

The Procedure based on D-H Convention for deriving the Forward Kinematics for any manipulator

Step 1: Locate and label the joint axes z_0, \dots, z_{n-1} .

Step 2: Establish the base frame. Set the origin anywhere on the z_0 axis. The x_0 and y_0 axes are chosen conveniently to form the right-hand frame.

For $i = 1, \dots, n-1$, perform steps 3 to 5

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i . If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i at joint i .

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i , or in the direction normal to the $z_{i-1} \times z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right hand frame.

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Set z_n along the direction z_{n-1} . Establish the origin o_n conveniently along z_n preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Establish x_n along the common normal between z_{n-1} and z_n through o_n .

Step 7: Create a table of link parameters $\theta_i, d_i, a_i, \alpha_i$.

θ_i : the angle between x_{i-1} and x_i measured about z_{i-1} . θ_i is variable if joint i is revolute.

d_i : distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

a_i : distance along x_i from o_i to the intersection of the x_i and z_{i-1} axes.

α_i : the angle between z_{i-1} and z_i measured about x_i .

Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters.

Step 9: Form ${}^0T_n = A_1 A_2 \dots A_n$. This then gives the position and orientation of the tool frame expressed in base coordinates.