The Convolution Sum Theory

Let a signal $x[n]$ be multiplied by the impulse sequence $\delta[n]$

$$x[n]\delta[n] = x[0]\delta[n]$$

For a shifted impulse

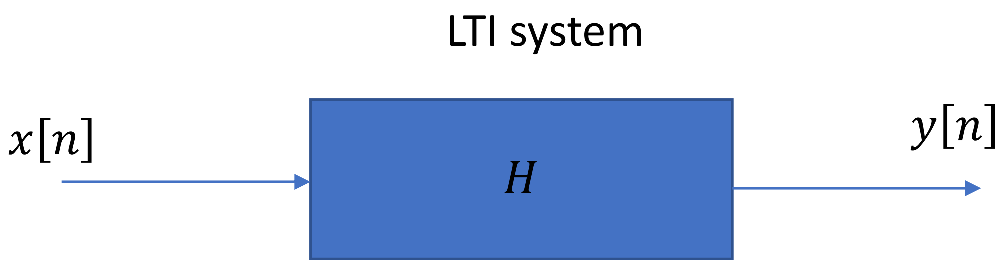
$$x[n]\delta[n-k] = x[k]\delta[n-k]$$

Hence $x[n]$ can be represented as a weighted sum of time-shifted impulses

$$x[n] = \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \cdots$$

Therefore

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$



$$y[n] = H\{x[n]\} = H\left\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right\}$$

$$y[n] = \sum_{k=-\infty}^{\infty} H\{\, x[k]\delta[n-k]\}$$

Since *x[k]* is constant with respect to the system operator *H*

$$y[n = \sum_{k=-\infty}^{\infty} x[k]H\{\, \delta[n-k]\}$$

$$H\{\,\delta[n-k]\} = h[n-k]$$

$$H\{\,\delta[n]\} = h[n]$$

$h[n]$ which is known as impulse response of a DT system is actually the response of the system to an input $x[n] = \delta[n]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Convolution Sum Formula

There are four basic steps to the calculation of the CONVOLUTION SUM

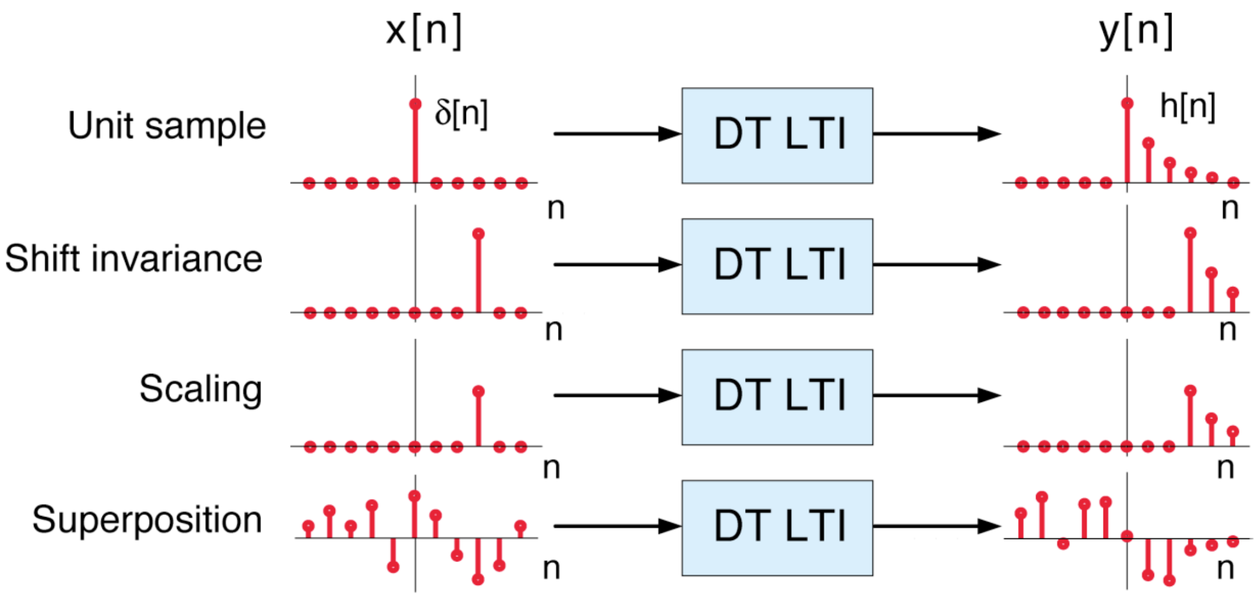
$$h[k] \xrightarrow{Flip} h[-k]$$

$$\xrightarrow{Shift} h[n-k]$$

$$\xrightarrow{Multiply} x[k]h[n-k]$$

$$\xrightarrow{Sum} \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

Convolution has a simple graphical interpretation



Convolution Integral for CT

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)$$

Properties of Convolution

- **Commutative:**

$$x[n]*h[n]=h[n]*x[n]$$

- **Distributive:**

$$x[n]*(h_1[n]+h_2[n])=$$
$$(x[n]*h_1[n])+(x[n]*h_2[n])$$

- **Associative:**

$$x[n]*h_1[n]*h_2[n]=$$
$$(x[n]*h_1[n])*h_2[n]=$$
$$(x[n]*h_2[n])*h_1[n]$$

- **Implications**