EENG/INFE 226 SIGNALS AND SYSTEMS
LAB 5
DIFFERENTIAL EQUATIONS

Objective

This experiment aims to introduce using MATLAB to evaluate the response of an LTI system, characterized by a linear constant-coefficient differential equation, to a certain input.As well as using MATLAB functions to calculate the step and impulse responses of such systems.

1. LTI Systems Described by Differential Equations:

MATLAB function (***lsim***) can be used to simulate the output of an LTI system characterized by a linear constant-coefficient differential equation of the form described in equation1.

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_k \frac{d^m x(t)}{dt^m} \qquad (1)$$

The latter equation can be programmed using the function (***lsim***), as

```
y=lsim(a,b,x,t)
```

We need to specify the following:

- The LTI system: is specified by providing the coefficients of y and x as row vectors (a) and (b), respectively.
- The input signal: specified as a row vector (x).
- The time interval: as a row vector (t) of equally-spaced time values.

Exercise 1

Consider the causal LTI system described by the first order differential equation:

$$\frac{dy(t)}{dt} = -\frac{1}{2} y(t) + x(t)$$

The latter equation can be rewritten as

$$\frac{dy(t)}{dt} + \frac{1}{2} y(t) = x(t)$$

Write a MATLAB code to simulate the step response of this system. This can be done as follows:

- Define the system coefficient vectors(a) and (b)
- Define the time vector t ranging from 0 to 10, with a 1 time increment.
- Define input as a row vector x of ones with the same length of *t*.
- Use the function (***lsim***) to evaluate the output *y*.
- Plot the output vector y versus time vector t with dashed lies (- -).

- Label the axes as the (output), (time), respectively.
- Title the figure as (Simulated Output).

To compare the simulated output to the actual one, plot the solution of the differential equation versus time as follows:

- Define the time vector t ranging from 0 to 10, with a 1 time increment.
- Plot the function y =2(1-$e^{-t/2}$), versus time.
- Label the axes and title the figure as(Exact Output)
- Your answer should be as indicated in Fig1 .

Exercise 2

Following a similar procedure to that of exercise 1, use (***lsim***) to compute the response of the system:

$$\frac{dy(t)}{dt} = -2y(t) + x(t)$$

To the input:

$$x(t) = u(t-2)$$

Your answer should be as indicated in Fig2.

2. Step Response:

The MATLAB function (***step***) can be used to evaluate the step function of a causal LTI system characterized by equation 1. This function can be called as follows:

```
y=step(b,a,t)
```

, where;
a: is the coefficient vector of *y*
b: is the coefficient vector of *x*
t: is the time vector
Similarly,

```
y=impulse(b,a,t)
```

generates the impulse response of the system characterized by coefficient vectors a and b, on time interval t.

Exercise 3

Plot the step and impulse responses of the system described by:

$$\frac{dy(t)}{dt} = -\frac{1}{2}y(t) + x(t)$$

Proceed as follows:

- Define the system coefficient vectors(a) and (b)
- Define the time vector t ranging from 0 to 10, with a 0.1 time increment.
- Define input as a row vector x of ones with the same length of t.
- Use the function (***step***) to evaluate the step response of the system, and name it as s.
- Use the function (***impulse***) to evaluate the step response of the system, and name it as i.
- Use the command (***subplot***) to plot the step impulse response(s) on the top, and impulse response (i) in the bottom of the same graph window.
- Label the axes and title the figures.
- Your answer should be as indicated in Fig2.

<u>Results</u>

The following four figures show the expected results of excersises 1,2,3 and4, respectively.

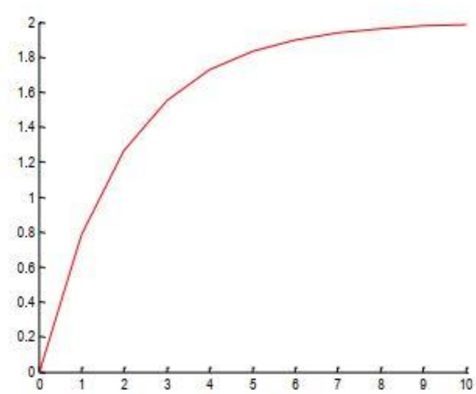

Fig 1: Exercise 1 Output

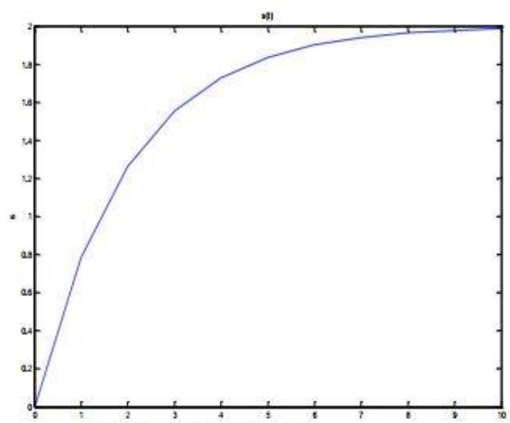

Fig2: Exercise 2 Output

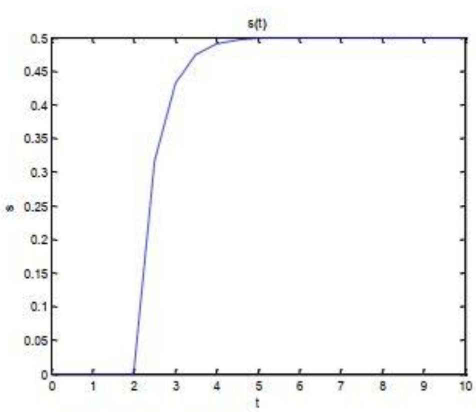

Fig 3: Exercise 3 Output

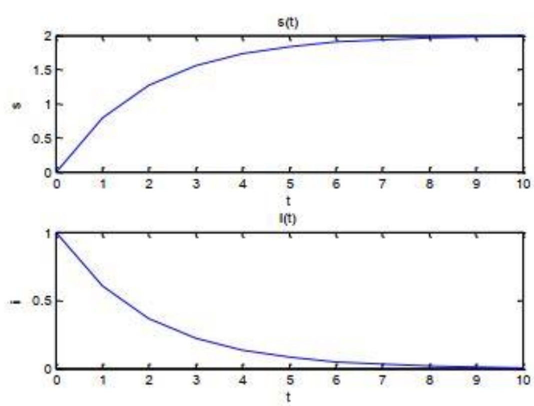

Fig 4: Exercise 4 Output

Assignment:
Use Matlab to compute the step and impulse responses of the causal LTI system:

$$\frac{d^3y(t)}{dt^3} - 2\frac{dy(t)}{dt} + y(t) = 4\frac{dx(t)}{dt} + x(t)$$