Tutorial 3
A: Motion in one Dimension

Tutorial Questions

1. The position of an object moving along an x axis is given by

$$x(\vec{t}) = 3t - 4t^2 + t^3,$$

where x is in meters and t in seconds. Find the position of the object at the following values of t:

(a) 1 s,

(b) 2 s,

(c) 3 s, and

(d) 4 s.

(e) What is the object's displacement between $t = 0$ and $t = 4$s?

(f) What is its average velocity for the time interval from $t = 2$s to $t = 4$s?

(g) Graph x versus t for $0 \leq t \leq 4$s and indicate how the answer for 1f can be found on the graph.

2. The position of a particle moving along the x axis is given in centimeters by $x(\vec{t}) = 9.75 + 1.50t^3$, where t is in seconds.
Calculate

(a) the average velocity during the time interval $t = 2.00$s to $t = 3.00$s;

(b) the instantaneous velocity at $t = 2.00$s; the instantaneous velocity at $t = 3.00$s; the instantaneous velocity at $t = 2.50$s; and

(c) the instantaneous velocity when the particle is midway between its positions at $t = 2.00$s and $t = 3.00$s.

(d) Graph x versus t and indicate your answers graphically.

3. The position of a particle is given by $x(\vec{t}) = 20t - 5t^3$, where x is in meters and t is in seconds.
   (a) when, if ever, is the particle's velocity zero?
   (b) When is its acceleration zero?
   (c) For what time range (positive or negative) is the acceleration $a(t)$ negative or positive?
   (d) Graph $x(\vec{t})$, $v(\vec{t})$, and $a(\vec{t})$.

4. A car moves along an x axis through a distance of 900 m, starting at rest (at $x = 0$) and ending at rest (at $x = 900$m). Through the first $\frac{1}{4}$ of that distance, its acceleration is $+2.25\text{m/s}^2$. Through the rest of that distance, its acceleration is -0.750m/s^2. What are
   (a) its travel time through the 900 m and
   (b) its maximum speed?

5. A brick dropped from rest from the top of a building. The brick sticks the ground after $4s$.
   (a) How tall, in meter is the building?
   (b) What is the velocity of the brick just before it hits the ground?

6. A plane touches the ground for landing with a speed of $100m/s$ and slow down at a rate of $5m/s^2$ before it stops.
   (a) What is the time it takes the plane to stop?
   (b) Can this plan safely land on a small airport when the runway is only $0.8km$ long?

Solutions

1.

(a) Evaluation of $x(t)$ at $t = 1s$ yields $x(1s) = 3 - 4 + 1 = 0$.

(b) Evaluation of $x(t)$ at $t = 2s$ we get $x(2s) = 3(2) - 4(2)^2 + (2)^3 = -2m$.

(c) Evaluation of $x(t)$ at $t = 3s$ we have $x(3s) = 0$.

(d) Evaluation of $x(t)$ at $t = 4s$ gives $x(4s) = 12m$.

(e) In order to find the displacement in the time between $t = 0$ and $t = 4s$ we calculate the difference of the positions at these times as $\Delta x = x(4s) - x(0) = 12m$.

(f) Firstly we need to calculate the position in the time $t = 4s$ and $t = 2s$:

$$x\vec{(t)} = (3t - 4t^2 + t^3)\hat{\mathbf{i}}\text{ m}$$

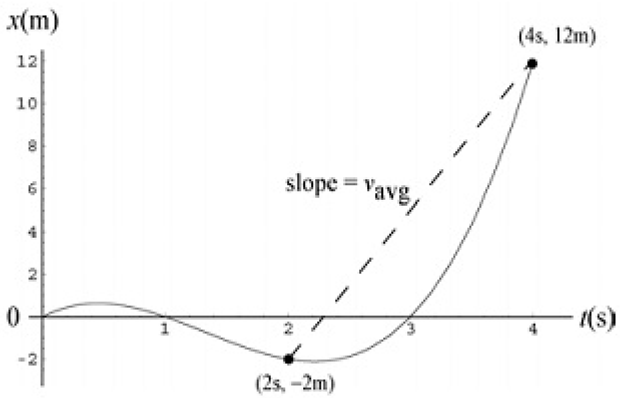
$$x\,(t \vec{=} 4s) = 12\,\hat{\mathbf{i}}\,m$$

$$x\,(t \vec{=} 2s) = -2\,\hat{\mathbf{i}}\,m$$

Then we calculate the average velocity by

$$v_{\vec{avg}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{x(\vec{4}s) - x(\vec{2}s)}{4s - 2s} = \frac{14m}{2s}\hat{\mathbf{i}} = 7\,\hat{\mathbf{i}}\,m/s$$

(g) The position of the object for the internal $0 \leq t \leq 4$ is plotted below. The slope of the straight line drawn from the point at $(t, x) = (2s, -2m) to (4s, 12m)$ represents the average velocity calculated in part 1f.



2.

(a) The average velocity during the time interval $2.00 \leq t \leq 3.00s$ is

$$\vec{v_{avg}} = \frac{\vec{\Delta x}}{\vec{\Delta t}} = \frac{x(3.\vec{0}0s) - x(2.\vec{0}0s)}{3.00s - 2.00s} = \frac{50.25cm - 21.75cm}{3.00s - 2.00s}\hat{\mathbf{i}} = 28.5\hat{\mathbf{i}}\,cm/s = 0.29\hat{\mathbf{i}}\,m/s$$

(b) The instantaneous velocity for all t is

$$v(\vec{t}) = \frac{dx(\vec{t})}{dt} = 4.5t^2\hat{\mathbf{i}}$$

In order to determine $v(t)$ at $t = 2.00s$,$t = 3.00s$, and $t = 2.50s$ we just have to evaluate $v(t)$ at these values and get :

$$\vec{v}(2.00s) = 4.5(2.00s)^2 cm/s\hat{\mathbf{i}} = 18.0cm/s\hat{\mathbf{i}} = 0.18\,\hat{\mathbf{i}}\,m/s$$
$$\vec{v}(3.00s) = 4.5(3.00s)^2 cm/s\hat{\mathbf{i}} = 40.5cm/s\hat{\mathbf{i}} = 0.40\,\hat{\mathbf{i}}\,m/s$$
$$\vec{v}(2.50s) = 4.5(2.50s)^2 cm/s\hat{\mathbf{i}} = 28.1cm/s\hat{\mathbf{i}} = 28.13\,\hat{\mathbf{i}}\,m/s$$

(c) In order to calculate the instantaneous velocity on the midway between $x(2.00s)$ and $x(3.00s)$ we first have to calculate x_m as following:

$$x_m = \frac{x(2.00s) + x(3.00s)}{2} = \frac{21.75cm + 50.25cm}{2} = 36cm = 0.36m$$

Now we have to calculate the time t_m, where $x(t_m) = 36cm = 0.36m$.

$$x_m = 9.75 + 1.5t_m^3 = 36cm \Longleftrightarrow t_m = \sqrt[3]{\frac{36 - 9.75}{1.5}} = 2.60s$$

Then we get for the velocity $v(t_m)$

$$v(\vec{t_m}) = 4.5(2.60s)^2 = 30.42cm/s = 0.30\,\hat{\mathbf{i}}\,m/s$$

3. We use the functional notation x(t), v(t), and a(t) in this solution, where the latter two quantities are obtained by differentiation:

$$x(t) = 20t - 5t^3$$

$$v(t) = \frac{dx(t)}{dt} = 20 - 15t^2$$

and

$$a(t) = \frac{dv(t)}{dt} = -30t$$

(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.15$ s.

(b) From 0 = 30t, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30$ t is negative for $t > 0$. The acceleration $a(t) = -30$ t is positive for $t < 0$.

4.

(a) For the first part of the movement: we have to find the time for the first 1/4 of the trip's

$$\vec{r}_1 = \frac{900}{4}\hat{\mathbf{i}} = 225\hat{\mathbf{i}} \text{ m}$$

$$\vec{v}_{01} = 0$$

$$\vec{r}_1 = \vec{r}_0 + \vec{v}_{01}t_1 + \frac{1}{2}\vec{a}t_1^2$$

$$225\text{m}\hat{\mathbf{i}} = 0 + 0 + \frac{1}{2}(2.25\text{m/s}^2\hat{\mathbf{i}})t_1^2$$

$$matching : 225\text{m} = (1.125\text{m/s}^2)t_1^2$$

$$t_1 = \sqrt{\frac{225}{1.125}}s = 14.14s$$

For the second part of the movement:

$$\vec{v}_{02} = \vec{v}_{01} + \vec{a}_1 t_1$$

$$\vec{v}_{02} = 0 + (2.25\text{m/s}^2).(14.14\text{s})\hat{\mathbf{i}} = 31.82\text{m/s}\hat{\mathbf{i}}$$

$$\vec{v}_f = \vec{v}_{02} + \vec{a}_2 t_2$$

$$t_2 = \frac{31.82\text{m/s}}{0.75\text{m/s}^2} = 42.43\text{s}$$

So the total time is:

$$t = t_1 + t_2 = 14.14s + 42.43s = 57.57s$$

5. Before we start solving this problem, we need define the frame of reference:

(a)

$$\vec{r}_f = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}t_1^2$$

$$0 = h\hat{\mathbf{j}} + 0 + \frac{1}{2}(-9.8\text{m/s}^2)\hat{\mathbf{j}}\ (4\text{s})^2$$

With matching we can calculate the height of the building

$$h = 78.4m.$$

(b)

$$\vec{v}_{bf} = \vec{v}_0 + \vec{g}t$$

$$\vec{v}_{bf} = 0 + g\hat{\mathbf{j}}t = (-9.8)(4)\hat{\mathbf{j}}$$

$$\vec{v}_{bf} = -39.2\hat{\mathbf{j}}\text{m/s}$$

6.

(a)

$$\vec{v} = \vec{v}_0 + \vec{a}t$$
$$0 = 100\text{m/s}\hat{\mathbf{i}} - 5\text{m/s}^2\hat{\mathbf{i}}t$$

With matching we can calculate the time

$$t = 20\text{s}$$

(b)

$$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t_1^2$$
$$\vec{r} - \vec{r}_0 = (100\text{m/s})\hat{\mathbf{i}}.(20\text{s}) - \frac{1}{2}(5\text{m/s}^2)\hat{\mathbf{i}}.(20\text{s})^2$$
$$\Delta r = \vec{r} - \vec{r}_0 = 1000\hat{\mathbf{i}}\text{m} = 1\text{km}$$

Therefore, the plane cannot land on this small airport.

