



Faculty of Engineering

DEPARTMENT of ELECTRICAL AND ELECTRONIC ENGINEERING

EENG (INFE) 115 | Instructor:

Introduction to Logic Design | G. YEMİŐCIOĐLU

Final EXAMINATION | Duration: 120 minutes

June 12, 2017 | Number of Questions: 4

Good Luck

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STUDENT'S

NUMBER

NAME

SURNAME

GROUP NO.

**SOLUTIONS**

Question	Achieved	Points
1		30
2		30
3		30
4		30
TOTAL		120

Read the following instructions carefully:

1. Calculators are not allowed.
2. Switch off mobile phones and do not borrow any stationery from your friends.
3. In your solutions, show all details you claim credit for.

**Question 1:**

Design a **combinational circuit** that accepts a **four-bit number (A,B,C,D)** and generates an output which is equal to the **sum of the binary numbers formed by the input (AB) and (CD)**.

(30 pts)

AB + CD = ??

Inputs				Outputs		
A	B	C	D	x	y	z
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	1
1	1	1	1	1	1	0

$$x(A,B,C,D) = \Sigma(7, 10, 11, 13, 14, 15)$$

$$y(A,B,C,D) = \Sigma(2, 3, 5, 6, 8, 9, 12, 15)$$

$$z(A,B,C,D) = \Sigma(1, 3, 4, 6, 9, 11, 12, 14)$$

x'sK-MAP	y'sK-MAP																																																		
<table border="1" style="margin: auto;"> <tr> <td style="border: none;">CD AB</td> <td style="border: none;">00</td> <td style="border: none;">01</td> <td style="border: none;">11</td> <td style="border: none;">10</td> </tr> <tr> <td style="border: none;">00</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="border: none;">01</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="border: none;">11</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">1</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="border: none;">10</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">1</td> </tr> </table>	CD AB	00	01	11	10	00	0	0	0	0	01	0	0	1	0	11	0	1	1	1	10	0	0	1	1	<table border="1" style="margin: auto;"> <tr> <td style="border: none;">CD AB</td> <td style="border: none;">00</td> <td style="border: none;">01</td> <td style="border: none;">11</td> <td style="border: none;">10</td> </tr> <tr> <td style="border: none;">00</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="border: none;">01</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="border: none;">11</td> <td style="text-align: center;">1</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">0</td> </tr> <tr> <td style="border: none;">10</td> <td style="text-align: center;">1</td> <td style="text-align: center;">1</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> </tr> </table>	CD AB	00	01	11	10	00	0	0	1	1	01	0	1	0	1	11	1	0	1	0	10	1	1	0	0
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$x = AC + ABD + BCD$	$y = AC'D' + AB'C' + A'B'C + A'CD' + A'BC'D + ABCD$ $y = AC'D' + A'CD' + AB'C' + A'B'C + A'BC'D + ABCD$ $y = D'(AC' + A'C) + B'(AC' + A'C) + BD(A'C' + AC)$ $y = D'(A \oplus C) + B'(A \oplus C) + BD(A \oplus C)'$																																																		
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10	0	1	1	0																																															
$z = BD' + B'D$																																																			

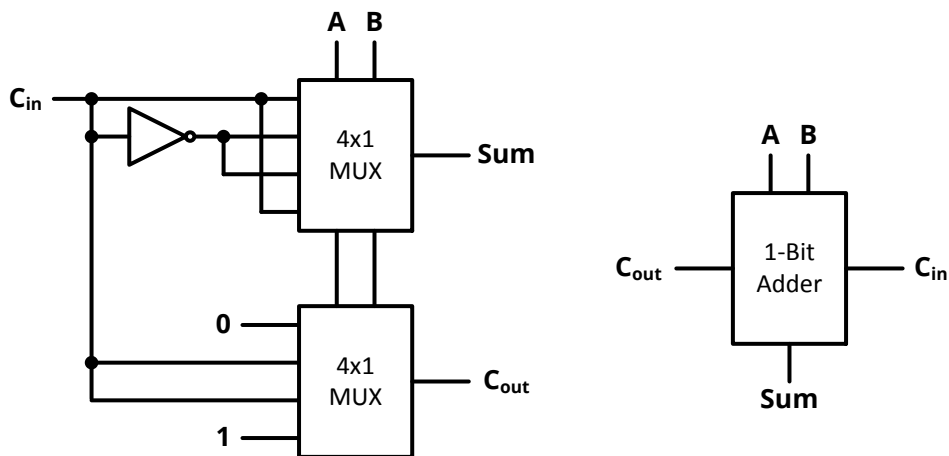
**Question 2:**

- a) Implement a 1-bit full adder circuit by using 4 x 1 Multiplexer and an inverter.
- b) Construct 4-bit ripple carry adder using 1-bit full adder implemented in (a).
- c) Implement a 4-bit BCD adder using (b) to add two BCD digits. Make sure that the circuit also detects invalid BCD digits and generates the correct BCD output sum.

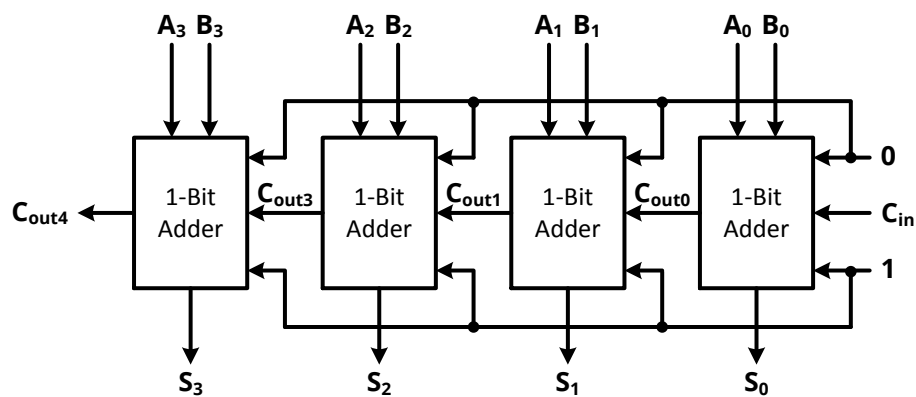
(30 pts)

a) (10 pts)

A	B	C <sub>in</sub>	C <sub>out</sub>	Mux 1 IN	Sum	Mux 2 IN
0	0	0	0	0	0	C <sub>in</sub>
0	0	1	0	0	1	C <sub>in</sub>
0	1	0	0	C <sub>in</sub>	1	C <sub>in</sub> '
0	1	1	1	C <sub>in</sub>	0	C <sub>in</sub> '
1	0	0	0	C <sub>in</sub>	1	C <sub>in</sub> '
1	0	1	1	C <sub>in</sub>	0	C <sub>in</sub> '
1	1	0	1	1	0	C <sub>in</sub>
1	1	1	1	1	1	C <sub>in</sub>



b) (5 pts)



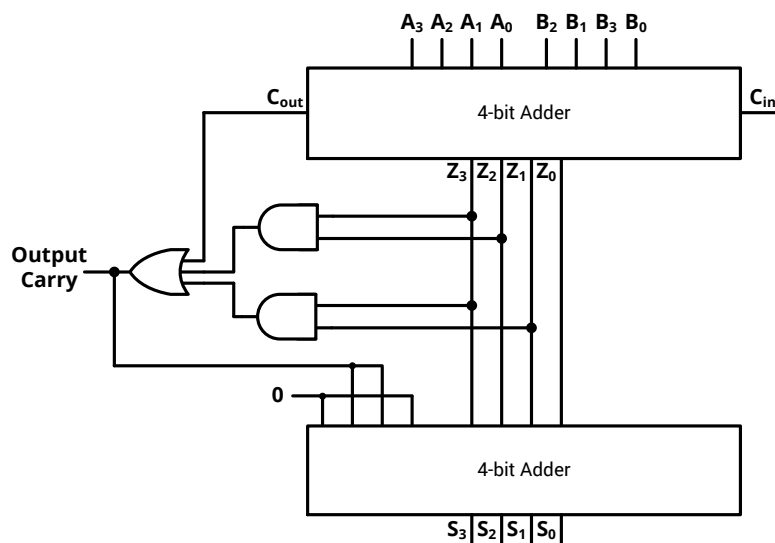
c) (15 pts)

Z3	Z2	Z1	Z0	Error
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Error's K-MAP					
$Z_3Z_2$	$Z_1Z_0$	00	01	11	10
00		0	0	0	0
01		0	0	0	0
11		1	1	1	1
10		0	0	1	1

$Err = Z_3Z_2 + Z_3Z_1$

**Output Carry =  $C_{out} + Z_3Z_2 + Z_3Z_1$**

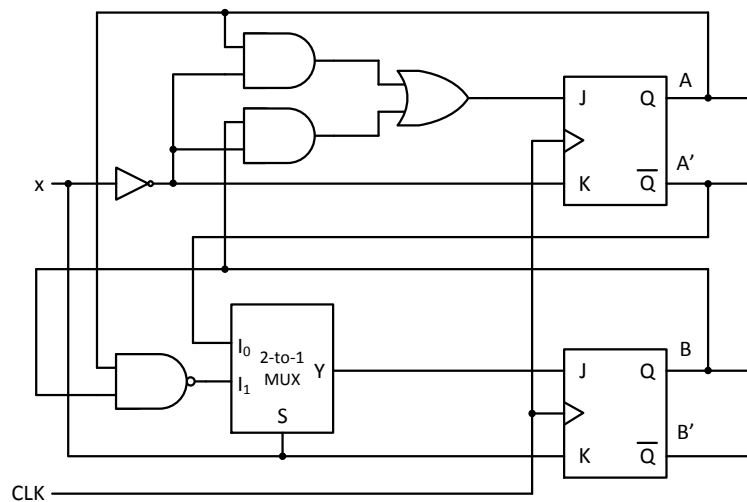


**Question 3:**

Sequential circuit shown below has two flip-flops A and B and one input x. It consists of a combinatorial logic connected to the flip-flops, as shown in Figure below. Analyse the circuit:

- a) Derive the next state equations.
- b) Derive the state table of the sequential circuit.
- c) Draw the corresponding state diagram.

**(30 pts)**



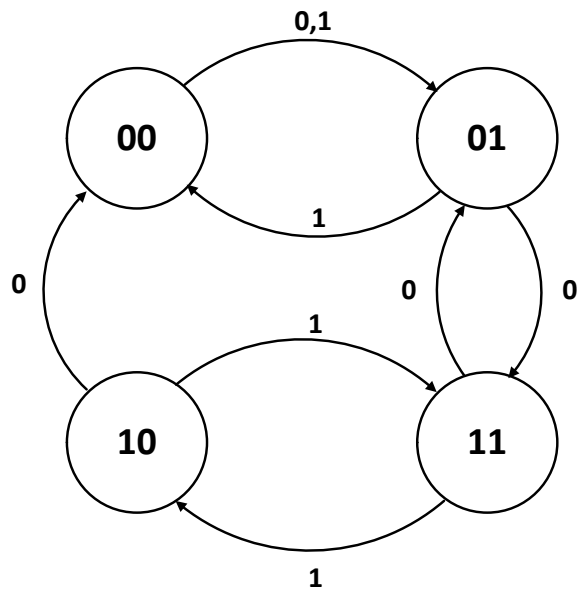
**a) (10 pts)**

$J_A = A.x' + B.x'$	$K_A = x'$	$J_B = A'.x' + (A.B)'x$ $J_B = A'.x' + (A' + B')x$ $J_B = A'.x' + A'x + B'.x$ $J_B = A'(x' + x) + B'.x$ $J_B = A' + B'.x$	$K_B = x$
$A(t+1) = J_A.A' + K_A'.A$ $A(t+1) = (A.x' + B.x')A' + (x')'A$ $A(t+1) = A'.A.x' + A'.B.x' + Ax$ $A(t+1) = A'.B.x' + Ax$		$B(t+1) = J_B.B' + K_B'.B$ $B(t+1) = (A'+B'.x)B' + B.x'$ $B(t+1) = A'B' + B'.B'x + B.x'$ $B(t+1) = A'B' + B'x + Bx'$ $B(t+1) = A'B' + B \text{ XOR } x$	

b) (10 pts)

Present		Input x	Next	
A	B		A	B
0	0	0	0	1
0	0	1	0	1
0	1	0	1	1
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	0	1
1	1	1	1	0

c) (10 pts)



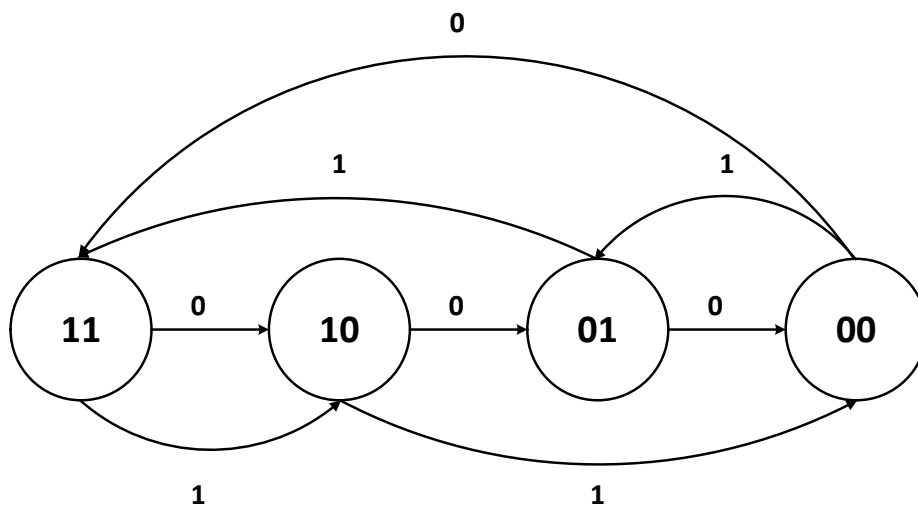
**Question 4:**

Design a **2-bit counter** using **JK-Flip flops** with one input. When the input is **0**, the counter counts down, with the repeated sequence (**11-10-01-00**). When the input is **1**, the counter counts repeated random sequence (**00-01-11-10**).

- Draw a state diagram for the sequential circuit.
- Derive the state table for the sequential circuit.
- Derive the simplified flip flops input equations.
- Draw the logic circuit diagram of a 2-bit counter.

(30 pts)

a) (7.5 pts)



b) (7.5 pts)

Step1: Characteristic and Excitation Table of JK-Flip Flop

Characteristic			Excitation			
J	K	Q	Q(t)	Q(t+1)	J	K
0	0	Q	0	0	0	X
0	1	0	0	1	1	X
1	0	1	1	0	X	1
1	1	Q'	1	1	X	0



Step2: Stata Table

Present State		Input	Next State		Flip-Flops Inputs			
A	B	x	A	B	J <sub>A</sub>	K <sub>A</sub>	J <sub>B</sub>	K <sub>B</sub>
0	0	0	1	1	1	X	1	X
0	0	1	0	1	0	X	1	X
0	1	0	0	0	0	X	X	1
0	1	1	1	1	1	X	X	0
1	0	0	0	1	X	1	1	X
1	0	1	0	0	X	1	0	X
1	1	0	1	0	X	0	X	1
1	1	1	1	0	X	0	X	1

c) (7.5 pts)

<p><b>JA's K-MAP</b></p> <p>JA = B'x' + Bx</p>	<p><b>KA's K-MAP</b></p> <p>KA = B'</p>
<p><b>JB's K-MAP</b></p> <p>JB = A' + x'</p>	<p><b>KB's K-MAP</b></p> <p>KB = A + x'</p>

d) (7.5 pts)

