,g} = \frac{\left(0.35\frac{m}{s}\right)^2}{0.1m \cdot 9.8\frac{m}{s^2}} = 0.125$$

b) Using the coefficient of static friction determined in part a), how far from the axis can the coin be placed, without slipping, if the record rotates now at $0.47\ m/s$?(2P)

$$\mu_s m\,g = m\frac{v^2}{R} \Rightarrow R = \frac{v^2}{\mu_s\,g} = \frac{\left(0.47\frac{m}{s}\right)^2}{0.125 \cdot 9.8\frac{m}{s^2}} = 0.18\ m$$