

# **EENG226**

## **Signals and Systems**

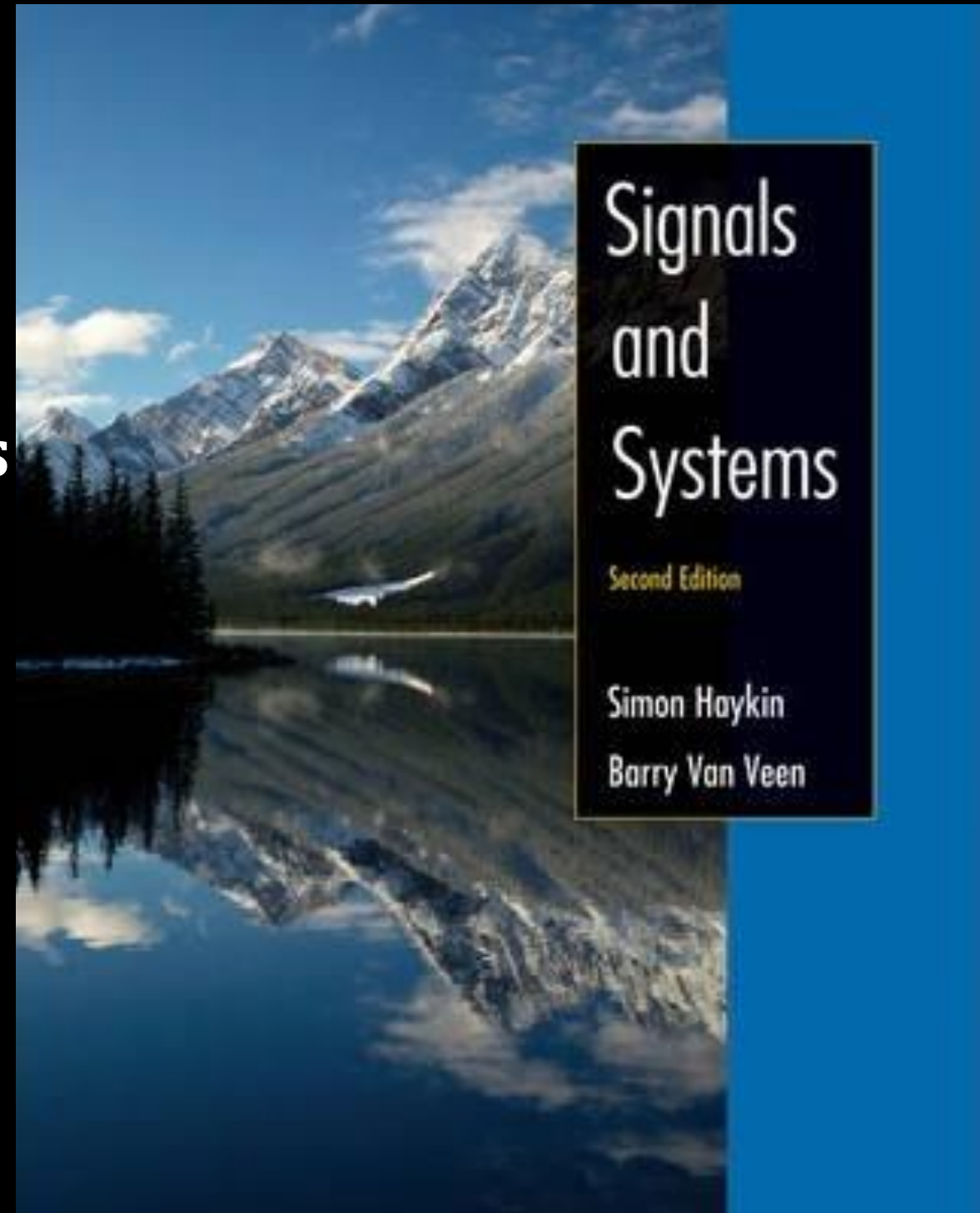
### **Chapter 3**

### **Fourier Representations of Signals and Linear Time-Invariant Systems**

## **Introduction**

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*Signals and Systems, 2/E* by Simon Haykin and Barry Van Veen  
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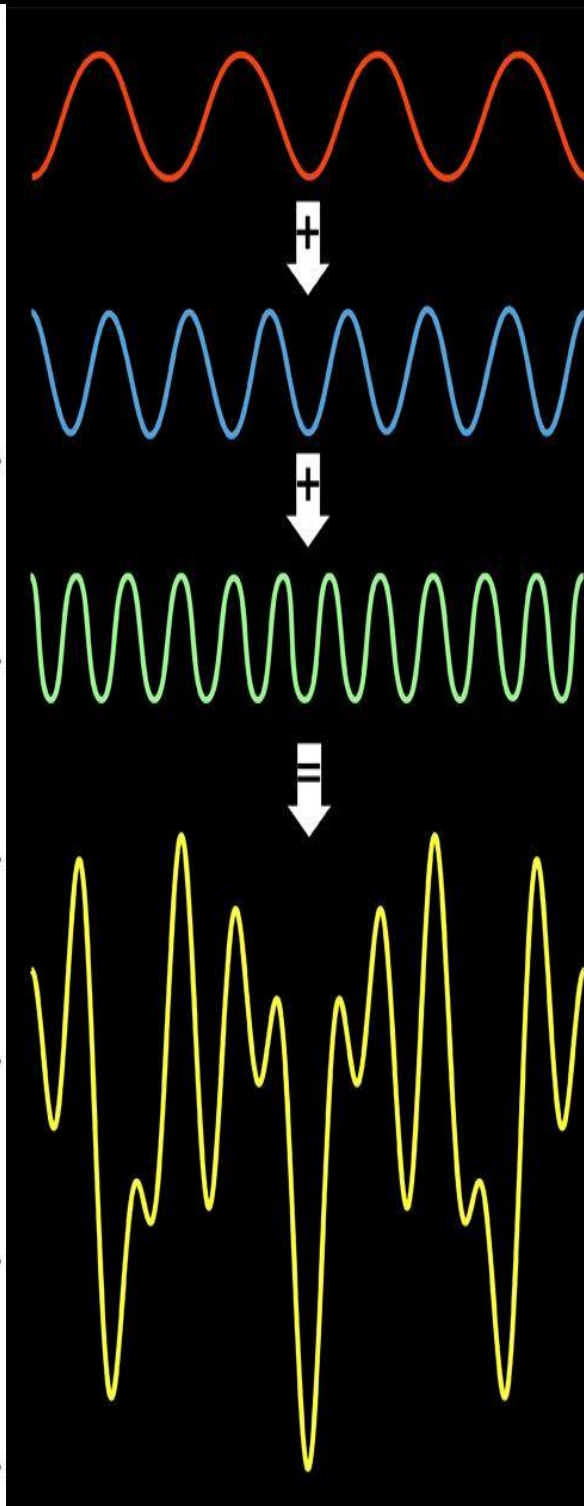
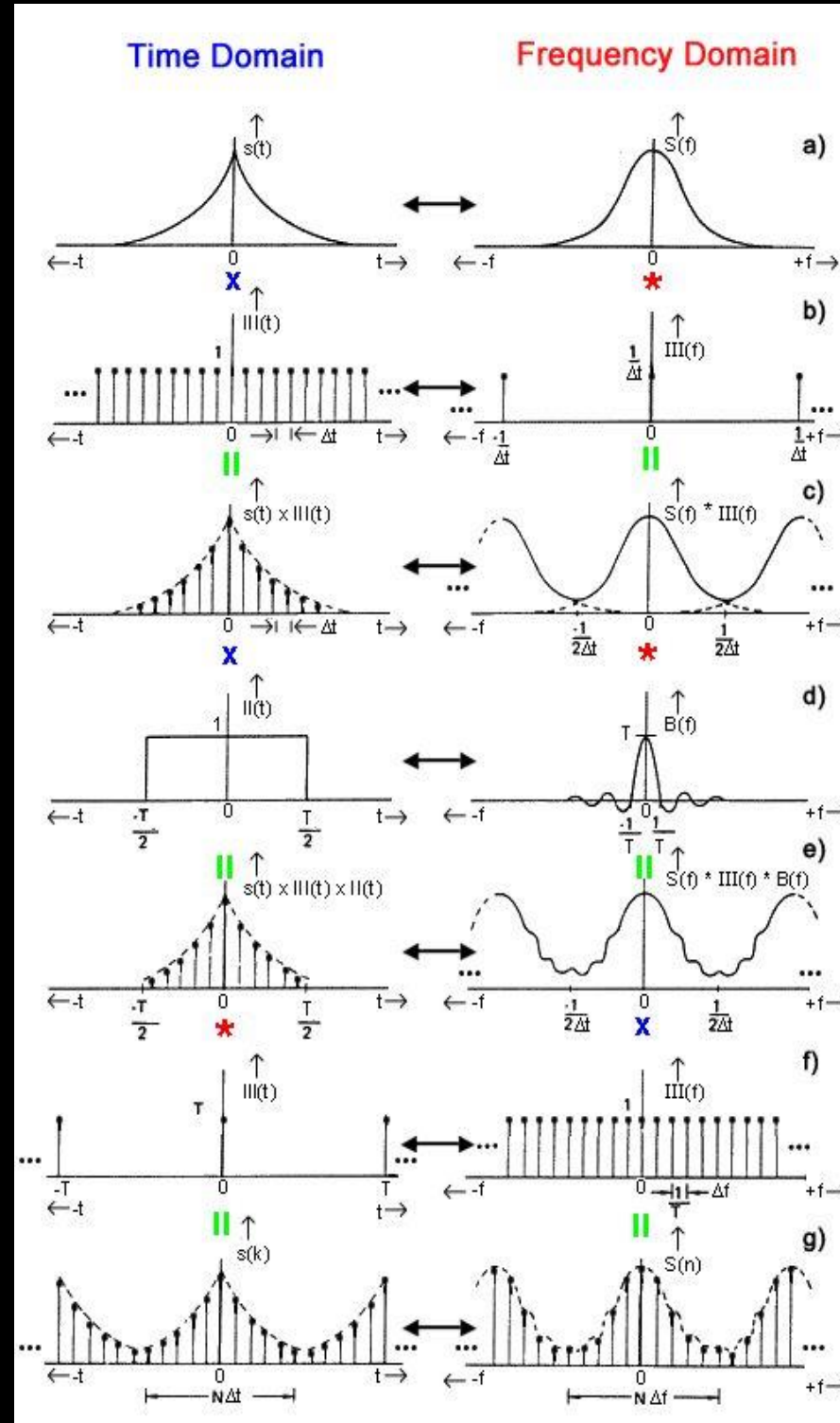


# Chapter 3

## Fourier Representations of Signals and Linear Time-Invariant Systems

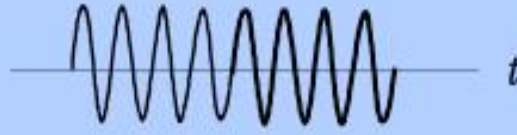
### Objectives of this chapter

- Introduction
- Complex Sinusoids and Frequency Response of LTI Systems
- Fourier Representations for Four Classes of Signals
- Discrete-Time Periodic Signals: The Discrete-Time Fourier Series
- Continuous-Time Periodic Signals: The Fourier Series
- Discrete-Time Nonperiodic Signals: The Discrete-Time Fourier Transform
- Continuous-Time Nonperiodic Signals: The Fourier Transform
- Properties of Fourier Representations
- Linearity and Symmetry Properties
- Convolution Property
- Differentiation and Integration Properties
- Time- and Frequency-Shift Properties
- Finding Inverse Fourier Transforms by Using Partial-Fraction Expansions
- Multiplication Property
- Scaling Properties
- Parseval Relationships
- Time-Bandwidth Product
- 3.18 Duality
- 3.19 Exploring Concepts with MATLAB 312



# Time-Domain versus Frequency-Domain Representation of Well-Known Signals

Signal  $s(t)$



*cosine wave*



*sinc function*



*Gaussian*



*double exponential*

Fourier Transform  $S(\omega)$



*single frequency*



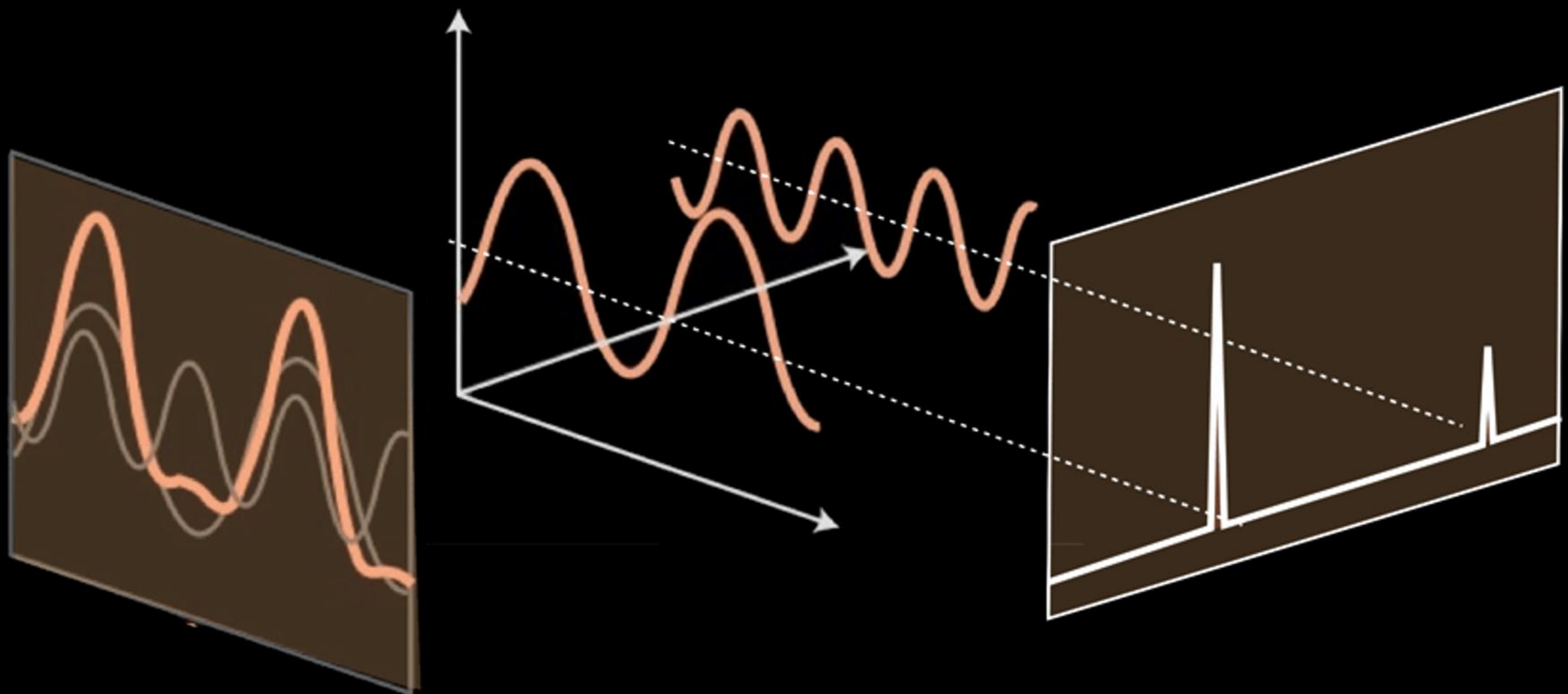
*uniform band of  
frequencies*



*Gaussian*



*Lorentzian*



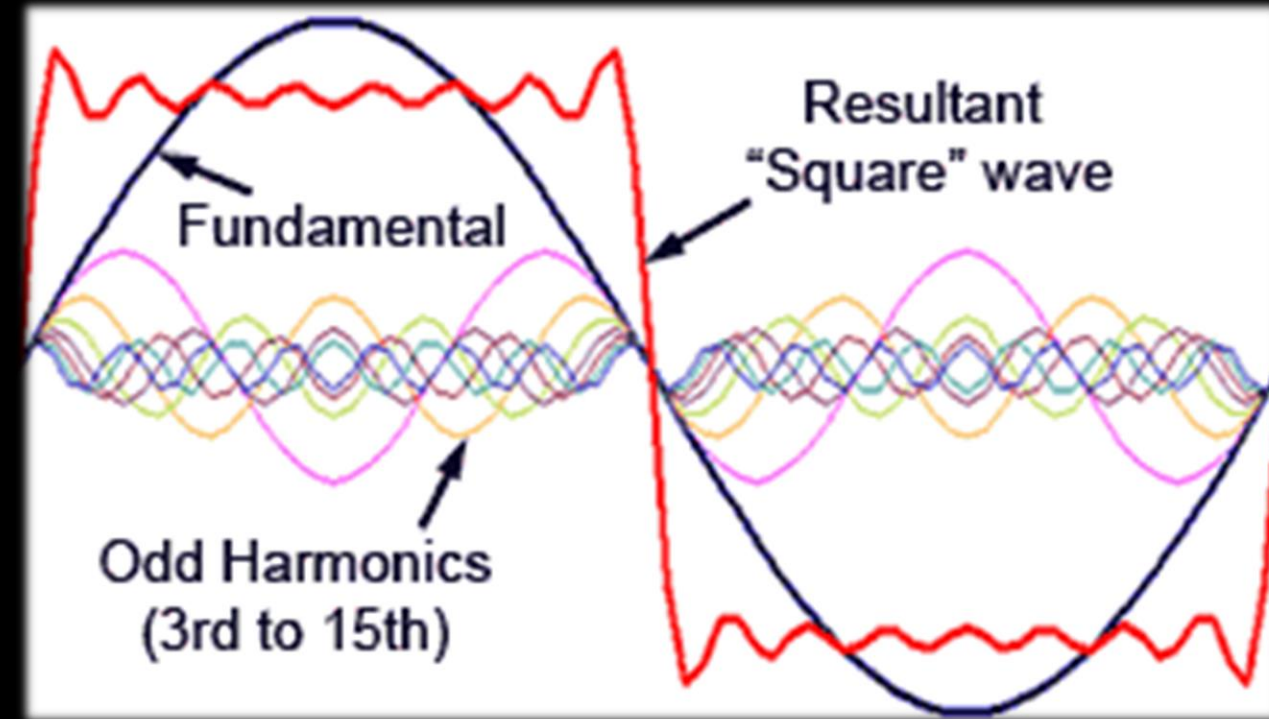
*Time Domain*  
 $s(t)$

**FT**  
→

*Frequency Domain*  
 $S(\omega)$

# 3.1 Introduction

- A signal can be represented as a weighted superposition of complex sinusoids
- If such a signal is applied to an LTI system, then the system output is a weighted superposition of the system response to each complex sinusoid



- Expressing signals in terms of impulses was termed “convolution”
- By representing signals in terms of sinusoids, we will obtain an alternative expression for the input-output behavior of an LTI system

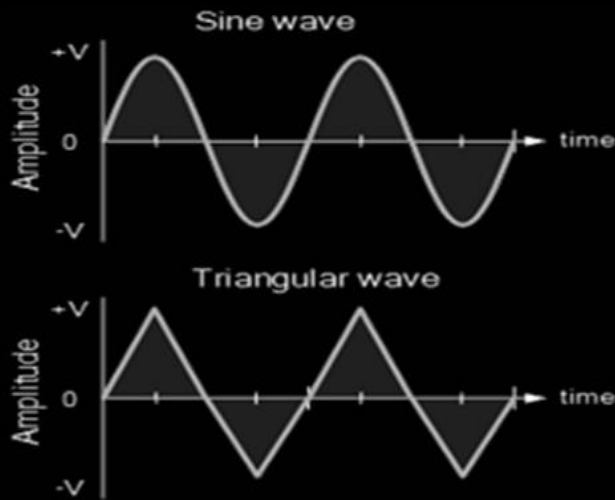
# 3.1 Introduction

- Representing signals as superpositions of complex sinusoids not only leads to a useful expression for the system output, but also provides an insightful characterization of signals and systems.
- The sound that we hear when listening to an orchestra is a superposition of sounds generated by different instruments
  - score for a choir contains bass, tenor, alto, and soprano parts, each of which contributes to a different frequency range in the overall sound

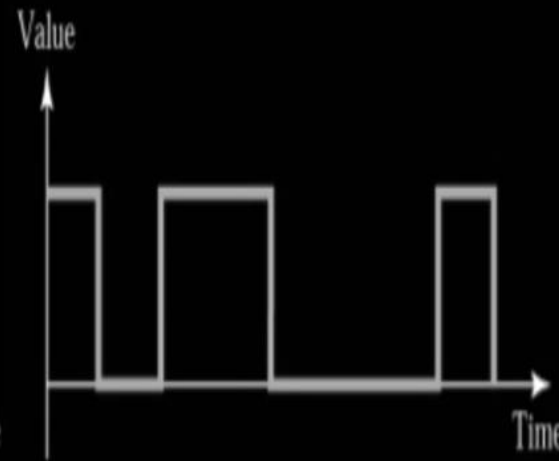


# 3.1 Introduction

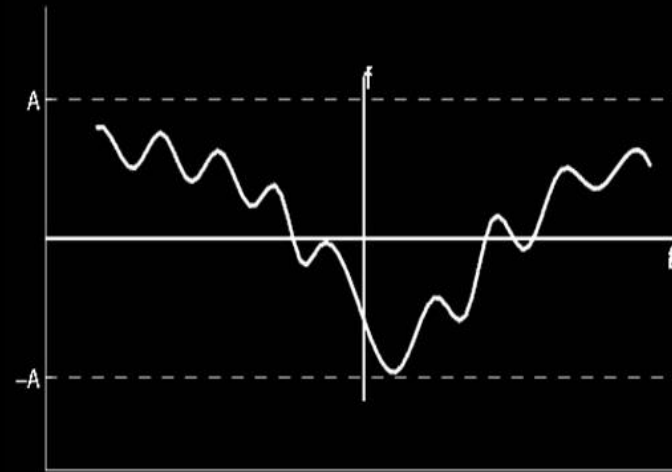
- The weight associated with a sinusoid of a given frequency represents the contribution of that sinusoid to the overall signal
- The study of signals and systems using sinusoidal representations is termed Fourier Analysis, after Joseph Fourier (1768-1830) who developed the theory
- There are 4 distinct Fourier representations, periodic, nonperiodic, discrete, continuous



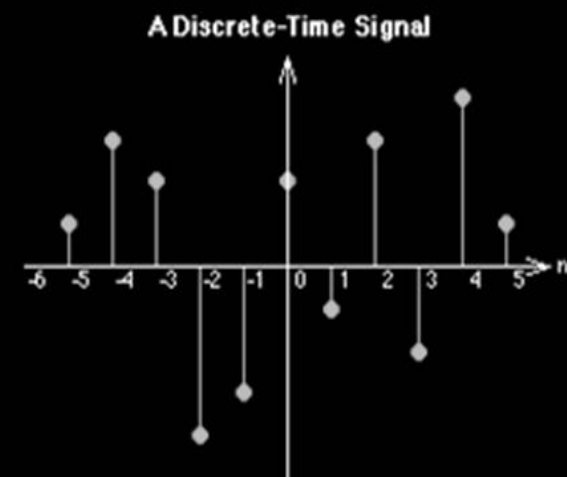
Periodic Signal



Aperiodic Signal



Continuous Signal



Discrete Signal 7

## 3.2 Complex Sinusoids and Frequency Response of LTI Systems

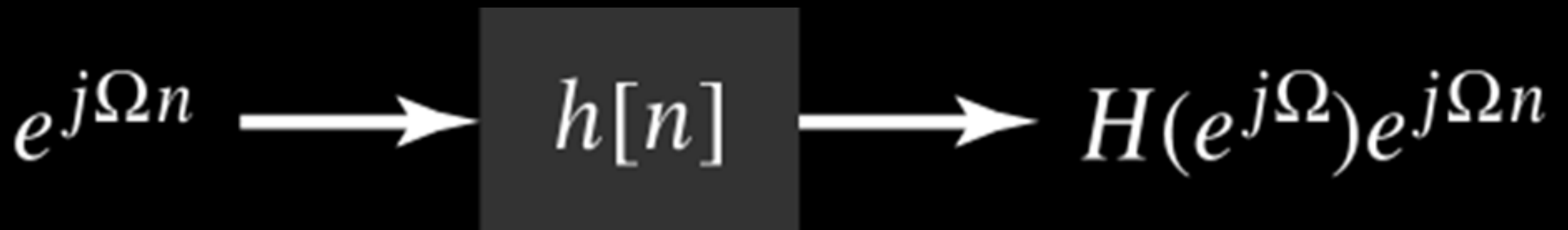
- Consider the output of a discrete-time LTI system with impulse response  $h[n]$  and unit amplitude complex sinusoidal input  $x[n] = e^{j\Omega n}$
- The output is given by

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]e^{j\Omega(n-k)} = e^{j\Omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k} \\ &= H(e^{j\Omega})e^{j\Omega n} \end{aligned}$$

- Where  $H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}$  as shown in Fig. 3.1
- The complex scaling factor  $H(e^{j\Omega})$  is not a function of time  $n$  but is a function of frequency  $\Omega$  and is termed the frequency response of the discrete-time system

### Figure 3.1 (p. 196)

The output of a complex sinusoidal input to an LTI system is a complex sinusoid of the same frequency as the input, multiplied by the frequency response of the system.



- For continuous-time LTI systems, we have:

$$y(t) = \int_{-\infty}^{\infty} h(\tau)e^{jw\tau} d\tau = e^{jw t} \int_{-\infty}^{\infty} h(\tau)e^{jw\tau} d\tau = H(jw)e^{jw t}$$

- Where

$$H(jw) = \int_{-\infty}^{\infty} h(\tau)e^{jw\tau} d\tau$$

- $H(jw)$  is a function of only the frequency  $w$  and not the time  $t$  and is termed as the frequency response of the continuous-time system.
- Writing  $c = a + jb$  in polar form as  $c = |c|e^{j\arg\{c\}}$ ,  
where  $|c| = \sqrt{a^2 + b^2}$  and  $\arg\{c\} = \arctan(b/a)$
- Hence, we have

$$H(jw) = |H(jw)|e^{j\arg\{H(jw)\}}$$

- Where  $|H(jw)|$  is now termed the magnitude response and  $\arg\{H(jw)\}$  is termed the phase response

- Substituting this polar form into Eq.(3.2), we may express the output as

$$y(t) = |H(j\omega)|e^{j\omega t + \arg\{H(j\omega)\}}$$

$$h(t) = \int_{-\infty}^{\infty} H(j\omega)e^{j\omega t} d\omega$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau} d\tau$$

**EXAMPLE 3.1 RC CIRCUIT: FREQUENCY RESPONSE** The impulse response of the system relating the input voltage to the voltage across the capacitor in Fig. 3.2 is derived in Example 1.21 as

$$h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t).$$

Find an expression for the frequency response, and plot the magnitude and phase response.

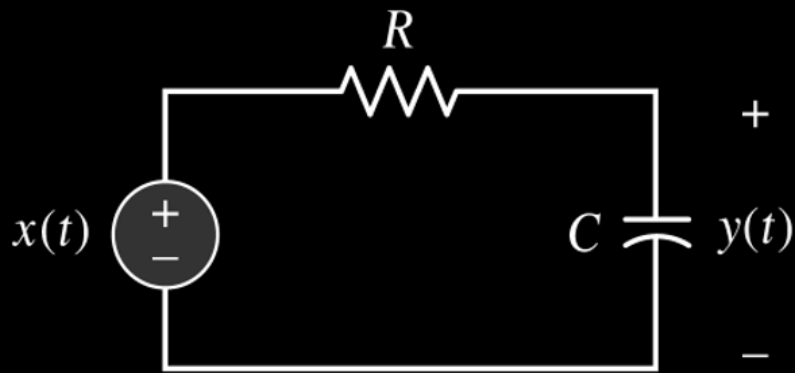
**Solution:** Substituting  $h(t)$  into Eq. (3.3) gives

$$\begin{aligned} H(j\omega) &= \frac{1}{RC} \int_{-\infty}^{\infty} e^{-\frac{\tau}{RC}} u(\tau) e^{-j\omega\tau} d\tau = \frac{1}{RC} \int_0^{\infty} e^{-(j\omega + \frac{1}{RC})\tau} d\tau \\ &= \frac{1}{RC} \frac{-1}{(j\omega + \frac{1}{RC})} e^{-(j\omega + \frac{1}{RC})\tau} \Big|_0^{\infty} = \frac{1}{RC} \frac{-1}{(j\omega + \frac{1}{RC})} (0 - 1) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}. \end{aligned}$$

The magnitude response is  $|H(j\omega)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + (\frac{1}{RC})^2}}$ ,

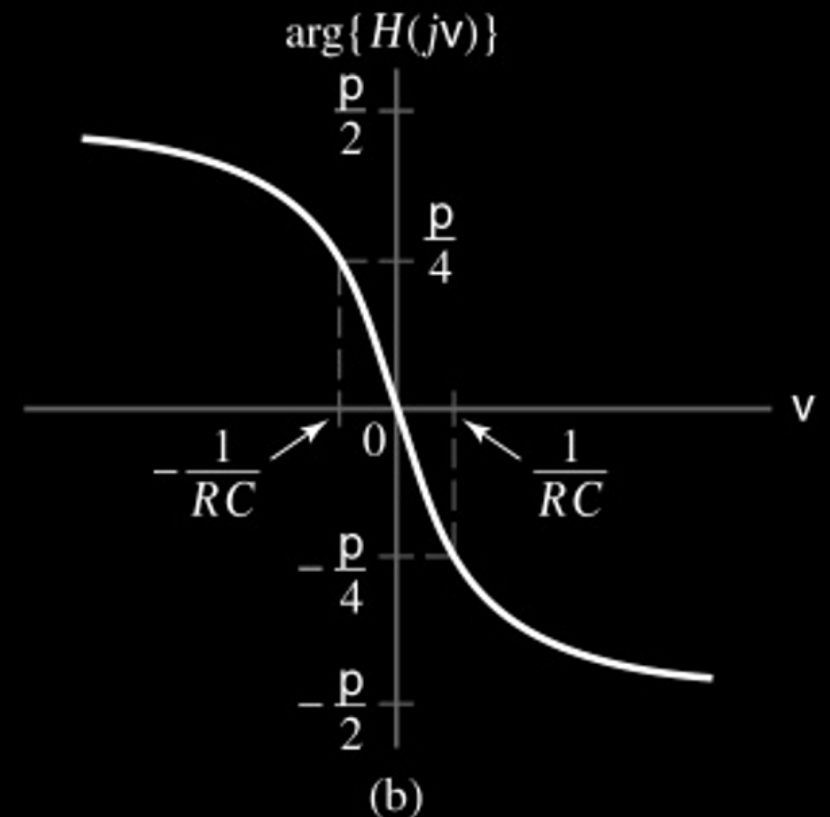
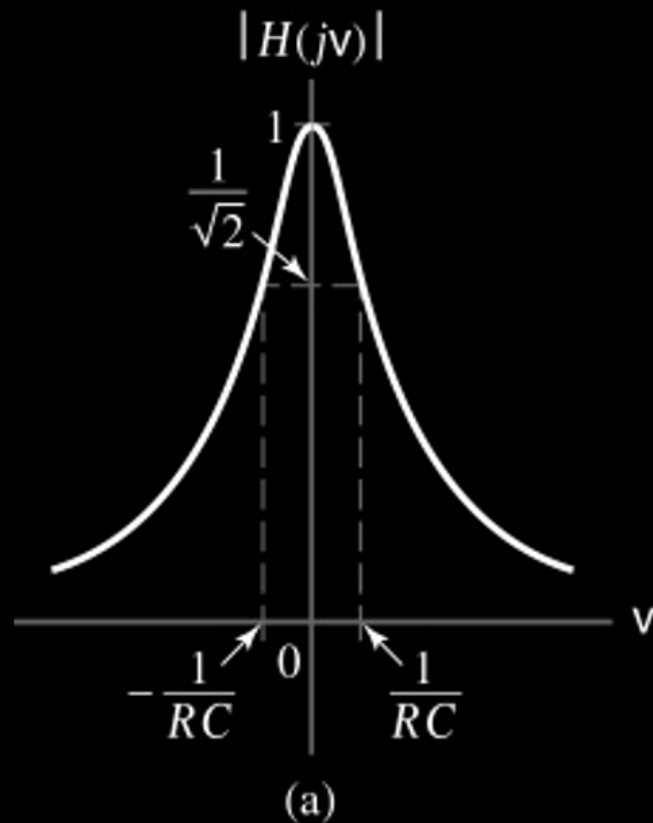
while the phase response is

$$\arg\{H(j\omega)\} = -\arctan(\omega RC). \quad \text{As shown in Figs. 3.3} \quad 12$$



**Figure 3.2 (p. 197):** *RC* circuit for Example 3.1.

The magnitude response indicates that the *RC* circuit tends to attenuate highfrequency ( $\omega > 1/RC$ ) Sinusoids: Hence, Low Pass Filter (LPF)

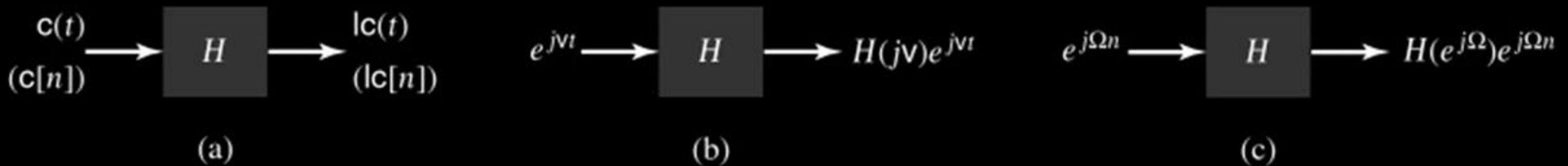


**Figure 3.3 (p. 198):** Frequency response of the *RC* circuit in Fig. 3.2. (a) Magnitude response. (b) Phase response. <sup>13</sup>

- We say that the complex sinusoid  $\psi(t) = e^{j\omega t}$  is an eigenfunction of the LTI system  $H$  associated with the eigenvalue  $\lambda = H(j\omega)$  because  $\psi$  satisfies an eigenvalue problem described by

$$H\{\psi(t)\} = \lambda\psi(t)$$

- As shown in Fig. 3.4



**Figure 3.4 (p. 198):** Illustration of the eigenfunction property of linear systems. The action of the system on an eigenfunction input is multiplication by the corresponding eigenvalue.

(a) General eigenfunction  $\psi(t)$  or  $\psi[n]$  and eigenvalue  $\lambda$ .

(b) Complex sinusoidal eigenfunction  $e^{j\omega t}$  and eigenvalue  $H(j\omega)$ .

(c) Complex sinusoidal eigenfunction  $e^{j\Omega n}$  and eigenvalue  $H(e^{j\Omega})$ .

- Expressing the input to an LTI system as the weighted sum of  $M$  complex sinusoids

$$x(t) = \sum_{k=1}^M a_k e^{j\omega_k t}$$

- If  $e^{j\omega_k t}$  is an eigenfunction of the system with eigenvalue  $H(j\omega_k)$ , then each term in the input  $a_k e^{j\omega_k t}$ , produces an output term  $a_k H(j\omega_k) e^{j\omega_k t}$ . Hence, we express the output of the system as

$$y(t) = \sum_{k=1}^M a_k H(j\omega_k) e^{j\omega_k t}$$