EEE 360 Communications Systems I

Lecture Presentation 15

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☞ **Complex Envelope Representation of Bandpass Waveforms:** *Sections 4.1-4.3*

The bandpass communication signal is obtained by modulating a baseband signal onto a carrier.

- ➢ **Baseband Signal:** has a spectral magnitude near the origin (i.e. $f = 0$) and zero elsewhere.
- ➢ **Bandpass signal:** has spectral magnitude which is nonzero in a band of frequencies (i.e. $\pm f_c$) where $f_c \gg 0$. f_c is the carrier frequency.

Modulation is the process of putting the source information on a bandpass signal with carrier frequency f_c by the introduction of amplitude or phase changes or both.

The resulting bandpass signal is the modulated signal $s(t)$ and the baseband source signal is called the modulating signal $m(t)$.

Complex Envelope Representation

Assume that $v(t)$ represents the bandpass signal. Any physical bandpass waveform can be represented by

$$v(t) = \Re\{g(t)e^{j\omega_c t}\} \tag{1}$$

$g(t)$ is called the complex envelope of $v(t)$.

The other two representations are given by

$$v(t) = R(t)\cos[\omega_c t + \theta(t)] \tag{2}$$

$$v(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c(t) \tag{3}$$

where

$$g(t) = x(t) + jy(t) = |g(t)|e^{j\angle g(t)} \equiv R(t)e^{j\theta(t)} \tag{4}$$

$$x(t) = \Re\{g(t)\} \equiv R(t)\cos\theta(t) \tag{5}$$

$$y(t) = \Im\{g(t)\} \equiv R(t)\sin\theta(t) \tag{6}$$

$$R(t) = |g(t)| \equiv \sqrt{x^2(t) + y^2(t)} \tag{7}$$

and

$$\theta(t) = \angle g(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right) \tag{8}$$

The waveforms $g(t)$, $x(t)$, $y(t)$, $R(t)$ and $\theta(t)$ are all baseband signals.

Additionally, all of them except $g(t)$ are real.

- $x(t)$ is said to be the in-phase modulation associated with $v(t)$.
- $y(t)$ is the quadrature modulation.
- $R(t)$ is the amplitude modulation(AM) on $v(t)$
- $\theta(t)$ is the phase modulation (PM) on $v(t)$.

Here, $\Re\{\cdot\}$ denotes the real part and $\Im\{\cdot\}$ the imaginary part of $\{\cdot\}$.

☞ Representation of Modulated Signals

Modulated signal is just a special application of the bandpass representation. The modulated signal is given by

$$s(t) = \Re\{g(t)e^{j\omega_c t}\} \tag{9}$$

the complex envelope $g(t)$ is a function of the modulating signal $m(t)$:

$$g(t) = g[m(t)] \tag{10}$$

Spectrum of Bandpass Signals

The spectrum of a bandpass signal is directly related to the spectrum of its complex envelope.

If a bandpass waveform is represented by

$$v(t) = \Re\{g(t)e^{j\omega_c t}\} \tag{11}$$

then the spectrum of the bandpass waveform is

$$V(f) = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)] \tag{12}$$

and the PSD of the waveform is

$$\mathcal{P}_v(f) = \frac{1}{4}[\mathcal{P}_g(f - f_c) + \mathcal{P}_g(-f - f_c)] \tag{13}$$

where $G(f) = \mathcal{F}[g(t)]$ and $\mathcal{P}_g(f)$ is the PSD of $g(t)$