EEE 360 Communications Systems I

Lecture Presentation 3

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Fourier Transform and Spectra: Section 2.2 of the textbook.

☞ The Fourier Transform

How do we find the frequencies which are present in waveforms ?

The Fourier Transform (FT) of a waveform $w(t)$ is given by

$$W(f) = \mathcal{F}[w(t)] = \int_{-\infty}^{\infty} [w(t)] e^{-j2\pi f t} dt \tag{1}$$

$W(f)$ is the two sided spectrum of $w(t)$ since both the negative and positive frequencies are obtained from the Fourier Transform.

☞ Inverse Fourier Transform

The inverse Fourier Transform is given by

$$w(t) = \mathcal{F}^{-1}[W(f)] = \int_{-\infty}^{\infty} [w(t)] e^{j2\pi f t} df \tag{2}$$

☞ **The waveform $w(t)$ is Fourier transformable if it satisfies the two Dirichlet conditions (not necessary but sufficient):**

1. Over any time interval of finite width, the function $w(t)$ is single valued with a finite number of maxima and minima, and the number of discontinuities is finite.
2. $w(t)$ is absolutely integrable $\left(\int_{-\infty}^{\infty} |w(t)|\, dt < \infty\right)$

☞ **A weaker sufficient condition is given by**

$$E = \int_{-\infty}^{\infty} |w(t)|^2\, dt < \infty \tag{3}$$

where E is the normalized energy.

☞ **Example 2.2 Spectrum of an Exponential Pulse. Page 46**

☞ Properties of Fourier Transform

If $w(t)$ is real, then $W(f)$ is conjugate symmetric:

$$W(-f) = W^*(f) \tag{4}$$

which implies that the magnitude spectrum is even about the origin

$$|W(-f)| = |W(f)| \tag{5}$$

whereas the phase spectrum (angle) is odd about the origin

$$\theta(-f) = -\theta(f) \tag{6}$$

☞ Parseval's Theorem and Energy Spectral Density

➢ Parseval's Theorem

$$\int_{-\infty}^{\infty} w_1(t)w_2^*(t)\,dt = \int_{-\infty}^{\infty} W_1(f)W_2^*(f)\,df \tag{7}$$

➢ Rayleigh's Theorem (for $w_1(t) = w_2(t) = w(t)$)

$$\int_{-\infty}^{\infty} |w(t)|^2\,dt = \int_{-\infty}^{\infty} |W(f)|^2\,df \tag{8}$$

➢ Energy Spectral Density (ESD)

$$\mathcal{E}(f) = |W(f)|^2 \tag{9}$$

ESD has the units of $joules/hertz$. The area under it gives the total normalized energy.

$$E = \int_{-\infty}^{\infty} \mathcal{E}(f)\,df \tag{10}$$

☞ *Table 2.1 can be used to evaluate the Fourier Transforms. After obtaining the result, these properties must be validated to ensure correct evaluation.*

1. $W(-f) = W^*(f)$ **or** $|W(f)|$ **is even and** $\theta(f)$ **is odd**
2. $W(f)$ **is real when** $w(t)$ **is even.**
3. $W(f)$ **is imaginary when** $w(t)$ **is odd.**

☞ Dirac Delta Function and Unit Step Function

The dirac delta function $\delta(x)$ is defined by

$$\int_{-\infty}^{\infty} w(x)\delta(x)\, dx = w(0) \tag{11}$$

where $w(x)$ is any function which is continuous at $x = 0$.

Alternatively, the delta function is defined using

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1 \tag{12}$$

and

$$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0 & x \neq 0 \end{cases} \quad (13)$$

The unit step function $u(t)$ is

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \quad (14)$$

The unit step and the delta function are related by the following equation:

$$\frac{du(t)}{dt} = \delta(t) \quad (15)$$