EEE 360 Communications Systems I

Lecture Presentation 4

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Fourier Transform and Spectra

☞ **Section 2.2 of the textbook.(Continued...)**

☞ **Rectangular and Triangular Pulses**

The rectangular pulse is defined as

$$\Pi\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \leq \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases} \tag{1}$$

☞ **The Sa$(\cdot)$ function is defined as**

$$\mathrm{Sa}(x) = \frac{\sin x}{x} \tag{2}$$

and similarly the triangle function is defined as

$$\Lambda\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases} \tag{3}$$

☞ **Convolution**

The convolution of a waveform $w_1(t)$ with a waveform $w_2(t)$ to produce $w_3(t)$ is given by

$$w_3(t) = w_1(t) * w_2(t) = \int_{-\infty}^{\infty} w_1(\lambda) w_2(t - \lambda)\, d\lambda. \tag{4}$$

☞ **The convolution operation can be described by the following steps:**

1. time reverse w_2 to obtain $w_2(-\lambda)$
2. time shift w_2 by t seconds to obtain $w_2(-\lambda + t)$
3. multiply this with w_1 *pointwise*
4. integrate the result to obtain w_3

Power Spectral Density and Autocorrelation Function

☞ **Section 2.3 of the textbook.**

☞ **Power Spectral Density**

The power spectral density (PSD) for a deterministic power waveform is

$$\mathcal{P}_w(f) = \lim_{T\to\infty} \left(\frac{|W_T(f)|^2}{T} \right) \tag{5}$$

where $w_T(t) \leftrightarrow W_T(f)$ and the unit of PSD is watts/hertz. The PSD is always a *real* and *nonnegative* function of frequency. It is not sensitive to the phase spectrum of $w(t)$. The normalized average power in terms of PSD is given by

$$P = \langle w^2(t) \rangle = \int_{-\infty}^{\infty} \mathcal{P}_w(f)\, df. \tag{6}$$

☞ Autocorrelation Function

The autocorrelation of a real waveform is

$$R_w(\tau) = \langle w(t)w(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} w(t)w_2(t+\tau)\,dt. \tag{7}$$

The PSD and the autocorrelation function form a Fourier Transform pair:

$$R_w(\tau) \leftrightarrow \mathcal{P}_w(f) \tag{8}$$

and hence the total average power for the waveform $w(t)$ can be evaluated by using any four of the techniques which are given below:

$$P = \langle w^2(t)\rangle = W_{rms}^2 = \int_{-\infty}^{\infty} \mathcal{P}_w(f)\,df = R_w(0). \tag{9}$$