Faculty of Engineering
ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 Circuit Theory I

Spring 2006-07

Instructor:
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Final EXAMINATION

June 08, 2007

Duration : 150 minutes

Number of Problems: 6

Good Luck

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1. For the circuit in Fig. P1, find the value of $v_0$ using
   a. mesh analysis. (10 pts.)
   b. nodal analysis. (10 pts.)

KCL at $v_0$:
\[
\left(1 + \frac{1}{3}\right)v_0 - (1)(8 + v_x) = -14
\]
\[
\frac{4}{3}v_0 - v_x = -6 \quad \text{or} \quad 4v_0 - 3v_x = -18 \quad \cdots \cdots (1)
\]

KCL at $(8 + v_x)$:
\[
-1v_0 + \left(1 + \frac{1}{2}\right)(8 + v_x) - \frac{8}{2} - 2v_x = 0
\]
\[
-v_0 - \frac{1}{2}v_x = -8 \quad \text{or} \quad -2v_0 - v_x = -16 \quad \cdots \cdots (2)
\]

Multiply Eq.(2) by (-3) and add to (1) yields:
\[
10v_0 = 30 \Rightarrow v_0 = 3 \text{ V}
\]

Mesh (I) and mesh (II) constitute a SUPERMESH.
KVL around the supermesh:
\[
2i_1 + 8 + 3(i_2 - i_3) + 1(i_3 - i_1) = 0
\]
\[
2i_1 + 3(14 + i_1 - 4i_3) + i_3 - 4i_1 = -8
\]
\[
-10i_1 = -50
\]
\[
i_1 = 5 \text{ A}
\]
\[
v_0 = 3(i_3 - i_2) = 3(4i_1 - 14 - i_1) = 3(15 - 14)
\]
\[
v_0 = 3 \text{ V}
\]
2. Find $v_x$, $i_a$, and $i_0$ in the circuit in Fig. P2. (15 pts.)

Figure P2

KCL at the inverting input terminal of OP AMP (1):

$$1.98 \left( \frac{1}{3.3k} + \frac{1}{4.7k} \right) - \frac{1}{4.7k} v_{o1} = 0$$

$$v_{o1} = \left( 1 + \frac{4.7}{3.3} \right) 1.98 = 4.8 \text{ V}$$

KCL at the inverting input terminal of OP AMP (2):

$$\frac{v_{o1}}{30k} + \frac{v_x}{20k} = 0$$

$$v_x = -\frac{2}{3} v_{o1} = -3.2 \text{ V}$$

$$i_a = \frac{4.8 - 1.98}{4.7k} = 0.6 \text{ mA}$$

$$i_0 = - \left( 0.6m + \frac{4.8}{30k} \right) = -0.76 \text{ mA}$$
3. Use the principle of superposition to find $i$ in the circuit in Fig. P3. (15 pts.)

**Figure P3**

17 V is active:

$$i_r = -\frac{17}{R_r} = -\frac{17}{4} \, A$$

by using current division principle:

$$i' = i_r \frac{2}{4} = -\frac{17}{8} \, A$$

6 V is active:

$$i'' = \frac{6}{R_r} = \frac{6}{(3/2 + 2)/3} = \frac{6}{48/31} = \frac{31}{8} \, A$$
2 A is active:

\[ i'' = 2 \frac{2}{2 + \frac{6}{5}} = \frac{20}{16} \text{ A} \]

\[ i = i' + i'' + i''' \]

\[ i = -\frac{17}{8} + \frac{31}{8} + \frac{20}{16} = 3 \text{ A} \]
4. The variable resistor in the circuit in Fig. P4 is adjusted for maximum power transfer to \( R \).

a. Find the value of \( R \). (10 pts.)

b. Find the maximum power that can be delivered to \( R \). (5 pts.)

When the value of \( R = R_{TH} \) then it will absorb maximum power. The maximum power is

\[
P_{\text{max}} = \frac{V_{TH}^2}{4R_{TH}}
\]

In order to find \( V_{TH} \) we will find \( V_{oc} \)

\[
V_{oc} = 40 \times 5i_1 = -200i_1
\]

\[
i_1 = \frac{12 - \frac{V_{oc}}{5}}{4} = \frac{60 - V_{oc}}{20}
\]

Therefore

\[
V_{oc} = -200i_1 = -10(60 - V_{oc})
\]

\[-9V_{oc} = -600\]

\[
V_{oc} = \frac{600}{9} V
\]
By using current division principle:

\[ i_{sc} = -5i_1 \frac{40}{48} = -\frac{25}{6} i_1 \]

\[ v_i = -40 \left( 5i_1 \frac{8}{48} \right) = -\frac{200}{6} i_1 \]

And

\[ i_1 = \frac{12 - \frac{v_i}{5}}{4} = \frac{60 - v_i}{20} = \frac{1}{20} \left( -\frac{200}{6} i_1 \right) \]

\[ i_1 = -\frac{18}{4} A \]

Therefore

\[ i_{sc} = -\frac{25}{6} \left( -\frac{18}{4} \right) = \frac{75}{4} A \]

\[ R_{TH} = \frac{V_{oc}}{i_{sc}} = \frac{600/9}{75/4} = \frac{32}{9} \Omega \]

When \( R = \frac{32}{9} \Omega \) it will absorb maximum power.

\[ P_{max} = \frac{\left( \frac{600}{9} \right)^2}{4 \left( \frac{32}{9} \right)} = 312.5 W \]
5. Find $i$ for $t \geq 0$ if the circuit in Fig. P5 is under dc conditions at $t = 0^-$. (15 pts.)

**Figure P5**

At $t = 0^-$

\[ i(0^-) = \frac{16}{4k} = 4 \text{ mA} = i(0^+) \]

For $t \geq 0$
Since the circuit contains a source $i$ will be:

$$i(t) = i(\infty) + \left[ i(0) - i(\infty) \right] e^{-\frac{t}{\tau}}$$

At $t = \infty$, the circuit is under dc conditions

$$i(\infty) = \frac{16}{8k} = 2 \text{ mA}$$

$$\tau = \frac{L}{R_{eq}}$$

where $R_{eq}$ is the equivalent resistance seen by the inductor.

$$R_{eq} = 8k \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{16k}$$

$$i(t) = i(\infty) + \left[ i(0) - i(\infty) \right] e^{-\frac{t}{\tau}} = 2 + \left( 4 - 2 \right) e^{-16000t} \text{ mA}$$

$$i(t) = 2 - 2e^{-16000t} \text{ mA}$$
6. Consider the circuit in Fig. P6. Find $v$ for $t \geq 0$ if $v(0) = 4 \text{ V}$ and $i(0) = 3 \text{ A}$. (20 pts.)

![Circuit Diagram]

KCL at $v$:

$$i = \frac{1}{4} \frac{dv}{dt} + \frac{v}{1} \quad \ldots \ldots (1)$$

KVL around the loop ($i$):

$$3i + \frac{1}{4} \frac{di}{dt} + v = 16 \quad \ldots \ldots (2)$$

Subst. Eq.(1) into (2) yields:

$$3 \left( \frac{1}{4} \frac{dv}{dt} + \frac{v}{1} \right) + \frac{1}{4} \frac{d}{dt} \left( \frac{1}{4} \frac{dv}{dt} + \frac{v}{1} \right) + v = 16$$

$$\frac{3}{4} \frac{dv}{dt} + 3v + \frac{1}{16} \frac{d^2v}{dt^2} + \frac{1}{4} \frac{dv}{dt} + v = 16$$

Multiply both sides by 16:

$$\frac{d^2v}{dt^2} + 16 \frac{dv}{dt} + 64v = 16^2$$

Characteristic equation:

$$s^2 + 16s + 64 = 0$$

$$\therefore s_{1,2} = -8$$

The natural response $v_n$:

$$v_n = (A + Bt)e^{-8t}$$

The force response $v_f$:
$v_f = K$ the trial forced response.

$64K = 16^2$

$K = 4$

$v_f = 4\, V$

The complete response

$v = v_n + v_f$

$v = (A + Bt)e^{-8t} + 4$

$v(0) = 4 = A + 4 \Rightarrow A = 0$

By writing Eq(1) at $t = 0$

$i(0) = \frac{1}{4} \frac{dv(0)}{dt} + \frac{v(0)}{1}$

$\frac{dv(0)}{dt} = 4(i(0) - v(0)) = 4(3 - 4) = -4\, V / s$

$$\frac{dv}{dt} = Be^{-8t} - 8(A + Bt)e^{-8t}$$

$$\frac{dv(0)}{dt} = -4 = B - 8(A) \Rightarrow B = -4$$

$\therefore v(t) = (-4te^{-8t} + 4)\, V$