EENG/INFE 226 SIGNALS AND SYSTEMS
LAB5
DIFFERENTIAL EQUATIONS

The function *lsim* can be used to simulate the output of continous-time, causal LTI system described by linear constant-coefficient differential equations of the form

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_k \frac{d^m x(t)}{dt^m} \quad . \tag{1.1}$$

To use lsim, the coefficients a_k and b_m must be stored in MATLAB vectors a and b, respectivly, in descending order of the indeces k and m. Rewriting Eg.(1.1) in terms of the vectors a and b gives

$$\sum_{k=0}^{N} a(N+1-k) \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b(M+1-m) \frac{d^m x(m)}{dt^m} . \tag{1.2}$$

Note that a must be contain N+1 elements. Which might require appending zeros to a to account for coefficients a_k that equal zero. Similarly, the vector b must contain M+1 elements. With *a* and *b* defined as in eq. (1.2), executing

```
lsim(b,a,x,t);
```

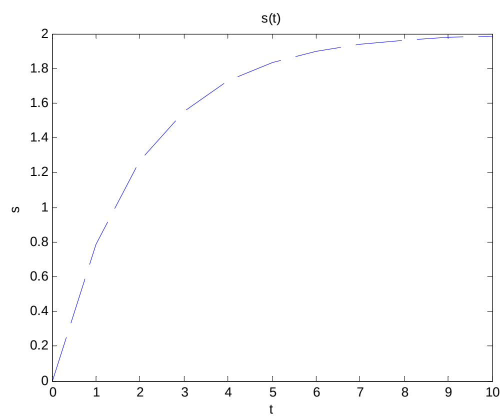
1) Consider the causal LTI system described by the first-order differential equation

$$\frac{dy(t)}{dt} = -\frac{1}{2} y(t) + x(t). \tag{1.3}$$

The step response of this system can be computed by first defining the input step function then the simulated step response can be computed and plotted by executing

Answer:

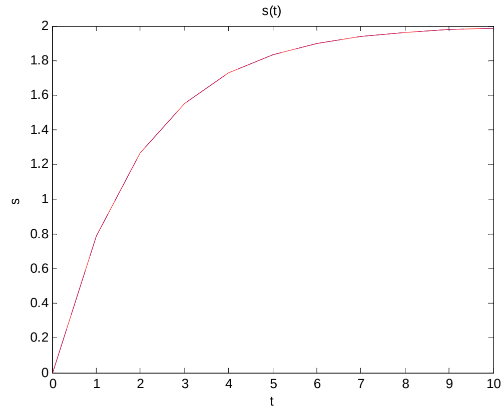
```
t=[0:10];
x=ones(1,length(t));
b=1;
a=[1 0.5];
s=lsim(b,a,x,t);
plot(t,s,'--')
xlabel('t')
ylabel('s')
title('s(t)')
```



Draw the actual step response
$s(t) = 2\left(1 - e^{-t/2}\right)u(t)$.

Answer:

```
hold on
s=2*(1-exp(-t/2));
plot(t,s,'r')
```



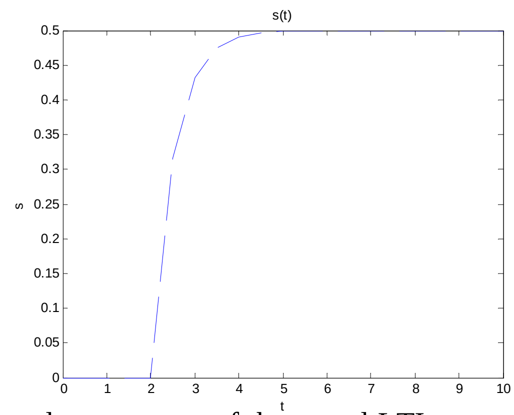
The function *lsim* will return more samples of s(t) if the samples in t are chosen more closely spaced, e.g., $t = [0:0.1:10]$.

2) i) On your own, use *lsim* to compute the response of the causal LTI system described by

$$\frac{dy(t)}{dt} = -2y(t) + x(t). \qquad x(t) = u(t-2). \qquad (1.4)$$

Answer:

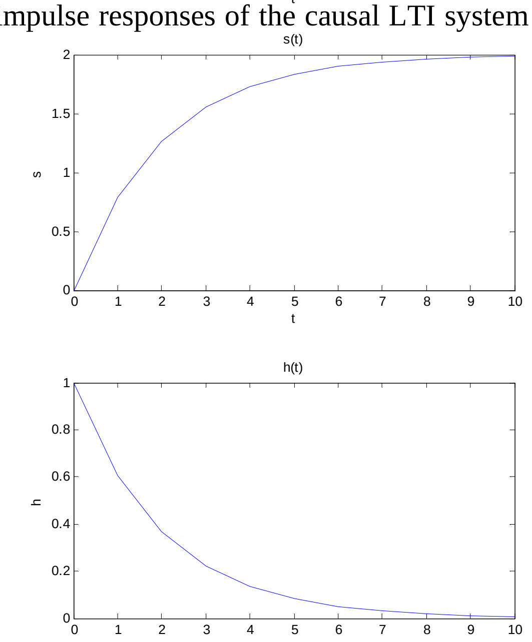
```
t1=[0:0.5:10];
x=[0 0 0 0 ones(1,length(t1)-4)];
b=1;
a=[1 2];
s=lsim(b,a,x,t1);
plot(t1,s,'--')
xlabel('t')
ylabel('s')
title('s(t)')
```



ii) Use *step* and *impulse* to compute the step and impulse responses of the causal LTI system characterized by Eq. (1.3).

Answer:

```
t=[0:10];
b=1;
a=[1 0.5];
s=step(b,a,t);
subplot(2,1,1), plot(t,s)
xlabel('t')
ylabel('s')
title('s(t)')
h=impulse(b,a,t);
subplot(2,1,2), plot(t,h)
xlabel('t')
ylabel('h')
title('h(t)')
```



Home work:

Use Matlab to compute the step and impulse responses of the causal LTI system:

$$\frac{d^3y(t)}{dt^3} - 2\frac{dy(t)}{dt} + y(t) = 4\frac{dx(t)}{dt} + x(t).$$