EEE 461 Communication Systems II

Lecture Presentation 11

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☞ *6.6 The Gaussian Random Process*

A random process $x(t)$ is said to be Gaussian if the random variables

$$x(t_1), x(t_2), \ldots, x(t_N)$$

have an N-dimensional Gaussian pdf for any N and any $t_1, t_2, \ldots, t_N$.

The N-dimensional Gaussian pdf in matrix notation is given compactly as:

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{(2\pi)^{N/2}|Det\ \mathbf{C}|^{1/2}} e^{-(1/2)[(\mathbf{x}-\mathbf{m})^T \mathbf{C}^{-1}(\mathbf{x}-\mathbf{m})]}$$

where $\mathbf{x}$ is a column vector and $\mathbf{m}$ is another column vector which contains the means;

$$\mathbf{m} = \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \vdots \\ \overline{x}_N \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_N \end{bmatrix}$$

The matrix $\mathbf{C}$ is the covariance matrix which is defined by

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1N} \\ c_{21} & c_{22} & \dots & c_{2N} \\ \vdots & \vdots & & \vdots \\ c_{N1} & c_{N2} & \dots & c_{NN} \end{bmatrix}$$

where the elements of the matrix are

$$c_{ij} = E[(x(t_i) - m_i)(x(t_j) - m_j)]$$

When the Gaussian random process is WSS and independent, the covariance matrix becomes diagonal.

The properties of Gaussian processes are as follows:

1. The N-dimensional Gaussian pdf is completely specified by the first and second order moments (means, variances and covariances).
2. Since $x(t_i)$ are jointly Gaussian then $x(t_i)$ are individually Gaussian.
3. The Gaussian random variables are independent when they are uncorrelated.
4. A linear transformation of a set of Gaussian random variables produces another set of Gaussian random variables.
5. A WSS Gaussian random process is also SSS.

➢ If the input to a linear system is a Gaussian random process, the system output is also a Gaussian random process.