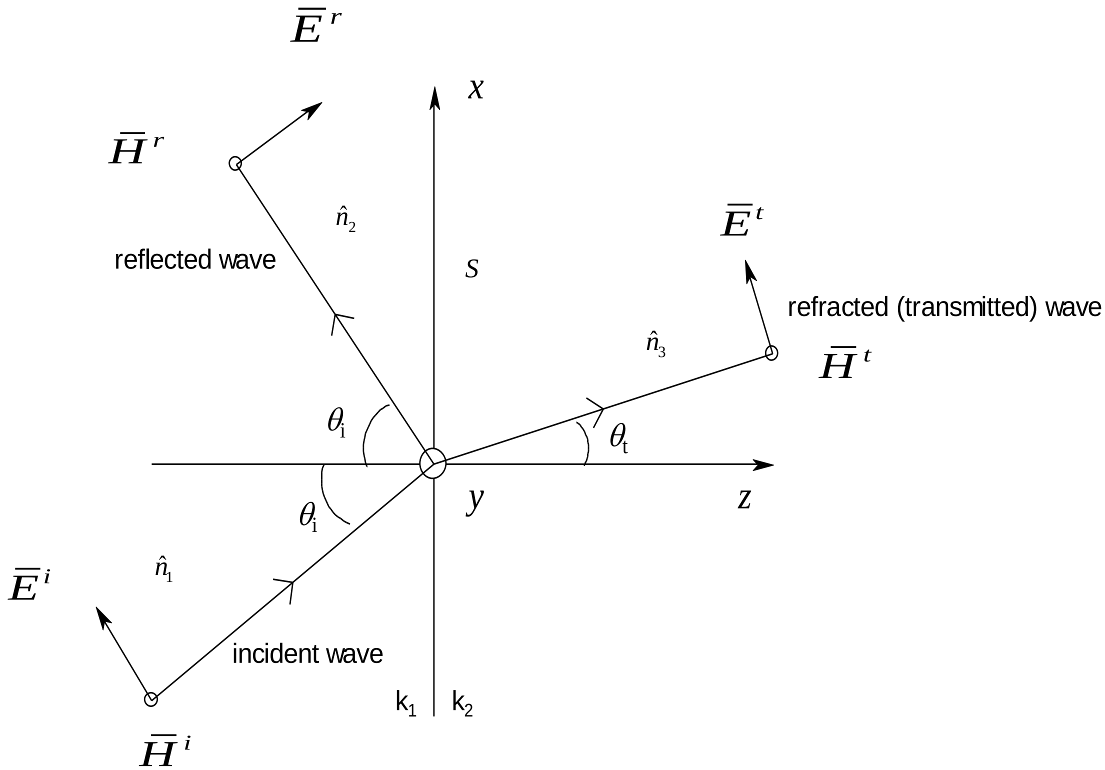
PARALLEL POLARIZATION



$$\bar{H}^i = H_1 \hat{a}_y e^{-jk_1 \hat{n}_1 . \bar{r}} \qquad \bar{E}^i = \eta_1 \bar{H}^i x \hat{n}_1$$

$$\bar{H}^r = -H_2 \hat{a}_y e^{-jk_1 \hat{n}_2 . \bar{r}} \qquad \bar{E}^r = \eta_1 \bar{H}^r x \hat{n}_2$$

$$\bar{H}^t = H_3 \hat{a}_y e^{-jk_2 \hat{n}_3 . \bar{r}} \qquad \bar{E}^t = \eta_2 \bar{H}^t x \hat{n}_3$$

$$\bar{E}^i = \eta_1 H_1 \left(\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z \right) e^{-jk_1 \hat{n}_1 . \bar{r}}$$

$$\bar{E}^r = \eta_1 H_2 \left(\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z \right) e^{-jk_1 \hat{n}_2 . \bar{r}}$$

$$\bar{E}^t = \eta_2 H_3 \left(\cos\theta_t \hat{a}_x - \sin\theta_t \hat{a}_z \right) e^{-jk_2 \hat{n}_3 . \bar{r}}$$

Let,

$$E_1 = \eta_1 H_1, \;\; E_2 = \eta_1 H_2, \;\; E_3 = \eta_2 H_3$$

Boundary Conditions (after phase- matching) yield:

i) $\left(\bar{H}^i + \bar{H}^r\right)_{\tan} = \left(\bar{H}^t\right)_{\tan} \quad at \quad z = 0$ gives,

$H_1 - H_2 = H_3$ or,

$$\frac{1}{\eta_1}\left(E_1 - E_2\right) = \frac{1}{\eta_2} E_3 \quad \text{.................(1)}$$

ii) $\left(\bar{E}^i + \bar{E}^r\right)_{\tan} = E^i_x + E^r_x = \left(\bar{E}^t\right)_{\tan} = E^t_x \qquad at \qquad z = 0$ gives

$$\left(E_1 + E_2\right)\cos\theta_i = E_3 \cos\theta_t \qquad \text{............}(2)$$

Define:

$\dfrac{E_2}{E_1} = \Gamma_{\|}, \qquad \dfrac{E_3}{E_1} = \mathrm{T}_{\|}$ From (1) and (2) we get, Frensel Formulas for Parallel Pololarization

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$\Gamma_{\parallel}$ is called the reflection coefficient for the paralel polarization.

$T_{\parallel}$ is called the transmission coefficient for the paralel polarization.

We see that,

$$1 + \Gamma_{\parallel} = \left(\frac{\cos\theta_t}{\cos_i} \right) T_{\parallel}$$

Frensel formulas are valid for ε_1 and ε_2 being compex (the two media are lossy). They are also valid for all frequencies.

TOTAL TRANSMISSION-BREWSTER ANGLE

Question: For a given set of material parameters of the two media forming an interface, is there an incidence angle that allows no reflection, i.e. $\Gamma = 0$?

a) Perpendicular Polarization

We will set $\Gamma_{\perp} = 0$ and see under what conditions can this be possible.

$$\Gamma_\perp = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}}_{1}\cos\theta_t} = 0$$

Or,

$$\cos\theta_i = \sqrt{\frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}}\cos\theta_t \;\; \dots\dots\dots\dots\dots\dots\dots(1)$$

We also have from the Snell's Law:

$$\sqrt{\varepsilon_1\mu_1}\sin\theta_i = \sqrt{\varepsilon_2\mu_2}\sin\theta_t \;\; \dots\dots\dots(2)$$

We can have write (1) as:

$$(1-\sin^2\theta_i) = \frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}\left(1-\sin^2\theta_t\right) \;\; \dots\dots\dots(3)$$

Let us now replace $\sin\theta_t$ on the RHS of (3) with its equivalent from (2):

$$\left(1-\sin^2\theta_i\right) = \frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}\left(1-\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_i\right)$$

$$\sin\theta_i = \left(\frac{\dfrac{\varepsilon_2}{\varepsilon_1} - \dfrac{\mu_2}{\mu_1}}{\dfrac{\mu_1}{\mu_2} - \dfrac{\mu_2}{\mu_1}} \right)^{1/2}$$

Since $\sin\theta_i \le 1$, we must have $\dfrac{\varepsilon_2}{\varepsilon_1} - \dfrac{\mu_2}{\mu_1} \le \dfrac{\mu_1}{\mu_2} - \dfrac{\mu_2}{\mu_1}$ or,

$\dfrac{\varepsilon_2}{\varepsilon_1} \le \dfrac{\mu_1}{\mu_2}$ (then there exists such a θ_i which makes $\Gamma_\perp = 0$)

If $\varepsilon_1 = \varepsilon_2$ and $\mu_1 \ne \mu_2$, then

$$\sin\theta_i = \left(\frac{1 - \dfrac{\mu_2}{\mu_1}}{\dfrac{\mu_1}{\mu_2} - \dfrac{\mu_2}{\mu_1}} \right)^{1/2} = \left(\frac{\dfrac{\mu_1 - \mu_2}{\mu_1}}{\left(\dfrac{\mu_1^2 - \mu_2^2}{\mu_1 \mu_2} \right)} \right) = \left(\frac{\mu_2}{\mu_1 + \mu_2} \right)^{1/2}$$

So,

$$\sin\theta_i = \sqrt{\frac{\mu_2}{\mu_1 + \mu_2}}$$

Or,

$$\theta_i = \theta_B = \sin^{-1}\left(\sqrt{\frac{\mu_2}{\mu_1 + \mu_2}} \right) = \cos^{-1}\left(\sqrt{\frac{\mu_1}{\mu_1 + \mu_2}} \right) = \tan^{-1}\left(\sqrt{\frac{\mu_2}{\mu_1}} \right)$$

$$\theta_B = Brewsters\ Angle$$

It the two media are electrically the same but magnetically different and if the incident wave is of perpendicular polarization, then, there exists an incidece angle for which $\Gamma_\perp = 0$.

b) Parallel Polarization

$$\Gamma_T = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t + \sqrt{\frac{\mu_1}{\varepsilon_1}}_1\cos\theta_i} = 0$$

or,

$$\cos\theta_i = \sqrt{\frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}\cos\theta_t \text{ ....(1)}$$

Using Snell's Law simultaneously with (1) gives:

$$\sin\theta_i = \left(\frac{\frac{\varepsilon_2}{\varepsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\varepsilon_2}{\varepsilon_1} - \frac{\varepsilon_1}{\varepsilon_1}}\right)^{1/2}$$

A real θ_i exists if $\frac{\varepsilon_2}{\varepsilon_1} - \frac{\mu_2}{\mu_1} \le \frac{\varepsilon_2}{\varepsilon_1} - \frac{\varepsilon_1}{\varepsilon_2}$ or,

$$\frac{\mu_2}{\mu_1} \ge \frac{\varepsilon_1}{\varepsilon_2}$$

If $\mu_1 = \mu_2$ and $\varepsilon_1 \neq \varepsilon_2$ then

$$\theta_i = \theta_B = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}\right) = \cos^{-1}\left(\sqrt{\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}}\right) = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

Polarization	**Medium 1**	**Medium 2**	Does Brewster's angle exists?	θ_B
Perpendicular	ε, μ_1	ε, μ_2	Yes	$\theta_B = \sin^{-1}\left(\sqrt{\frac{\mu_2}{\mu_1 + \mu_2}}\right) = \tan^{-1}\left(\sqrt{\frac{\mu_2}{\mu_1}}\right)$
Perpendicular	ε_1, μ	ε_2, μ	No	-
Parallel	ε, μ_1	ε, μ_2	No	-
Parallel	ε_1, μ	ε_2, μ	Yes	$\theta_B = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}\right) = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$