

PHYS101 Midterm Exam - Solution Set

Department of Physics

Fall 2013/2014 - December 2, 2013

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Questions:

1. An object is moving with $x(t) = 5t^2 + 3t - 2$ and $y(t) = -3t^2 + 6t + 2$.

(a) Write the position vector at any time t .

Solution:

$$\vec{r}(t) = (5t^2 + 3t - 2)\hat{i} + (-3t^2 + 6t + 2)\hat{j}$$

(b) Find the velocity vector.

Solution:

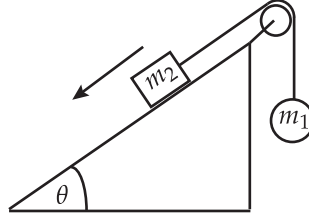
$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = (10t + 3)\hat{i} + (-6t + 6)\hat{j}$$

(c) Find the average acceleration between $t = 1.0$ s and $t = 2.0$ s.

Solution:

$$\vec{a}_{avg} = \frac{\vec{v}(2s) - \vec{v}(1s)}{2s - 1s} = \frac{(23\hat{i} - 6\hat{j}) \frac{m}{s} - (13\hat{i}) \frac{m}{s}}{1s} = (10\hat{i} - 6\hat{j}) \frac{m}{s^2}$$

2. Two objects are connected by a light string that passes over a frictionless pulley as shown in the figure below. Assume the incline is a rough surface with friction and take $m_1 = 2.0 \text{ kg}$ and $m_2 = 6.0 \text{ kg}$, and the angle is $\theta = 60^\circ$.



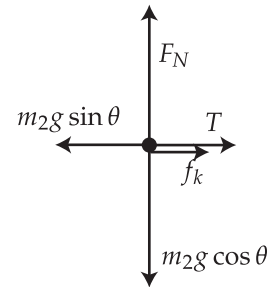
- (a) Draw the free body diagrams for the masses m_1 and m_2 .

Solution:

free body diagram for m_1



free body diagram for m_2



- (b) For a given $f_k = 4.0 \text{ N}$, determine the magnitude and direction of the acceleration a .

Solution:

From the free body diagram of m_1 we get

$$T - m_1g = m_1a. \quad (1)$$

Solving (1) for T we get

$$T = m_1(a + g). \quad (2)$$

From the free body diagram for m_2 we get

$$\sum F_x = T + f_k - m_2g \sin \theta = -m_2a, \text{ and} \quad (3)$$

$$\sum F_y = F_N - m_2g \cos \theta = 0. \quad (4)$$

Substituting T in (4) by (2) we get

$$m_1(a + g) + f_k - m_2g \sin \theta = -m_2a. \quad (5)$$

Solving now (5) for a we get:

$$a = \frac{m_1g - f_k + m_2g \sin \theta}{m_1 + m_2} = \frac{2.0 \text{ kg} \cdot 9.80 \frac{\text{m}}{\text{s}^2} - 4.0 \text{ N} + 6.0 \text{ kg} \cdot 9.80 \frac{\text{m}}{\text{s}^2} \sin 60^\circ}{2.0 \text{ kg} + 6.0 \text{ kg}} = 3.4 \frac{\text{m}}{\text{s}^2}. \quad (6)$$

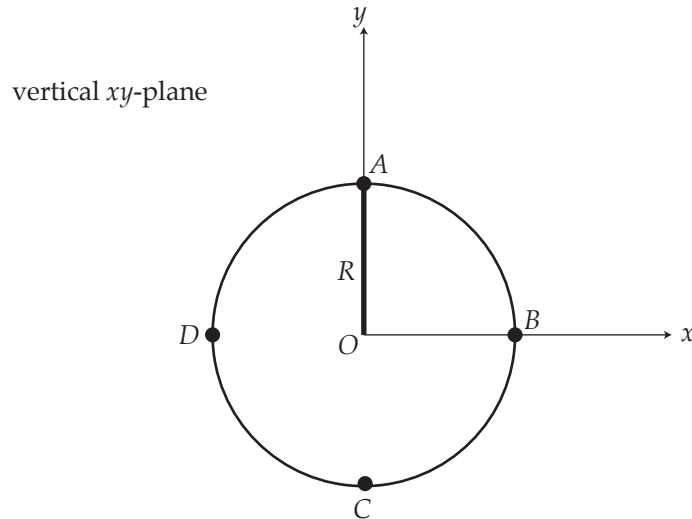
- (c) If the length of the incline is $l = 2.0 \text{ m}$, how long does it take the block to reach the bottom of the incline?

Solution:

Assuming that m_2 is at rest, we get:

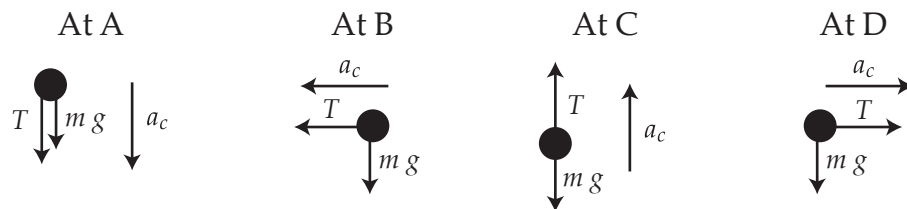
$$l = \frac{1}{2}at^2 \implies t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2 \cdot 2.0 \text{ m}}{3.4 \frac{\text{m}}{\text{s}^2}}} = 1.1 \text{ s}.$$

3. A small sphere of mass $m = 10.0 \text{ g}$ is attached to the end of a massless cord of length $R = 1.0 \text{ m}$ and set into motion in a **vertical** circle about a fixed point O as given in the figure below.



- (a) Draw the free body diagrams at the positions A,B,C, and D.

Solution:



- (b) Find the tension in the string at the points A and C if the speed at A is 2.0 m/s and the speed at C is 4.0 m/s .

Solution:

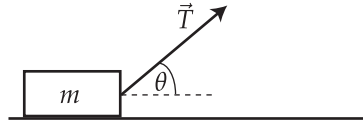
From the free body diagram at A we get

$$-T_A - mg = m \frac{v^2}{R} \implies T_A = mg - m \frac{v^2}{R} = 0.01 \text{ kg} \left(9.80 \frac{\text{m}}{\text{s}^2} - \frac{(2.0 \frac{\text{m}}{\text{s}})^2}{1.0 \text{ m}} \right) = 0.058 \text{ N}$$

From the free body diagram at C we get

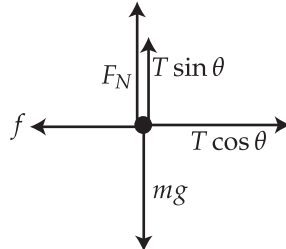
$$T_C - mg = m \frac{v^2}{R} \implies T_C = mg + m \frac{v^2}{R} = 0.01 \text{ kg} \left(9.80 \frac{\text{m}}{\text{s}^2} + \frac{(4.0 \frac{\text{m}}{\text{s}})^2}{1.0 \text{ m}} \right) = 0.258 \text{ N}$$

4. The mass $m = 1.0 \text{ kg}$ is pulled by a massless string with a force $T = 5.0 \text{ N}$ making an angle $\theta = 37^\circ$ with the horizontal as shown in the figure below. The coefficients of friction are $\mu_s = 0.5$ and $\mu_k = 0.4$.



- (a) Draw the free body diagram for the mass m .

Solution:



- (b) Find the acceleration of m , if there is any at all.

Solution:

From the free body diagram we can calculate the normal force:

$$F_N + T \sin \theta = mg \implies F_N = mg - T \sin \theta = 1.0 \text{ kg} \cdot 9.80 \frac{\text{m}}{\text{s}^2} - 5.0 \text{ N} \sin 37^\circ = 6.8 \text{ N}$$

So we can now calculate the value for the static friction

$$f_s \leq \mu_s F_N = 0.5 \cdot 6.8 \text{ N} = 3.4 \text{ N}$$

As the tension in x -direction is

$$T \cos \theta = 5.0 \text{ N} \cos 37^\circ = 4.0 \text{ N}$$

As the tension of the cord in x -direction is greater than the maximum static friction, the mass starts sliding and then the kinetic friction becomes effective

$$f_k = \mu_k F_N = 0.4 \cdot 6.8 \text{ N} = 2.7 \text{ N}$$

Therefore the effective force in x -direction is

$$T \cos \theta - f_k = ma \implies a = \frac{T \cos \theta - f_k}{m} = \frac{4 \text{ N} - 2.7 \text{ N}}{1.0 \text{ kg}} = 1.3 \frac{\text{m}}{\text{s}^2}$$

- (c) What is the static frictional force the mass feels?

Solution:

As the mass is in motion, it has overcome the static friction, and therefore the mass just feels the kinetic friction.