ELECTRIC FIELDS IN MATERIAL SPACE

DIELECTRICS:

An ideal dielectric material is one which has no free charge. All the charges present in the material are bound charges (bound to the atom). Under the action of an electric field, (+) charges move in the direction of the electric field and (–) charges move opposite to the field. Hence, (+) and (-) parts of each molecule or (atom) are displaced from their equilibrium positions. These displacements, however, are limited by strong restoring forces which are set up by changing charge configuration in the molecule. The dielectric is said to be polarized. A polarized dielectric is neutral as a whole, but it produces an electric field, both at exterior points and inside the dielectric as well. The polarization of the dielectric depends on the total electric field in the medium. A part of the total electric field is produced by the dielectric itself.

Polarization Vector:

We define the electric polarization or simply the polarization of a medium by means of a vector $\overline{P}$ defined as:

$$\overline{P} = \lim_{\Delta v \to 0} \frac{\Delta \overline{P}}{\Delta v} = \lim_{\Delta v \to 0} \frac{\sum_i \overline{P}_i}{\Delta v} \quad \frac{C.m^2}{m^3} = \frac{C}{m^2}$$

$\overline{P}$ is a point vector function. In each volume element it has the direction of $\Delta\overline{P}$. ($\Delta\overline{P}$ is the resultant dipole moment associated with the volume element Δv).

Electric Field Intensity due to a Polarized Medium:

The electric field intensity due to a polarized medium can be shown to have the following form:

$$\overline{E}(\overline{r}) = \frac{1}{4\pi\varepsilon_0} \oint_s \frac{\rho_{s_p}(\overline{r}\,')(\overline{r} - \overline{r}\,')}{|\overline{r} - \overline{r}\,'|^3} ds' + \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho_p(\overline{r}\,')(\overline{r} - \overline{r}\,')}{|\overline{r} - \overline{r}\,'|^3} dv'$$

Define:

$\rho_{s_p}(\bar{r}') = \bar{P}.\hat{n}'$, surface polarization charge density in (C/m^2).

$\rho_{v_p}(\bar{r}') = -\nabla.\bar{P}$, volume polarization charge density in (C/m^3).

$\hat{n}'$: Outward unit normal vector from the surface element ds'.

This equation says that, the dielectric medium can be replaced by a volume distribution of charge density $-\nabla.\bar{P}$ and surface distribution of charge density $\bar{P}.\hat{n}'$ situated in free space.

The total polarization charge is:

$$Q_P = \oint_s \bar{P}.\hat{n}'ds' + \int_v -\nabla'.\bar{P}\,dv' = \oint_s \bar{P}.\hat{n}'ds' - \oint_s \bar{P}.\hat{n}'ds' = 0$$

This is expected, because the dielectric is neutral.

Gauss Law for a Dielectric Medium
$\bar{D}$ Vector

We modify the differential form of the Gauss's Law as:

$$\nabla.\bar{E} = \frac{1}{\varepsilon_0}\left(\rho_f + \rho_P\right)$$

ρ_f : Free charge density (true charge density),

$\rho_P = -\nabla.\bar{P}$: Polarization charge density.

$$\nabla.\bar{E} = \frac{\rho_f}{\varepsilon_0} - \frac{\nabla.\bar{P}}{\varepsilon_0}$$

$$\nabla.\left(\bar{E} + \frac{\bar{P}}{\varepsilon_0}\right) = \frac{\rho_f}{\varepsilon_0}$$

$$\nabla.\left(\varepsilon_0\overline{E}+\overline{P}\right)=\rho_f$$

Let, $\overline{D}=\varepsilon_0\overline{E}+\overline{P}$, electric displacement vector or the electric flux density vector in C/m^2.

Then,

$\nabla.\overline{D}=\rho_f$ Differential form of the Gauss Law.

$\oint_s \overline{D}.d\overline{s}=Q_f$ Integral form of the Gauss Law.

<u>Electric Susceptibility, Permittivity and Dielectric Constant</u>

For most materials, $\overline{P}$ vanishes when $\overline{E}$ vanishes (ferroelectric materials are exceptions). We will consider those materials for which $\overline{E}$ and $\overline{P}$ are in the same direction (<u>isotropicity</u>). If $\overline{E}$ and $\overline{P}$ are not in the same direction, then the dielectric is said to be <u>anisotropic</u>.

$$\overline{P}=\varepsilon_0\chi_e\overline{E}$$

Where χ_e is the electric susceptibility. Since,

$$\begin{aligned}\overline{D}&=\varepsilon_0\overline{E}+\overline{P}\\&=\varepsilon_0\overline{E}+\varepsilon_0\chi_e\overline{E}\\&=\varepsilon_0\left(1+\chi_e\right)\overline{E}\end{aligned}$$

Define the permittivity of the medium in *(F/m)* as:

$$\varepsilon=\varepsilon_0\left(1+\chi_e\right)$$

Then,

$$\overline{D}=\varepsilon\overline{E}$$

It is found experimentally that, χ and ε are usually independent of the electric field, except perhaps for very intense fields. So, χ and ε are constant characteristic of the dielectric. Materials of this type are called ***linear dielectrics***.

We define the dielectric constant (relative permittivity) (dimensionless) as:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{1+\chi}{\varepsilon_0}$$

If ε_r is independent of the position the medium is said to be homogeneous. A linear, homogeneous and isotropic medium is called a simple medium.

ε_r , of a simple medium is a constant number.