Tutorial Questions Related to Physics and Measurement

1. Earth is approximately a sphere of radius 6.37×10^6 m.
   What are

   (a) its circumference in kilometers,

   (b) its surface area in square kilometers, and

   (c) its volume in cubic kilometers?

Solution:

(a) The radius of the Earth in km is

$$R = (6.37 \times 10^6\text{m})(10^{-3}\text{km/m}) = 6.37 \times 10^3\text{km}$$

so its circumference is

$$s = 2\pi R = 2\pi(6.37 \times 10^3\text{km}) = 4.00 \times 10^4\text{km}.$$

(b) The surface area of Earth is

$$A = 4\pi R^2 = 4\pi(6.37 \times 10^3\text{km})^2 = 5.10 \times 10^8\text{km}^2.$$

(c) The volume of Earth is

$$V = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(6.37 \times 10^3\text{km})^3 = 1.08 \times 10^{12}\text{km}^3.$$

2. The fastest growing plant on record is a Hesperoyucca whipplei that grew 3.7 m in 14 days. What was its growth rate in micrometers per second?

Solution: A day is equivalent to

$$1\text{d} = 1\text{d}\frac{24\text{h}}{1\text{d}} \cdot \frac{60\text{min}}{1\text{h}} \cdot \frac{60\text{s}}{1\text{min}} = 86400\text{s}$$

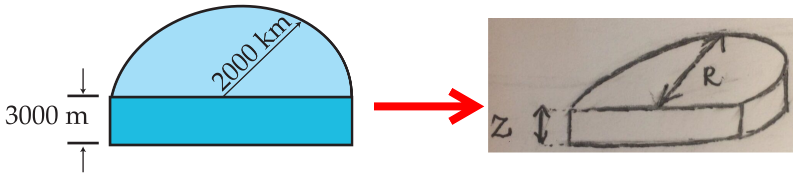
and

$$1\text{m} = 10^6\mu\text{m},$$

so the growing speed becomes

$$\frac{(3.7\text{m})(10^6\mu\text{m/m})}{(14\text{d})(86400\text{s/d})} = 3.1\mu\text{m/s}.$$

3. Antarctica is roughly semicircular, with a radius of 2000 km (see figure below). The average thickness of its ice cover is 3000 m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of Earth.)



Solution:
The volume of ice is given by the product of the semicircular surface area and the thickness. The area of the semicircle is

$$A = \pi r^2/2,$$

where r is the radius. Therefore, the volume is

$$V = z\pi r^2/2$$

where z is the ice thickness. Therefore

$$V = \frac{\pi}{2}(2000 \times 10^5\text{cm})^2(3000 \times 10^2\text{cm}) = 1.9 \times 10^{22}\text{cm}^3.$$

4. Earth has a mass of 5.98×10^{24} kg. The average mass of the atoms that make up Earth is 40 u (1 u $= 1.66053886 \times 10^{-27}$ kg). How many atoms are there in Earth?

Solution:
If M_E is the mass of Earth, m is the average mass of an atom in Earth, and N is the number of atoms, then

$$M_E = N \cdot m \text{ or } N = M_E/m.$$

Thus,

$$N = \frac{M_E}{m} = \frac{5.98 \times 10^{24}\text{kg}}{(40\text{u})(1.661 \times 10^{-27}\text{kg/u})} = 9.0 \times 10^{49}$$

5. Iron has a density of $7.87\text{g}/\text{cm}^3$, and the mass of an iron atom is 9.27×10^{-26}kg. If the atoms are spherical and tightly packed,
   (a) what is the volume of an iron atom and
   (b) what is the distance between the centers of adjacent atoms?

Solution:

(a) The density ρ of a sample of iron is

$$\rho = m/V = (7.87\text{g}/\text{cm}^3)(\frac{1\text{kg}}{1000\text{g}})(\frac{100\text{cm}}{1\text{m}})^3 = 7870\text{kg}/\text{m}^3.$$

if M is the mass and V is the volume of an atom, then

$$V = \frac{M}{\rho} = \frac{9.27 \times 10^{-26}\text{kg}}{7.87 \times 10^3\text{kg}/\text{m}^3} = 1.18 \times 10^{-29}\text{m}^3.$$

(b) We set $V = 4/3\pi R^3$. Solving for R, we find

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(3(1.18 \times 10^{-29}\text{m}^3)/(4\pi)\right)^{1/3} = 1.41 \times 10^{-10}\text{m}.$$

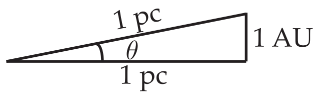
The center-to-center distance between atoms is twice the radius, or 2.82×10^{-10}m.

6. An astronomical unit (AU) is the average distance between Earth and the Sun, approximately 1.50×10^8 km. The speed of light is about $3.0 \times 10^8\text{m}/\text{s}$. Express the speed of light in astronomical units per minute.

Solution:
We convert meters to astronomical units, and seconds to minutes, using 1000m = 1km , 1AU = 1.50×10^8km and 60s = 1min . Thus, $3.0 \times 10^8\text{m}/\text{s}$ becomes

$$\frac{3.0 \times 10^8\text{m}/\text{s}}{1000\text{m}/\text{km}} \frac{60\text{s}/\text{min}}{1.50 \times 10^8\text{km}/\text{AU}} = 0.12\text{AU}/\text{min}.$$

7. An astronomical unit (AU) is the average distance between Earth and the Sun, approximately 1.50×10^8 km. A parsec (pc) is the distance at which a length of 1 AU would subtend an angle of exactly 1second of arc (see figure below).



$\theta = 1''$ is 1/3600's of 1°

A light-year (ly) is the distance that light, traveling through a vacuum with a speed of 3.0×10^8m/s, would cover in 1.0 year. Express the Earth – Sun distance in

(a) parsecs and

(b) light-years.

Solution:

(a) Note that when θ is measured in radians, it is equal to the arc length s divided by the radius R. For a very large radius circle and small value of θ, the arc can be approximated as the straight line-segment. Thus,

$$\theta = 1\text{arcsec} = (1\text{arcsec})\frac{\text{arcmin}}{60\text{arcsec}}\frac{1^\circ}{60\text{arcmin}}\frac{2\pi\text{rad}}{360^\circ} = 4.85 \times 10^{-6}\text{rad}.$$

Therefore,one parsec is

$$1pc = R_0 = \frac{s}{\theta} = \frac{1\text{AU}}{4.85 \times 10^{-6}\text{rad}} = 2.06 \times 10^5\text{AU}.$$

(b) Next, we relate AU to light-year (ly). Since a year is about 3.16×10^7s, we have

$$1\text{ly} = (3.00 \times 10^8\text{m/s})(3.16 \times 10^7\text{s}) = 9.46 \times 10^{15}\text{m}.$$

Since $1pc = 2.06 \times 10^5$AU, inverting the relationship gives,

$$R = 1\text{AU} = (1\text{AU})\frac{1\text{pc}}{2.06 \times 10^5} = 4.9 \times 10^{-6}\text{pc}.$$

Next part, given that $1\text{ly} = 9.46 \times 10^{15}$m and $1\text{AU} = 149.6 \times 10^9$m , the two expressions together lead to

$$1\text{AU} = 149.6 \times 10^9\text{m} = (149.6 \times 10^9\text{m})\frac{1\text{ly}}{9.46 \times 10^{15}\text{m}} = 1.6 \times 10^{-6}\text{ly}.$$