CONSERVATION OF CHARGE, CONTINUITY EQUATION AND KIRCHHOFF'S CURRENT LAW

It is the fundamental property of nature that charge is conserved. This is verified without doubt experimentally.

Equation of continuity is the mathematical expression of this fundamental fact. Consider an arbitrarily volume *V* bounded by surface *S*. Let the total charge inside *S* be Q_{in} at time *t*. Then, any positive current leaving S must be accompanied by a decrease in the charge in *S*. (Conservation of Charge.) This fact is expressed mathematically as:

$$I = \oint_S \bar{J}.d\bar{s} = -\frac{dQ_{in}}{dt}$$

But

$$Q_{in} = \int_v \rho_v dv$$

So,

$$\oint_S \bar{J}.d\bar{s} = -\frac{d}{dt}\int_v \rho_v dv$$

This is the integral form of the Continuity Equation.

Now, on the left-hand side we can use the Divergence Theorem.

$$\int_v \nabla.\overline{J}dv = -\frac{d}{dt}\int_v \rho dv$$

For a stationary volume, we can take the operator $\frac{d}{dt}$ into the volume integral.

$$\int_v \nabla.\overline{J}dv = -\int_v \frac{d\rho}{dt}dv$$

This equation is always valid for any volume V, hence

$$\nabla.\overline{J} + \frac{d\rho}{dt} = 0$$

This is the Continuity Equation.

For steady currents $\frac{d}{dt} \equiv 0$ and $\nabla.\overline{J} = 0$

Field Theory expression of K.C.L. : $\oint_S \overline{J}.d\overline{s} = 0$

Circuit Theory expression of K.C.L. : $\sum_j I_j = 0$

BOUNDARY CONDITIONS FOR STEADY CURRENT DENSITY

We will consider a boundary which separates two different conducting media with conductivities σ_1, σ_2 and permittivities ε_1, ε_2 respectively. We will find the conditions for the normal and tangential components of the vector $\bar{J}$ as we cross the boundary.

Assume that the positive unit normal is from medium 1 into medium 2.

Steady current satisfies $\nabla.\bar{J} = 0$ or $\oint_S \bar{J}.d\bar{s} = 0$. Applying this equation to a Gaussian Cylindrical Surface on the interface results:

$$J_{2_n} = J_{1_n}$$

Which also implies, $\sigma_2 E_{2_n} = \sigma_1 E_{1_n}$

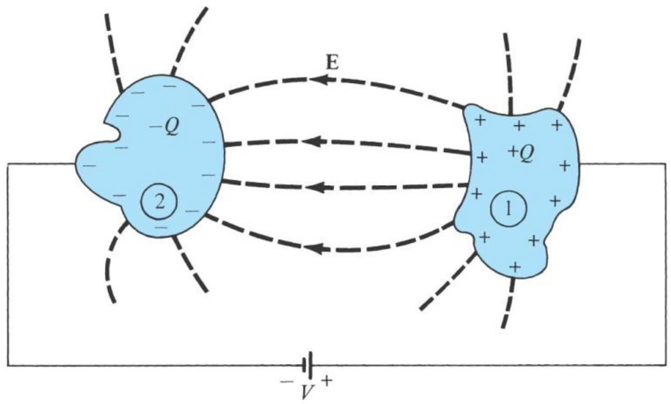
For static fields electric field intensity is conservative.

$$\nabla X\bar{E} = 0 \text{ or } \oint_C \bar{E}.d\bar{l} = 0$$

Applying this equation to the rectangular path on the crossing the boundary results:

$$E_{2_t} = E_{1_t} \qquad \text{or} \qquad \frac{J_{2_t}}{J_{1_t}} = \frac{\sigma_2}{\sigma_1}$$

RESISTANCE AND CAPACITANCE BETWEEN TWO ELECTRODES EMBEDDED IN A HOMOGENEOUS LOSSY DIELECTRIC



We define the capacitance of this structure as

$$C = \frac{Q}{V} \quad (F)$$

The resistance between the two electrodes:

$$R = \frac{V}{I}$$

Let us look at the value of the product:

$$RC = \frac{Q}{V}\frac{V}{I} = \frac{Q}{I} = \frac{\oint_S \varepsilon \bar{E}.d\bar{s}}{\oint_S \sigma \bar{E}.d\bar{s}} = \frac{\varepsilon}{\sigma}$$

$$RC = \frac{\varepsilon}{\sigma}$$

This is an important result since knowing C gives R immediately.