UNIFORM PLANE WAVES IN UNBOUNDED LOSSLESS MEDIA

PROPAGATION IN z DIRECTION

Consider the case which was investigated before. Assume a homogeneous, irotropic, losless medium. i.e. take free space into consideration. Then,

$$\nabla^2 \bar{E} + k_0^2 \bar{E} = 0 \quad \text{..............(1)}$$

Where $k_0 = \omega\sqrt{\varepsilon_0 \mu_0} = \dfrac{\omega}{c}\ (rad\ /\ m)$

If we assume, $\bar{E} = E_x(z)\hat{a}_x$, then the solution of (1):

$$E_x(z) = E_0^+ e^{-jk_0 z} + E_0^- e^{jk_0 z}$$

$$E_x^+ = E_0^+ e^{-jk_0 z} \qquad E_x^+(z,t) = E_0^+ \cos(\omega t - k_0 z)$$

$$E_x^- = E_0^- e^{jk_0 z} \qquad E_x^-(z,t) = E_0^- \cos(\omega t + k_0 z)$$

This is called a plane wave because the phase $(k_0 z)$ of $\bar{E}$ and $\bar{H}$ is constant over a set of planes (defined by $z = const.$) called equiphase surfaces.

It is called a uniform plane wave because the amplitudes E_0^+, $\dfrac{E_0^+}{\eta_0}$ of E_x^+ and H_x^+ are constant over the equiphase planes.

The angular frequency is ω in $(rad\ /\ s)$, the frequency is $f = \dfrac{\omega}{2\pi}(Hz)$ and the wavenumber is

$$k_0 = \omega\sqrt{\varepsilon_0 \mu_0} = 2\pi f \sqrt{\varepsilon_0 \mu_0}$$

$$\frac{k_0}{2\pi} = \frac{f}{c}$$

Since the wavelength is $\lambda = \dfrac{c}{f}$ then we can write the wavelength as $\lambda = \dfrac{2\pi}{k_0}$ in meters.

The magnetic field intensity:

Consider the electric field intensity $\bar{E}(z) = E_0^+ e^{-jk_0 z}\hat{a}_x$. We use;

$$\nabla X\,\bar{E} = -j\omega\mu_0\bar{H}$$

$$\nabla X\bar{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_0^+ e^{-jk_0 z} & 0 & 0 \end{vmatrix}$$

$$\bar{H} = \frac{k_0}{\omega\mu_0}\hat{a}_y E_0 e^{-jk_0 z}$$

$$\frac{k_0}{\omega\mu_0} = \frac{\omega\sqrt{\varepsilon_0\mu_0}}{\omega\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

Denote:

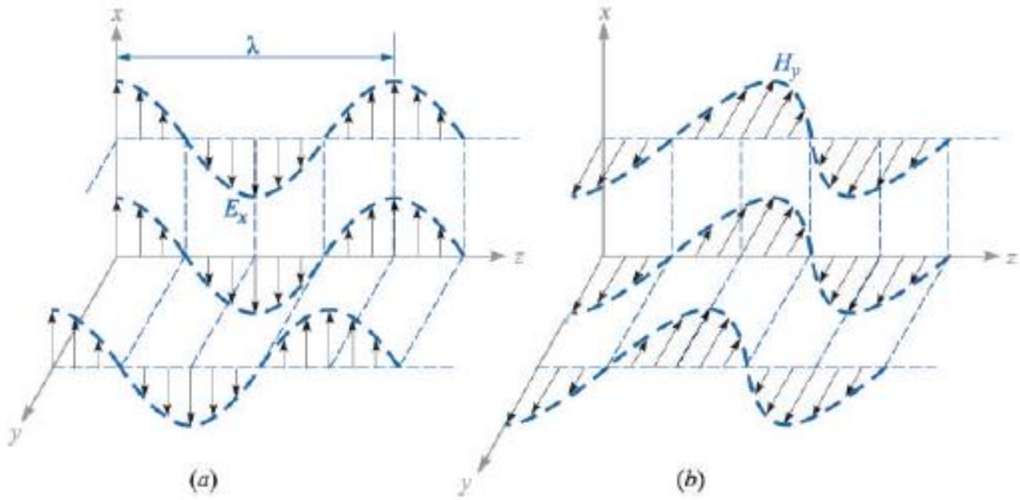
$\eta_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} \approx 120\pi \approx 377\Omega$, the intrinsic impedance of the free space. The real physical fields:

$$\bar{E}(z,t) = E_0^+ \cos(\omega t - k_0 z)\hat{a}_x$$

$$\bar{H}(z,t) = \frac{E_0^+}{\eta_0}\cos(\omega t - k_0 z)\hat{a}_y$$

Some useful results:

1) $\bar{E}$ is $\perp$ to $\bar{H}$.
2) $\left|\bar{H}\right| = \dfrac{\left|\bar{E}\right|}{\eta_0}$
3) $\bar{E}$ and $\bar{H}$ are in phase.
4) Both $\bar{E}$ and $\bar{H}$ are $\perp$ to the direction of the propagation.



(*a*) Arrows represent the instantaneous values of $E_{x0}\cos[\omega(t - z/c)]$ at $t = 0$ along the z axis, along an arbitrary line in the $x = 0$ plane parallel to the z axis, and along an arbitrary line in the $y = 0$ plane parallel to the z axis. (*b*) Corresponding values of H_y are indicated. Note that E_x and H_y are in phase at any point at any time.

Reference:

www.ie.itcr.ac.cr/chapt11.pdf