Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EEE 223 Circuit Theory I

Spring 2005-06

Instructor:

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Final EXAMINATION

June 15, 2006

Duration : 120 minutes

Number of Problems: 6

Good Luck

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1. Use nodal analysis to find $v$.

KCL at the supernode:
\[
\frac{v - 12}{1} + \frac{v + 12}{1} + \frac{v}{1} = 3
\]

$3v = 3$

$v = 1 \text{ V}$
2. Use mesh analysis to find \( v \).

KVL around the supermesh:

\[
2i_1 + 4(i_1 + 6) + 2(i_1 + 6) + 4(i_1 + 6 - i_3) + 2(i_1 - 4) = 24
\]
\[
14i_1 - 4i_3 = 24 - 60 + 8 = -28\ldots\ldots(1)
\]

KVL around \( i_3 \):

\[
2(i_3 - 4) + 4(i_3 - i_1 - 6) + 4i_3 = -12
\]
\[
10i_3 - 4i_1 = 20\ldots\ldots(2)
\]

By using Eqns(1) and (2), \( i_1 \) is obtained as

\[
i_1 = -1.613 \ A
\]

\[
v = 2(i_1 + 6) = 8.774 \ V.
\]
3. Find $R_L$ for maximum power transfer and the maximum power absorbed by the load.

When $R_L = R_{TH}$ it will absorb maximum power. The maximum power;

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

In order to find $R_{TH}$:

$$R_{TH} = \frac{6}{(4 + 2)} = 3\Omega$$
For $V_{TH}$:

KCL at $V_{oc}$:
\[
\frac{V_{oc} - 12}{6} + \frac{V_{oc} - V_1}{4} = 0
\]
\[5V_{oc} - 3V_1 = 24 \ldots (1)\]

KCL at $V_1$:
\[
\frac{V_1 - V_{oc}}{4} + \frac{V_1 - 12}{2} = 12
\]
\[-V_{oc} + 3V_1 = 72 \ldots (2)\]

From Eqns (1) and (2),
\[V_{oc} = 24 \text{ V}\]

Therefore, when $R_L = 3\Omega$ then it will absorb maximum power.
\[P_{max} = \frac{24^2}{4 \times 3} = 48W\]
4. Find $v_0$ in the circuit.

KCL at the inverting input terminal of OPAMP (2):

$$\frac{v_1}{10} + \frac{v_1 - v_0}{10} = 0$$

$$v_1 = \frac{v_0}{2} \quad \text{...(1)}$$

KCL at the inverting input terminal of OPAMP (1)

$$\frac{10}{10} + \frac{v_0/2}{20} + \frac{v_0}{40} = 0$$

$$2v_0 = -40$$

$$v_0 = -20 \text{ V}$$
5. If $v_c(0) = 100$ V, find $v_c(t)$ for $t \geq 0$. 

Since the circuit is a source-free RC circuit, the voltage across the capacitor is 

$$v_c(t) = v_c(0)e^{-\frac{t}{\tau}}$$

Where $\tau = R_{eq}C$ and $R_{eq}$ is the resistance seen by the capacitor.

KCL at the supernode:

$$i_i = \frac{v - 18i_i}{12} \Rightarrow \frac{5}{2}i_i = \frac{v}{12} \Rightarrow i_i = \frac{2}{60}v$$

$$v - 18i_i = v - 18\left( \frac{2}{60}v \right) = v - \frac{36}{60}v = \frac{24}{60}v = 0.4v$$

So, $v = 10$ V.
\[ R_{eq} = \frac{\gamma}{1} = 10\Omega \]

\[ \tau = R_{eq}C = 10 \times 0.01 = 0.1s \]

\[ v_c(t) = 100e^{-10t} \text{V for } t \geq 0 \]
6. Suppose that the switch has been closed for a long time and is opened at $t = 0$.

determine $v(t)$ for $t \geq 0$.

At $t = 0^-$ (The circuit is under dc conditions)

\[ i_L(0^-) = \frac{15}{5} = 3A \]
\[ v_C(0^-) = 0V \]

Since the inductor current and capacitor voltage cannot change instantaneously,
\[ i_L(0^-) = i_L(0^+) = 3A \]
\[ v_C(0^-) = v_C(0^+) = 0V \]
For $t \geq 0$

KCL at node $a$:

$$v - \frac{12}{2} + 0.125 \frac{dv}{dt} + i_L = 0 \ldots (1)$$

KVL around the loop I:

$$-v + 2i_L + 0.5 \frac{di_L}{dt} = 0 \ldots (2)$$

From Eqn (1):

$$i_L = -\frac{v - 12}{2} - 0.125 \frac{dv}{dt} \ldots (3)$$

At $t = 0$

$$i_L(0) = -\frac{v(0) - 12}{2} - 0.125 \frac{dv(0)}{dt}$$

$$3 = 6 - 0.125 \frac{dv(0)}{dt}$$

$$\frac{dv(0)}{dt} = 24V/s$$

Subst. Eqn. (3) into (2) gives:

$$-v + 2 \left( -\frac{v - 12}{2} - 0.125 \frac{dv}{dt} \right) + 0.5 \frac{d}{dt} \left( -\frac{v - 12}{2} - 0.125 \frac{dv}{dt} \right) = 0$$

$$-2v - 0.5 \frac{dv}{dt} - 0.0625 \frac{d^2v}{dt^2} = -12$$

multiply both sides $-1 \frac{0.0625}{0.0625}$ yields:

$$\frac{d^2v}{dt^2} + 8 \frac{dv}{dt} + 32v = 192$$

The characteristic equation

$$s^2 + 8s + 32 = 0$$

The natural frequencies:

$$s_{1,2} = -4 \pm j4$$

The natural response

$$v_n = e^{-4t} \left( K_1 \cos 4t + K_2 \sin 4t \right)$$

The forced response

$$v_f = \frac{192}{32} = 6$$

The complete response

$$v = v_n + v_f$$

$$v = e^{-4t} \left( K_1 \cos 4t + K_2 \sin 4t \right) + 6$$
\[ v(0) = 0 = e^{-0} (K_1 \cos 0 + K_2 \sin 0) + 6 \]

\[ K_1 + 6 = 0 \Rightarrow K_1 = -6 \]

\[ \frac{dv}{dt} = -4e^{-4t} (K_1 \cos 4t + K_2 \sin 4t) + e^{-4t} (-4K_1 \sin 4t + 4K_2 \cos 4t) \]

\[ \frac{dv(0)}{dt} = 24 = -4K_1 + 4K_2 \]

\[ K_2 = 0 \]

**Therefore**

\[ v(t) = -6e^{-4t} \cos 4t + 6 \quad \text{for} \quad t \geq 0 \]