GAUSS LAW-MAXWELL'S EQUATION

Gauss Law constitutes one of the fundamental laws of electromagnetism.

Gauss's Law: The total flux ψ of the electric field intensity over any closed surface in free space is equal to the total charge enclosed in the surface divided by ε_0 .

$$\psi = \frac{Q_{enc}}{\varepsilon_0}$$

$$\psi = \oint_s d\psi = \oint_s \bar{E}.d\bar{s}$$

Integral Form of the Gauss Law:

$$\oint_s \bar{E}.d\bar{s} = \frac{Q_{enc}}{\varepsilon_0}$$

Q_{enc} is the total charge contained in volume *V* bounded by surface *S.*

Applying the Divergence Theorem:

$$\oint_s \bar{E}.d\bar{s} = \int_v \left(\nabla.\bar{E}\right)dv$$

$$\int_v \left(\nabla . \bar{E}\right) dv = \frac{Q_{enc}}{\varepsilon_0} = \frac{\int_v \rho_v dv}{\varepsilon_0}$$

So, the differential form (point form) of the Gauss' Law is:

$$\nabla . \overline{E} = \frac{\rho_v}{\varepsilon_0}$$

Electric flux density (displacement vector) in free space:

The electric field intensity is dependent on the medium in which the charge is placed (free space in this case). A new vector can be defined as:

$$\bar{D} = \varepsilon_0 \bar{E}$$

The flux of $\bar{D}$ is:

$$\psi = \oint_s \bar{D} . d\bar{s}$$

This vector is known as the electric flux density vector or displacement vector. In SI units the electric flux is measured in coulombs and the electric flux density in coulombs per meter square.
Gauss Law for electric flux density can be written as:

$$\nabla . \overline{D} = \rho_v$$

This is the first of the four Maxwell's equations to be derived.

This equation states that the divergence of the electric flux density is equal to the volume charge density.

Note that;

1) Gauss Law is the alternative statement of the Coulomb's Law.
2) Gauss Law provides an easy means of finding $\overline{E}$ and $\overline{D}$ for the for symmetrical charge distributions.

Applications of Gauss's Law:

The procedure for applying the Gauss's Law to calculate the electric field intensity involves first whether symmetry exists. Once it has been found that the symmetry charge distribution exists, we construct a mathematical closed surface (known as the Gaussian Surface) such that the electric field intensity is the same everywhere on that surface and is perpendicular to that surface.