PHYS101 Midterm Exam - Solution Set

Department of Physics

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Questions:

1. An object is moving with $x(t) = 5t^2 + 3t - 2$ and $y(t) = -3t^2 + 6t + 2$.

   (a) Write the position vector at any time t.
   Solution:

$$\vec{r}(t) = (5t^2 + 3t - 2)\hat{\mathbf{i}} + (-3t^2 + 6t + 2)\hat{\mathbf{j}}$$

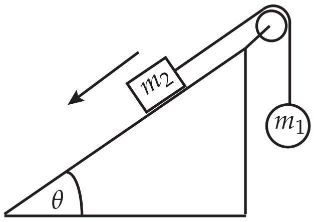
   (b) Find the velocity vector.
   Solution:

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = (10t + 3)\hat{\mathbf{i}} + (-6t + 6)\hat{\mathbf{j}}$$

   (c) Find the average acceleration between $t = 1.0\,\mathrm{s}$ and $t = 2.0\,\mathrm{s}$.
   Solution:

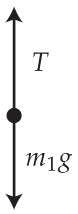
$$\vec{a}_{avg} = \frac{\vec{v}(2s) - \vec{v}(1s)}{2s - 1s} = \frac{(23\hat{\mathbf{i}} - 6\hat{\mathbf{j}})\,\frac{\mathrm{m}}{\mathrm{s}} - (13\hat{\mathbf{i}})\,\frac{\mathrm{m}}{\mathrm{s}}}{1s} = (10\hat{\mathbf{i}} - 6\hat{\mathbf{j}})\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

2. Two objects are connected by a light string that passes over a frictionless pulley as shown in the figure below. Assume the incline is a rough surface with friction and take $m_1 = 2.0\,\text{kg}$ and $m_2 = 6.0\,\text{kg}$, and the angle is $\theta = 60°$.

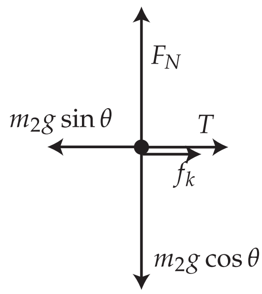


(a) Draw the free body diagrams for the masses m_1 and m_2.
Solution:

free body diagram for m_1



free body diagram for m_2



(b) For a given $f_k = 4.0\,\text{N}$, determine the magnitude and direction of the acceleration a.
Solution:
From the free body diagram of m_1 we get

$$T - m_1 g = m_1 a. \tag{1}$$

Solving (1) for T we get

$$T = m_1(a + g). \tag{2}$$

From the free body diagram for m_2 we get

$$\sum F_x = T + f_k - m_2 g \sin\theta = -m_2 a, \text{and} \tag{3}$$

$$\sum F_y = F_N - m_2 g \cos\theta = 0. \tag{4}$$

Substituting T in (4) by (2) we get

$$m_1(a + g) + f_k - m_2 g \sin\theta = -m_2 a. \tag{5}$$

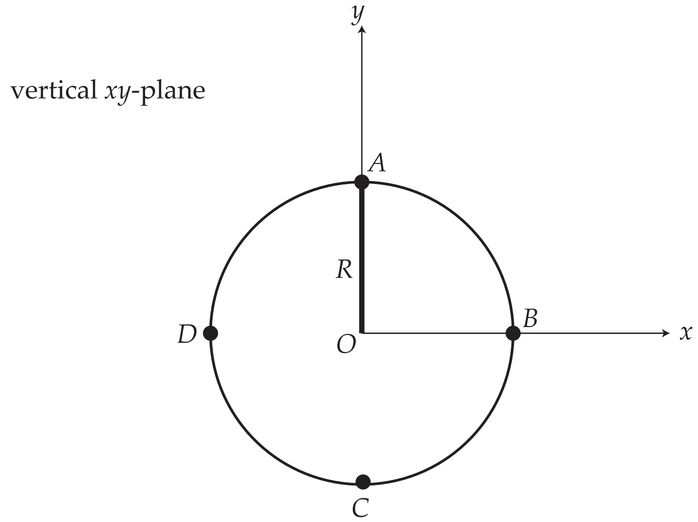
Solving now (5) for a we get:

$$a = \frac{m_1 g - f_k + m_2 g \sin\theta}{m_1 + m_2} = \frac{2.0\,\text{kg} 9.80\,\frac{\text{m}}{\text{s}^2} - 4.0N + 6.0\,\text{kg} 9.80\,\frac{\text{m}}{\text{s}^2}\sin 60°}{2.0\,\text{kg} + 6.0\,\text{kg}} = 3.4\,\frac{m}{s^2}. \tag{6}$$

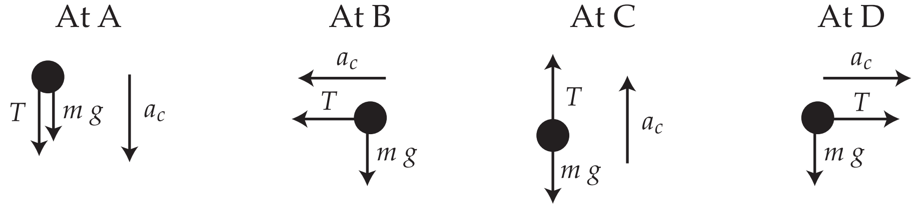
(c) If the length of the incline is $l = 2.0\,\text{m}$, how long does it take the block to reach the bottom of the incline?
Solution:
Assuming that m_2 is at rest, we get:

$$l = \frac{1}{2} a t^2 \Longrightarrow t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2 \cdot 2.0m}{3.4\,\frac{\text{m}}{\text{s}^2}}} = 1.1\,\text{s}.$$

3. A small sphere of mass $m = 10.0\,\text{g}$ is attached to the end of a massless cord of length $R = 1.0\,\text{m}$ and set into motion in a **vertical** circle about a fixed point O as given in the figure below.



(a) Draw the free body diagrams at the positions A,B,C, and D.
Solution:



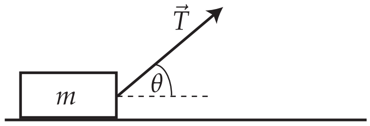
(b) Find the tension in the string at the points A and C if the speed at A is $2.0\,\text{m/s}$ and the speed at C is $4.0\,\text{m/s}$.
Solution:
From the free body diagram at A we get

$$-T_A - mg = m\frac{v^2}{R} \Longrightarrow T_A = mg - m\frac{v^2}{R} = 0.01\,\text{kg}\left(9.80\,\frac{\text{m}}{\text{s}^2} - \frac{\left(2.0\,\frac{\text{m}}{\text{s}}\right)^2}{1.0\,\text{m}}\right) = 0.058\,\text{N}$$

From the free body diagram at C we get

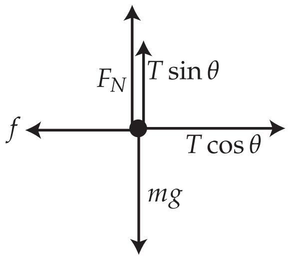
$$T_C - mg = m\frac{v^2}{R} \Longrightarrow T_B = mg + m\frac{v^2}{R} = 0.01\,\text{kg}\left(9.80\,\frac{\text{m}}{\text{s}^2} + \frac{\left(4.0\,\frac{\text{m}}{\text{s}}\right)^2}{1.0\,\text{m}}\right) = 0.258\,\text{N}$$

4. The mass $m = 1.0\,\text{kg}$ is pulled by a massless string with a force $T = 5.0\,\text{N}$ making an angle $\theta = 37°$ with the horizontal as shown in the figure below. The coefficients of friction are $\mu_s = 0.5$ and $\mu_k = 0.4$.



(a) Draw the free body diagram for the mass m.

Solution:



(b) Find the acceleration of m, if there is any at all.

Solution:

From the free body diagram we can calculate the normal force:

$$F_N + T\sin\theta = mg \Longrightarrow F_N = mg - T\sin\theta = 1.0\,\text{kg}\,9.80\,\frac{\text{m}}{\text{s}^2} - 5.0\,\text{N}\sin 37° = 6.8\,\text{N}$$

So we can now calculate the value for the static friction

$$f_s \leq \mu_s F_N = 0.5 \cdot 6.8\,\text{N} = 3.4\,\text{N}$$

As the tension in x-direction is

$$T\cos\theta = 5.0\,\text{N}\cos 37° = 4.0\,\text{N}$$

As the tension of the cord in x-direction is greater than the maximum static friction, the mass starts sliding and then the kinetic friction becomes effective

$$f_k = \mu_k F_N = 0.4 \cdot 6.8\,\text{N} = 2.7\,\text{N}$$

Therefore the effective force in x-direction is

$$T\cos\theta - f_k = ma \Longrightarrow a = \frac{T\cos\theta - f_k}{m} = \frac{4\,\text{N} - 2.7\,\text{N}}{1.0\,\text{kg}} = 1.3\,\frac{\text{m}}{\text{s}^2}$$

(c) What is the static frictional force the mass feels?

Solution:

As the mass is in motion, it has overcome the static friction, and therefore the mass just feels the kinetic friction.