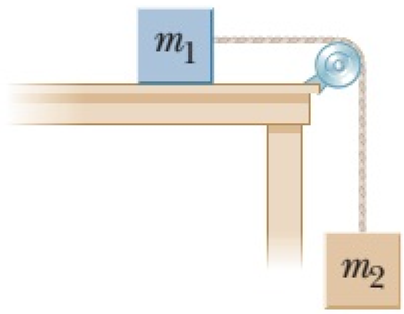
Tutorial 5: The Laws of Motion

1. An object of mass $m_1 = 55$ kg placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging object of mass $m_2 = 59$ kg as shown in Figure below.

   (a) Draw free-body diagrams of both objects.

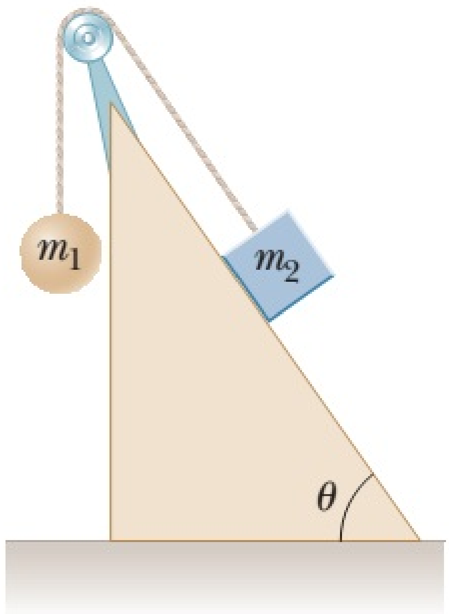


   (b) Find the magnitude of the acceleration of the objects

   (c) Find the tension in the string.

   (d) If there is a friction $\mu_k = 0.2$ on the surface, repeat the part a ,b, c, d

2. Two objects are connected by a light string that passes over a frictionless pulley as shown in Figure below. Assume the incline is frictionless and take $m_1 = 2$ kg, $m_2 = 6$ kg, and $\theta = 55$



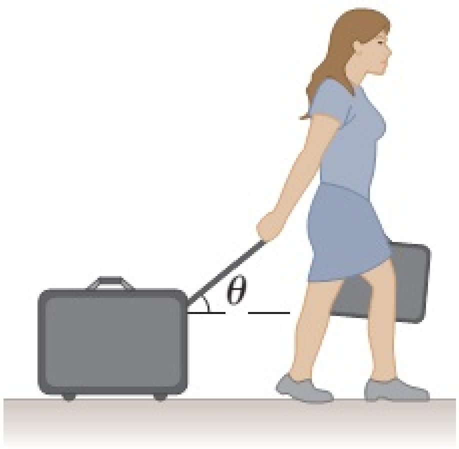
   (a) Draw free-body diagrams of both objects.

(b) Find the magnitude of the acceleration of the objects

(c) Find the tension in the string.

(d) Find the speed of each object 2 s after it is released from rest.

3. A woman at an airport is pulling her $20kg$ suitcase at constant speed by pulling on a strap at an angle above the horizontal. She pulls on the strap with a $35N$ force, and the friction force on the suitcase is $20N$.

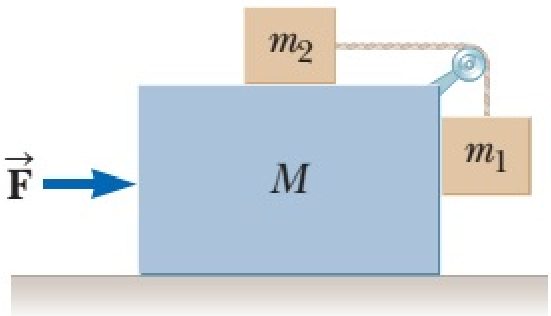


(a) Draw a freebody diagram of the suitcase.

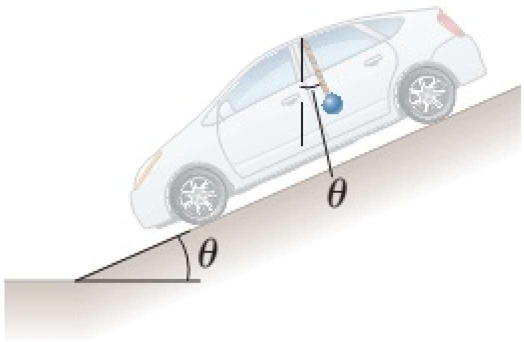
(b) What angle does the strap make with the horizontal?

(c) What is the magnitude of the normal force that the ground exerts on the suitcase?

4. What horizontal force must be applied to a large block of mass M shown in Figure below so that the tan blocks remain stationary relative to M? Assume all surfaces and the pulley are frictionless. Notice that the force exerted by the string accelerates m_2.



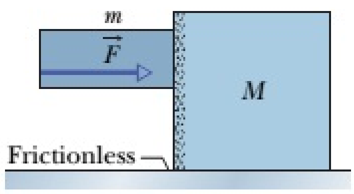
5. A car accelerates down a hill (see figure below), going from rest to $30 m/s$ in $6s$. A toy inside the car hangs by a string from the car's ceiling. The ball in the figure represents the toy, of mass $0.1kg$. The acceleration is such that the string remains perpendicular to the ceiling.



(a) Determine the angle θ?

(b) Determine the tension in the string?

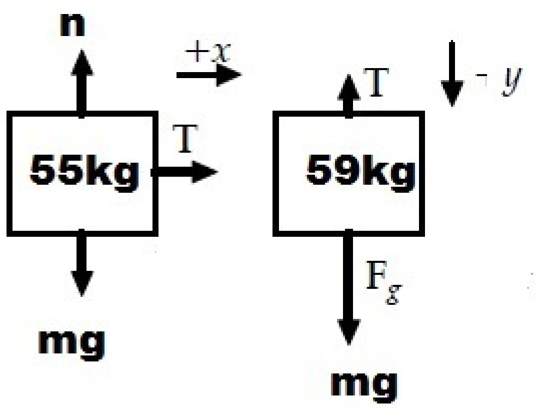
6. The two blocks $(m = 16kg)$ and $(M = 88kg)$ in Figure below are not attached to each other.The coefficient of static friction between the blocks is $\mu_k = 0.38$, but the surface beneath the larger block is frictionless. What is the minimum magnitude of the horizontal force required to keep the smaller block from slipping down the larger block?



Solutions

1.

a. Free-Body Diagrams (Notice the chosen directions for acceleration)



b. Applying the second law of motion $\Sigma\vec{F} = m\vec{a}$
for m_1

$$\Sigma\vec{F}_1 = m_1\vec{a} \Longrightarrow \vec{T} + \vec{n} + \vec{F}_g = m_1\vec{a} \quad (1)$$

$$T\hat{\mathbf{i}} + n\hat{\mathbf{j}} - m_1 g\hat{\mathbf{j}} = m_1 a\hat{\mathbf{i}} \quad (2)$$

By matching the x-components and y-components we can approach to two equations:

$$T = m_1 a \quad (3)$$

$$n = m_1 g \quad (4)$$

for m_2

$$\Sigma\vec{F}_2 = m_2\vec{a} \Longrightarrow \vec{T} + m_2\vec{g} = m_2\vec{a} \quad (5)$$

$$-m_2 g\hat{\mathbf{j}} + T\hat{\mathbf{j}} = -m_2 a\hat{\mathbf{j}} \Longrightarrow -m_2 g + T = -m_2 a \quad (6)$$

from (3) and (6)

$$-m_2 g + m_1 a = -m_2 a$$

$$m_2 g = (m_1 + m_2)a$$

$$a = \frac{m_2}{(m_1 + m_2)} g \quad (7)$$

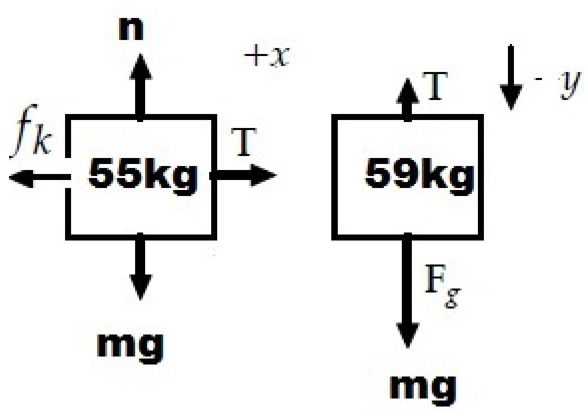
$m_1 = 55kg$ and $m_2 = 59kg$ and $g = 9.81ms^{-2}$

$$a = \frac{59kg}{59kg + 55kg} \times 9.81ms^{-2} = 5.08ms^{-2}$$

c. To find the tension we may use the equation (3):

$$T = m_1 a = 55kg \times 5.08ms^{-2} = 279.4N$$

d. Free-Body diagram when friction exists between the



for m_1

$$\Sigma\vec{F}_1 = m_1\vec{a} \Longrightarrow \vec{T} + \vec{f}_k + \vec{n} + m_1\vec{g} = m_1\vec{a} \tag{8}$$

$$T\hat{\mathbf{i}} - f_k\hat{\mathbf{i}} + n\hat{\mathbf{j}} - m_1 g\hat{\mathbf{j}} = m_1 a\hat{\mathbf{i}} \tag{9}$$

By matching the x-components and y-components we can approach to two equations:

$$T - f_k = m_1 a \qquad ; \qquad f_k = \mu_k n \tag{10}$$

$$n - m_1 g = 0 \qquad \Rightarrow \qquad n = m_1 g \tag{11}$$

from equations of (10) and (11), we can have

$$T - \mu_k m_1 g = m_1 a \tag{12}$$

for m_2

$$-m_2 g\hat{\mathbf{j}} + T\hat{\mathbf{j}} = -m_2 a\hat{\mathbf{j}} \Longrightarrow -m_2 g + T = -m_2 a \tag{13}$$

Now we add equations (10) and (11), we will obtain

$$m_2 g - \mu_k m_1 g = (m_1 + m_2)a$$

$$a = \frac{(m_2 - \mu_k m_1)}{m_1 + m_2} g \tag{14}$$

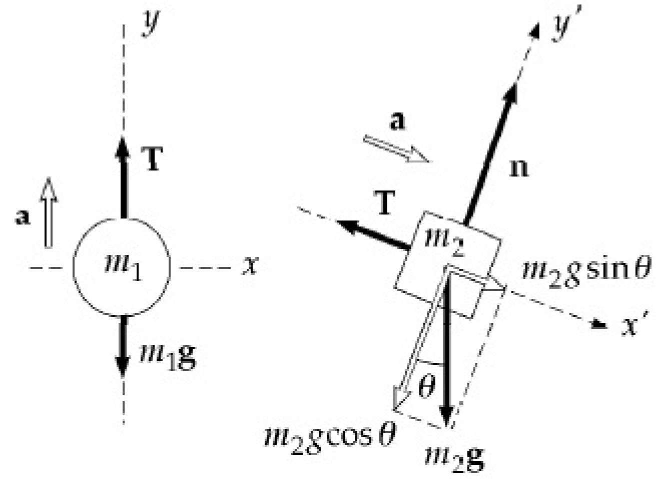
$$a = \frac{(59kg - 0.2 \times 55kg) \times 9.81ms^{-2}}{55kg + 59kg} = 4.13ms^{-2}$$

and for tension we may use equation (14), as you can see below:

$$T = m_2(g - a) = 59kg \times (9.81ms^{-2} - 4.13ms^{-2}) = 335.12N$$

2.

a. Free-Body Diagram (Notice the chosen directions for acceleration)



b. Apply the second law for m_1

$$\Sigma\vec{F}_1 = m_1\vec{a} \tag{15}$$

$$\Sigma\vec{F}_1 = \vec{T} + m_1\vec{g} = m_1\vec{a} \tag{16}$$

$$T\hat{\mathbf{j}} - m_1 g\hat{\mathbf{j}} = m_1 a\hat{\mathbf{j}} \tag{17}$$

By matching:

$$T - m_1 g = m_1 a \tag{18}$$

for m_2

$$\Sigma\vec{F}_2 = m_2\vec{a} \tag{19}$$

$$m_2\vec{g}\sin\theta + \vec{T} + \vec{n} + m_2\vec{g}\cos\theta = m_2\vec{a} \tag{20}$$

$$m_2 g\sin\theta\hat{\mathbf{i}} - T\hat{\mathbf{i}} + n\hat{\mathbf{j}} - m_2 g\cos\theta\hat{\mathbf{j}} = m_2 a\hat{\mathbf{i}} \tag{21}$$

By matching for x-components and y-components we can approach to two equations:

$$m_2 g\sin\theta - T = m_2 a \tag{22}$$

$$n - m_2 g\cos\theta = 0 \tag{23}$$

Adding (18) and (22), we get

$$m_2 g\sin\theta - m_1 g = (m_1 + m_2)a$$

$$a = \frac{(m_2\sin\theta - m_1)}{m_1 + m_2}g \tag{24}$$

$$a = \frac{(6kg \times \sin 55 - 2kg)}{2kg + 6kg} \times 9.81ms^{-2} = 3.57ms^{-2}$$

c. To calculate tension we use equation (18)

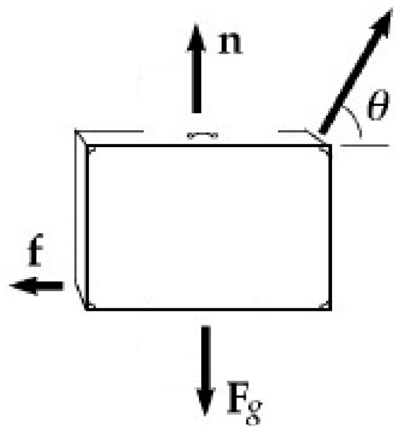
$$T = m_1(g + a) = 2kg \times (9.81ms^{-2} + 3.57ms^{-2}) = 26.76N$$

d. For both objects we have $v_i = 0$ and we may use $v_f = v_i + at$

$$v_f = at = 3.57ms^{-2} \times 2.00s = 7.14ms^{-1}$$

3.

a. Free-Body Diagram



b. Apply the second law to the suitcase but note that the right-hand side of the equation on the horizontal must be zero since the suitcase is moved at constant speed.

$$\Sigma\vec{F} = 0 \Rightarrow \qquad \vec{n} + \vec{F} + m\vec{g} + \vec{f_k} = 0 \tag{25}$$

$$n\hat{\mathbf{j}} + Fsin\theta\hat{\mathbf{j}} + Fcos\theta\hat{\mathbf{i}} - mg\hat{\mathbf{j}} - f_k\hat{\mathbf{i}} = 0 \tag{26}$$

By matching the $x - components$, we have

$$F\cos\theta - f_k = 0 \qquad \Rightarrow \qquad \cos\theta = \frac{f_k}{F} \tag{27}$$

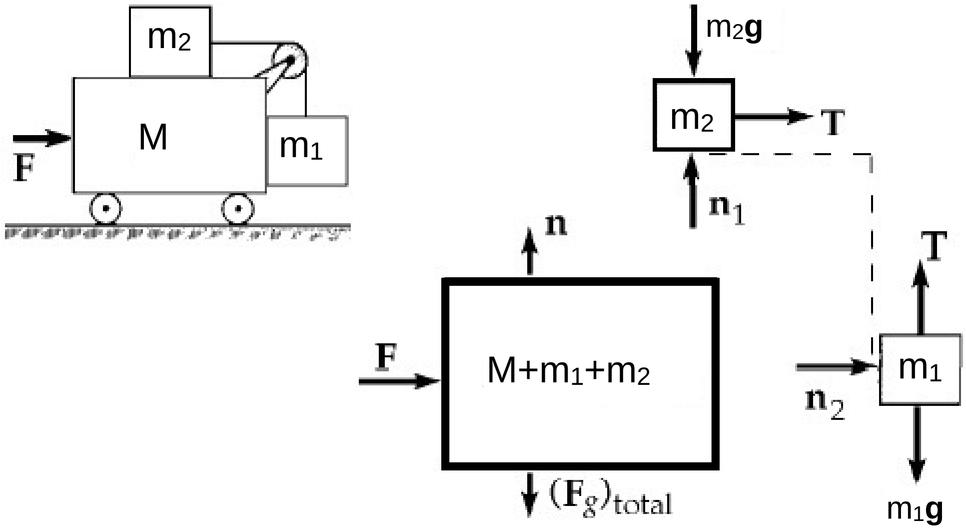
$$\cos\theta = \frac{20N}{35N} = 0.57 \qquad \Rightarrow \qquad \theta = \arccos(0.57) = 55.15^\circ$$

c. By matching the $y - components$, we can obtain

$$n + F\sin\theta - mg = 0 \tag{28}$$

$$n = (20kg)(9.81ms^{-2}) - 35N \times \sin(55.15) = 167.48N$$

4.



In accordance with the second low of motion $\Sigma\vec{F} = m\vec{a}$ for entire system, we have

$$F = (M + m_1 + m_2)a \tag{29}$$

for m_2

$$\vec{F_2} = m_2\vec{a} \qquad \Rightarrow \qquad m_2\vec{g} + \vec{n}_1 + \vec{T} = m_2\vec{a}m_2g\hat{\mathbf{j}} - n_1\hat{\mathbf{j}} + T\hat{\mathbf{i}} = m_2a\hat{\mathbf{i}} \tag{30}$$

By matching the $x - components$, we have

$$T = m_2a \tag{31}$$

for m_1

$$\Sigma\vec{F_2} = 0 \qquad \Rightarrow \qquad \vec{n_2} + \vec{T} + m_1\vec{g} = 0 \tag{32}$$

By matching for $y - components$, one can obtain

$$T - m_1g = 0 \tag{33}$$

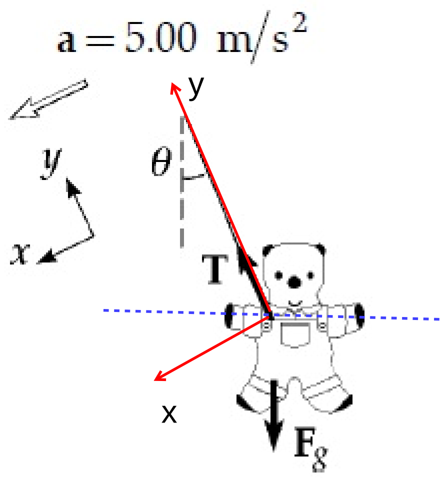
By eliminating T from equations of (31) and (33), we get

$$m_2a - m_1g = 0 \qquad \Rightarrow \qquad a = \frac{m_1}{m_2}g \tag{34}$$

for the entire system

$$F = (M + m_1 + m_2)(\frac{m_1}{m_2})g \tag{35}$$

5.



a. Using the coordinate choice seen in the figure, we can derive the following equations:

$$v_f = v_i + at$$

$$30ms^{-1} = 0 + a(6ms^{-2}) \qquad \Rightarrow \qquad a = 5ms^{-2}$$

$$\Sigma\vec{F} = m\vec{a} \qquad \Rightarrow \qquad \vec{T} + m\vec{g} = m\vec{a} \tag{36}$$

$$T\hat{} - mgcos\theta\hat{} - mgsin\theta\hat{} = -ma\hat{} \tag{37}$$

By matching the $x - components$, we obtain

$$mg\sin\theta = ma \qquad \Rightarrow \qquad \sin\theta = \frac{a}{g} \tag{38}$$

$$\sin\theta = \frac{5.00ms^{-2}}{9.81ms^{-2}} = 0.51 \qquad \Rightarrow \qquad \theta = \sin^{-1}(0.51) = 30.64^{\circ}$$

b. By matching the $y - components$, we have

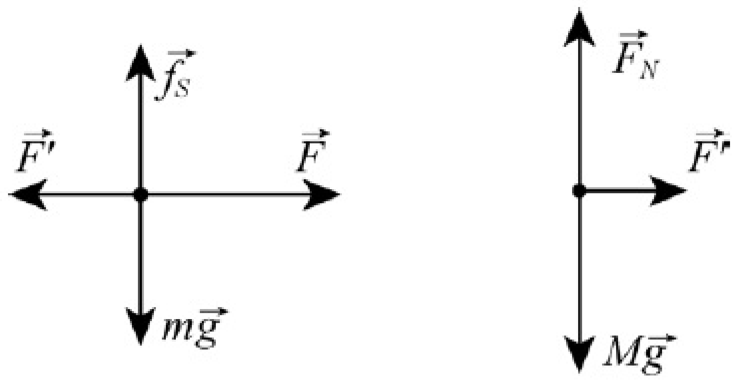
$$T - mg\cos\theta = 0 \tag{39}$$

$$T = (0.1kg)(9.81ms^{-2})\cos(30.64) = 0.844N$$

6. Firstly, with using the $\Sigma\vec{F} = (m+M)\vec{a}$:

$$F = (m+M)a \qquad \Rightarrow \qquad a = \frac{F}{m+M} \tag{40}$$

Secondly, we draw the free body diagram for m and M:



For m:

$$\Sigma\vec{F} = m\vec{a} \tag{41}$$

$$\vec{f_{s_{max}}} + m\vec{g} + \vec{F} + \vec{F'} = 0 \qquad \Rightarrow \qquad f_{s_{max}}\hat{\mathbf{j}} - mg\hat{\mathbf{j}} + F\hat{\mathbf{i}} - F'\hat{\mathbf{i}} = 0 \tag{42}$$

By matching the $y - components$, we have

$$mg - f_{s_{max}} = 0 \qquad ; \qquad f_{s_{max}} = \mu_s n \qquad \Rightarrow \qquad n = \frac{mg}{\mu_s} \tag{43}$$

and, by matching the $x - components$, one can obtain

$$F - n = ma \tag{44}$$

from equations (40), (43), and (44), we get

$$F = \frac{(m+M)mg}{M\mu_s} \tag{45}$$

$$F = \frac{(16kg + 88kg) \times 16kg \times 9.81ms^{-2}}{88kg \times 0.38} = 488.15N$$