Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EEE 223 Circuit Theory I

Spring 2005-06

Instructor:

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Midterm EXAMINATION

Apr 20, 2006

Duration : 100 minutes

Number of Problems: 6

Good Luck

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Consider the circuit shown below.

(a) (5 pts.) How much charge flows through the resistor in the center of the circuit in a 3 second interval of time?

\[ q_T = \int_0^3 6 \times 10^{-3} \, dt = 6 \times 10^{-3} t \bigg|_0^3 = 18 \, \text{mC} \]

(b) (5 pts.) How much energy does the upper-left current source supply in a 5 second interval of time?

\[ w = \int_0^5 v_i \, dt = \int_0^5 (6 \times 10^{-3}) \, dt = 36 \times 10^{-3} \bigg|_0^5 = 180 \, \text{mJ} \, \text{Absorbs} \]

\[ w_{\text{supply}} = -180 \, \text{mJ} \]

(c) (5 pts.) Find the voltage \( V_x \).

KVL around the loop:

\[ 6 \times \mu_2k - 6 + V_x = 0 \]

\[ V_x = -6V \]

(d) (5 pts.) Find the voltage \( V_y \).

KCL at node \( a \):

\[ I_y + 6mA + 6mA = 0 \]

\[ I_y = -12mA \]

\[ V_y = I_y \times 1k = -12V \]
2. (15 pts.) Use a Δ to Y transform on the 5kΩ, 3kΩ, and 7kΩ resistors (plus series and parallel combinations) to reduce this circuit to a single resistor connected to the voltage source.

\[ R_1 = \frac{3 \times 10^6}{(3 + 5 + 7) \times 10^3} = 1kΩ \]
\[ R_2 = \frac{21 \times 10^6}{15 \times 10^3} = 1.4kΩ \]
\[ R_3 = \frac{35 \times 10^6}{15 \times 10^3} = 7kΩ \]
\[ R_{eq} = \left( \frac{1k + R_1}{(9k + R_2) + R_3} \right) \]
\[ R_{eq} = 2k \parallel 10.4k + \frac{7}{3}k \]
\[ R_{eq} = \frac{2 \times 10.4 \times 10^6}{12.4 \times 10^3} + \frac{7}{3}k = 4.01kΩ \]
3. (15 pts.) Find $I_s$ and $V$ using nodal analysis.

KCL at the supernode:
$$-\left(-\frac{V_1}{5}\right) + 6 + \frac{V}{2} + \frac{V}{10} = 0$$
multiply both sides by 10 gives:
$$2V_1 + 6V = -60 \ldots \ldots (1)$$

where
$$V - V_1 = 4I_s = 4\left(-\frac{V_1}{5}\right)$$
$$V_1 = 5V \ldots \ldots (2)$$
Substitute Eq. (2) into (1) yields:
$$2(5V) + 6V = -60$$
$$16V = -60 \Rightarrow V = \frac{-60}{16} = -3.75V$$
$$I_s = -\frac{V_1}{5} = -\frac{5V}{5} = -V = 3.75A$$
4. (15 pts.) Use mesh analysis to find $I_x$.

\[ I_x = i_3 - i_2 \]

\[ i_2 - i_1 = 4 \]

\[ i_1 = i_2 - 4 \]

KVL around the supermesh:

\[ 2k(i_2 - 4) + 3k(i_2 - i_3) = 0 \]

\[ 5i_2 - 3i_3 = 8m \] \text{ .......(1)}

KVL around $i_3$:

\[ 3k(i_3 - i_2) + 4ki_1 + 5k(i_3 - i_2) = 0 \]

\[ -8i_2 + 12i_3 = 0 \] \text{ .......(2)}

From Eqs. (1) and (2)

\[ i_3 = 1.78mA \]

\[ i_2 = 2.67mA \]

\[ I_x = i_3 - i_2 = -0.89mA \]
In the circuit below, $V_1$, $V_2$, $V_3$, and $V_4$ are node voltages (relative to the reference node).

In this circuit, two of the node voltages are known: $V_2 = 1.34\ \text{V}$ and $V_3 = 5.08\ \text{V}$. Using this information, complete the following parts of this problem.

(a) (5 pts.) Find the value of node voltage $V_1$.

$$V_3 - V_1 = 8 \Rightarrow V_1 = V_3 - 8 = 5.08 - 8 = -2.92\ \text{V}$$

(b) (5 pts.) Find the value of current $I_z$.

$$I_z = \frac{V_3 - V_1}{2k} = \frac{5.08 - 1.34}{2} = 1.87\ mA$$

(c) (5 pts.) Find the value of the voltage $V_a$.

Using voltage division principle:

$$V_a = -\frac{2k}{2k + 3k}V_3 = (-0.4)(5.08) = -2.03\ \text{V}$$
5. (15 pts.) Find the value of $I_n$ in the circuit below using superposition.

\[
\begin{align*}
I'_n &= -\frac{8}{7k} = -\frac{8}{7} \text{mA} \\
I''_n &= 4 \frac{5k}{7} = \frac{20}{7} \text{mA} \\
I_n &= I'_n + I''_n = -\frac{8}{7} + \frac{20}{7} = \frac{12}{7} \text{mA} \\
&= 12/7 \text{mA}
\end{align*}
\]