Wireless Communications

Chapter 4

Exercises, Examples, Problems

Spring 2012-13

Example 4.1

Find the far-field distance for an antenna with maximum dimension of 1 m and operating frequency of 900 MHz.

Solution 4.1

Given:

Largest dimension of antenna, D = 1 m

Operating frequency f = 900 MHz,

$$\lambda = c / f = (3 * 10^8 \text{ m/s}) / (9 * 10^8 \text{ /s}) = 1/3 \text{ m}$$

Using equation (4.7.a), far-field distance is obtained as

$$d_f = {2D^2}/{\lambda} = {2\,1^2}/{1/3} = 6\,m$$

Example 4.2

If a transmitter produces 50 watts of power, express the transmit power in units of

a) dBm,

b) dBW.

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna.

What is P_r (10 km)? Assume unity gain for the receiver antenna.

Solution 4.2

Solution to Example 3.2

Given:

Transmitter power, $P_t = 50$ W.

Carrier frequency, $f_c = 900$ MHz

Using equation (3.9),

(a) Transmitter power,

$$P_t(\text{dBm}) = 10\log[P_t(\text{mW})/(1\text{ mW})]$$
$$= 10\log[50 \times 10^3] = 47.0\text{ dBm}.$$

(b) Transmitter power,

$$P_t(\text{dBW}) = 10\log[P_t(\text{W})/(1\text{ W})]$$
$$= 10\log[50] = 17.0\text{ dBW}.$$

The received power can be determined using equation (3.1).

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} = \frac{50(1)(1)(1/3)^2}{(4\pi)^2(100)^2(1)} = 3.5 \times 10^{-6}\text{ W} = 3.5 \times 10^{-3}\text{ mW}$$

$$P_r(\text{dBm}) = 10\log P_r(\text{mW}) = 10\log\left(3.5 \times 10^{-3}\text{ mW}\right) = -24.5\text{ dBm}.$$

The received power at 10 km can be expressed in terms of dBm using equation (3.9), where $d_0 = 100$ m and $d = 10$ km

$$P_r(10\text{ km}) = P_r(100) + 20\log\left[\frac{100}{10000}\right] = -24.5\text{ dBm} - 40\text{ dB}$$
$$= -64.5\text{ dBm}.$$

Example 4.3

Assume a receiver is located 10 km from a 50 W transmitter. The carrier frequency is 900 MHz, free space propagation is assumed, $G_t = 1$, and $G_r = 2$, find

a) the power at the receiver,
b) the magnitude of the E-field at the receiver antenna
c) the rms voltage applied to the receiver input assuming that the receiver antenna has a purely real impedance of 50 Ω and is matched to the receiver.

Solution 4.3

Given:

- Transmitter power, $P_t = 50\ \text{W}$
- Carrier frequency, $f_c = 900\ \text{MHz}$
- Transmitter antenna gain, $G_t = 1$
- Receiver antenna gain, $G_r = 2$
- Receiver antenna resistance $= 50\ \Omega$

Solution 4.3

(a) Using equation (3.5), the power received at a distance $d = 10$ km is

$$P_r(d) = 10\log\left(\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2}\right) = 10\log\left(\frac{50 \times 1 \times 2 \times (1/3)^2}{(4\pi)^2 10000^2}\right)$$

$$= -91.5 \text{ dBW} = -61.5 \text{ dBm}$$

(b) Using equation (3.15), the magnitude of the received E-field is

$$|E| = \sqrt{\frac{P_r(d)120\pi}{A_e}} = \sqrt{\frac{P_r(d)120\pi}{G_r\lambda^2/4\pi}} = \sqrt{\frac{7 \times 10^{-10} \times 120\pi}{2 \times 0.33^2/4\pi}} = 0.0039 \text{ V/m}$$

(c) Using equation (3.16), the open circuit rms voltage at the receiver input is

$$V_{ant} = \sqrt{P_r(d) \times 4R_{ant}} = \sqrt{7 \times 10^{-10} \times 4 \times 50} = 0.374 \text{ mV}$$

Example 4.4

Demonstrate that if medium 1 is free space and medium 2 is a dielectric, both $|\Gamma_{||}|$ and $|\Gamma_{\perp}|$ approach 1 as θ_i approaches 0^0 regardless of ε_r.

Example 4.4

Background on Reflection

- When a radio wave propagating in one medium impinges upon another medium having different electrical properties, the wave is partially reflected and partially transmitted

- If the plane wave is incident on a perfect dielectric, part of the energy is transmitted into the second medium and part of the energy is reflected back into the first medium, and there is no loss of energy in absorption.

Example 4.4

- If the second medium is a perfect conductor, then all incident energy is reflected back into the first medium without loss of energy
- The electric field intensity of the reflected and transmitted waves may be related to the incident wave in the medium of origin through the Fresnel reflection coefficient (Γ)
- The reflection coefficient is a function of the material properties, and generally depends on the wave polarization, angle of incidence, and frequency of the propagating wave

Example 4.4

- **Reflection from Dielectrics**
- Figure 4.4 shows an electromagnetic wave incident at an angle θ_i with the plane of the boundary between two dielectric media.
- As shown in the figure, part of the energy is reflected back to the first media at an angle θ_r and part of the energy is transmitted (refracted) into the second media at an angle θ_t The nature of reflection varies with the direction of polarization of the E-field.
- The behavior for arbitrary directions of polarization can be studied by considering the two distinct cases shown in Figure4.4.

Example 4.4

- The plane of incidence is defined as the plane containing the incident, reflected, and transmitted rays
- In Figure 4.4a, the E-field polarization is parallel with the plane of incidence (that is, the E-field has a vertical polarization, or normal component, with respect to the reflecting surface) and in Figure 4.4b, the E-field polarization is perpendicular to the plane of incidence (that is, the incident E-field is pointing out of the page towards the reader, and is perpendicular to the page and parallel to the reflecting surface).

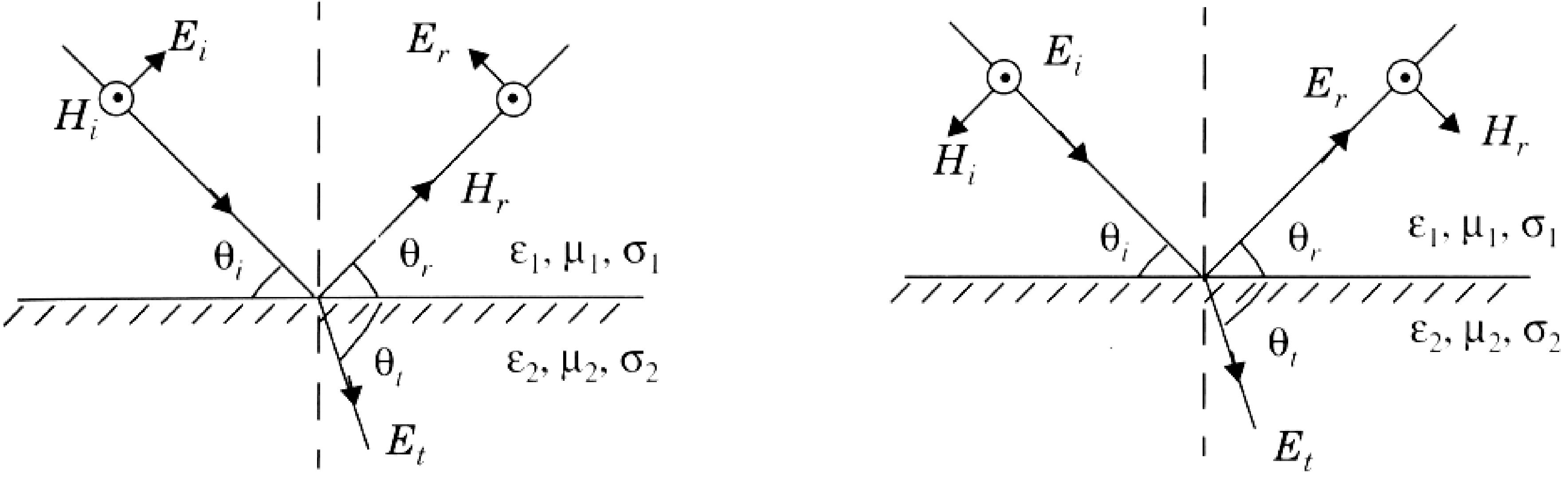
Example 4.4

- In Figure 4.4, the subscripts *i*, *r*, *t* refer to the incident, reflected and transmitted fields, respectively. Parameters ε_1, μ_1, σ_1 and ε_2, μ_2, σ_2 represent the permittivity, permeability and conductance of the two media, respectively
- Often, the dielectric constant of a perfect (lossless) dielectric is related to a relative value of permittivity, ε_r such that

$$\varepsilon = \varepsilon_o \varepsilon_r$$

- where ε_o is a constant given by $8.85*10^{-12}$ F/m

Reflection from smooth surface



(a) E-field in the plane of incidence (b) E-field normal to the plane of incidence

Figure 4.4 Geometry for calculating the reflection coefficients between two dielectrics.

Solution 4.4

Substituting the angel of incidence $\theta_i = 0^0$ in equation (4.24)

$$\Gamma_{\parallel} = \frac{-\varepsilon_r \sin 0 + \sqrt{\varepsilon_r - \cos^2 0}}{\varepsilon_r \sin 0 + \sqrt{\varepsilon_r - \cos^2 0}}$$

$$\Gamma_{\parallel} = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r - 1}}$$

$$= 1$$

Substituting $\theta_i = 0^0$ in equation (4.25)

$$\Gamma_{\perp} = \frac{\sin 0 - \sqrt{\varepsilon_r - \cos^2 0}}{\sin 0 + \sqrt{\varepsilon_r - \cos^2 0}}$$

$$\Gamma_{\perp} = \frac{-\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r - 1}}$$

$$= -1.$$

Example 4.5

Calculate the Brewster angle for a wave impinging on ground having a permitivity $\varepsilon_r = 4$

Solution 4.5

The Brewster angle is the angle at which no reflection occurs in the medium of origin. It occurs when the incident angle θ_B is such that the reflection coefficient $\Gamma_{||} = 0$. See Figure 4.6

The Brewster angle is given by the value of θ_B which satisfies

$$\sin(\theta_B) = \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}} \qquad (4.27)$$

For the case when the first medium is free space and the second medium has a relative permittivity ε_r, 4.27 can be expressed as

$$\sin(\theta_B) = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r^2 - 1}}$$

Note: Brewster angle occurs only for vertical (i.e. parallel) polarization.

Solution 4.5

The Brewster angle can he found by substituting the values for ε_r in equation (4.28)

$$\sin(\theta_i) = \frac{\sqrt{(4) - 1}}{\sqrt{(4)^2 - 1}} = \sqrt{\frac{3}{15}} = \sqrt{\frac{1}{5}}$$

$$\theta_i = \sin^{-1}\sqrt{\frac{1}{5}} = 26.56^\circ$$

Thus Brewster angle for $\varepsilon_r = 4$ is equal to 26.56°

Example 4.6

A mobile is located 5 km away from a base station and uses a vertical $\lambda/_4$ monopole antenna with a gain of 2.55 dB to receive cellular radio signals. The E-field at 1 km from the transmitter is measured to be 10^{-3} V/rn. The carrier frequency used for this system is 900 MHz.

a) Find the length and the gain of the receiving antenna.
b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5 m above ground.

Solution 4.6

Given:

T-R separation distance = 5 km

E-field at a distance of 1 km = 10^{-3} V/m

Frequency of operation, f = 900 MHz

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{900\times10^{6}} = 0.333 \text{ m.}$$

Length of the antenna, $L = \lambda/4 = 0.333/4 = 0.0833$ m = 8.33 cm.

Gain of $\lambda/4$ monopole antenna can be obtained using equation (3.2).

Gain of antenna =1.8 = 2.55 dB.

Solution 4.6

(b) Since $d \gg \sqrt{h_t h_r}$, the electric field is given by

$$E_R(d) \approx \frac{2E_0 d_0}{d} \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

$$= \frac{2 \times 10^{-3} \times 1 \times 10^{3}}{5 \times 10^{3}} \left[\frac{2\pi (50)(1.5)}{0.333 \left(5 \times 10^{3} \right)} \right]$$

$$= 113.1 \times 10^{-6} \text{ V/m.}$$

The received power at a distance d can be obtained using equation (3.15)

$$P_r(d) = \frac{\left(113.1 \times 10^{-6}\right)^2}{377} \left[\frac{1.8 (0.333)^2}{4\pi} \right]$$

$$P_r(d = 5 \text{ km}) = 5.4 \times 10^{-13} \text{ W} = -122.68 \text{ dBW or } -92.68 \text{ dBm.}$$